MATHEMATIC

SINDH BOARD NOTES

10 TH CLASS

Presented by:

Urdu Books Whatsapp Group

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EXERCISE 1.1

Q.1 Write the following quadratic equations in the Standard form and point out pure quadratic equations.

(i) $(x+7)(x-3)=-7$

Solution: $(x+7)(x-3)=-7$

$$x^2-3x+7x-21=-7$$
$$x^2+4x-21+7=0$$
$$x^2+4x-14=0$$

The standard form of quadratic equation is:

$$x^2+4x-14=0$$

(ii) $\frac{x^2+4}{3}-\frac{x}{7}=1$

Solution: $\frac{x^2+4}{3}-\frac{x}{7}=1$

$$\frac{7(x^2+4)-3x}{21}=1$$
$$7x^2+28-3x=21$$
$$7x^2-3x+28-21=0$$
$$7x^2-3x+7=0$$

The standard form of quadratic equation is:

$$7x^2-3x+7=0$$

(iii) $\frac{x}{x+1}+\frac{x+1}{x}=6$

Solution: $\frac{x}{x+1}+\frac{x+1}{x}=6$

$$\frac{x^2+(x+1)^2}{x(x+1)}=6$$
$$x^2+x^2+1+2x=6x(x+1)$$
$$2x^2+2x+1=6x^2+6x$$
$$0=6x^2+6x-2x^2-2x-1$$
$$0=4x^2+4x-1$$
$$\Rightarrow \quad 4x^2+4x-1=0$$

The standard form of quadratic equation is:

$$4x^2+4x-1=0$$

(iv) $\left(\frac{x+4}{x-2}\right)-\left(\frac{x-2}{x}\right)+4=0$

Solution: $\left(\frac{x+4}{x-2}\right)-\left(\frac{x-2}{x}\right)+4=0$

$$\frac{(x+4)x-(x-2)^2+4x(x-2)}{(x-2)(x)}=0$$
$$x^2+4x-[x^2+2^2-2(x)(2)]+4x^2-8x=0$$
$$\cancel{x^2}+4x-\cancel{x^2}-4+4x+4x^2-8x=0$$
$$4x^2+4x+4x-8x-4=0$$
$$4x^2+\cancel{8x}-\cancel{8x}-4=0$$
$$4x^2-4=0$$
$$4(x^2-1)=0$$
$$\therefore \quad x^2-1=0 \quad (\because 4\neq 0)$$

So, $x^2-1=0$ is Pure Quadratic Equation

(v) $\frac{x+3}{x+4}-\frac{x-5}{x}=1$

Solution: $\frac{x+3}{x+4}-\frac{x-5}{x}=1$

$$\frac{x(x+3)-(x+4)(x-5)}{x(x+4)}=1$$
$$x^2+3x-(x^2-5x+4x-20)=1x(x+4)$$
$$x^2+3x-(x^2-1x-20)=x^2+4x$$
$$\cancel{x^2}+3x-\cancel{x^2}+x+20=x^2+4x$$
$$3x+x+20=x^2+4x$$
$$4x+20=x^2+4x$$
$$\Rightarrow \quad x^2+\cancel{4x}-\cancel{4x}-20=0$$
$$x^2-20=0$$
$$x^2+0x-20=0$$

As, $b=0$

So, $x^2-20=0$ is Pure Quadratic Equation

(vi) $\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$

Solution: $\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$

$$\frac{(x+1)(x+3)+(x+2)(x+2)}{(x+2)(x+3)}=\frac{25}{12}$$

$$\frac{(x^2+3x+1x+3)+(x^2+2x+2x+4)}{(x+2)(x+3)}=\frac{25}{12}$$

$$\frac{(x^2+4x+3)+(x^2+4x+4)}{x^2+3x+2x+6}=\frac{25}{12}$$

$$\frac{2x^2+8x+7}{x^2+5x+6}=\frac{25}{12}$$

$$12(2x^2+8x+7)=25(x^2+5x+6)$$

$$24x^2+96x+84=25x^2+125x+150$$

$$25x^2-24x^2+125x-96x+150-84=0$$

$$x^2+29x+66=0$$

The standard form of quadratic equation is:

$$x^2+29x+66=0$$

Q.2 Solve by Factorization:

(i) $x^2-x-20=0$

Solution:

$$x^2-x-20=0$$

$$x^2-5x+4x-20=0$$

$$x(x-5)+4(x-5)=0$$

$$(x-5)(x+4)=0$$

$$x-5=0 \quad \text{or} \quad x+4=0$$

$$x=5 \quad \text{or} \quad x=-4$$

Solution set is $\{-4, 5\}$

(ii) $3y^2=y(y-5)$

Solution: $3y^2=y(y-5)$

$$3y^2=y^2-5y$$

$$3y^2-y^2+5y=0$$

$$2y^2+5y=0$$

$$y(2y+5)=0$$

Either $y=0$ or $2y+5=0$

$$y=0 \quad \text{or} \quad 2y=-5$$

$$y=0 \quad \text{or} \quad y=\frac{-5}{2}$$

Solution is $\left\{0, \frac{-5}{2}\right\}$

(iii) $4-32x=17x^2$

Solution: $4-32x=17x^2$

$$17x^2+32x-4=0$$

$$17x^2+34x-2x-4=0$$

$$17x(x+2)-2(x+2)=0$$

$$(x+2)(17x-2)=0$$

Either $x+2=0$ or $17x-2=0$

$$x=-2 \quad \text{or} \quad 17x=2$$

$$x=-2 \quad \text{or} \quad x=\frac{2}{17}$$

Solution set is $\left\{-2, \frac{2}{17}\right\}$

(iv) $x^2-11x=152$

Solution: $x^2-11x=152$

$$x^2-11x-152=0$$

$$x^2-19x+8x-152=0$$

$$x(x-19)+8(x-19)=0$$

$$(x-19)(x+8)=0$$

$$x-19=0 \quad \text{or} \quad x+8=0$$

$$x=19 \quad \text{or} \quad x=-8$$

Solution set is $\{-8, 19\}$

(v) $\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$

Solution: $\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$

$\frac{(x+1)^2+x^2}{x(x+1)}=\frac{25}{12}$

$\frac{(x)^2+(1)^2+2(x)(1)+x^2}{x^2+x}=\frac{25}{12}$

$\frac{x^2+1+2x+x^2}{x^2+x}=\frac{25}{12}$

$12(2x^2+2x+1)=25(x^2+x)$

$24x^2+24x+12=25x^2+25x$

$0=25x^2+25x-24x^2-24x-12$

$0=x^2+x-12$

$\Rightarrow\quad x^2+x-12=0$

$x^2+4x-3x-12=0$

$x(x+4)-3(x+4)=0$

$(x+4)(x-3)=0$

Either

$x+4=0$ or $x-3=0$

$\Rightarrow\quad x=-4$ or $x=3$

Solution set is $\{3,-4\}$

(vi) $\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$

Solution: $\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$

$\frac{2}{x-9}=\frac{(x-4)-(x-3)}{(x-3)(x-4)}$

$\frac{2}{x-9}=\frac{\cancel{x}-4-\cancel{x}+3}{x^2-4x-3x+12}$

$\frac{2}{x-9}=\frac{-1}{x^2-7x+12}$

$2(x^2-7x+12)=-1(x-9)$

$2x^2-14x+24=-1x+9$

$2x^2-14x+x+24-9=0$

$2x^2-13x+15=0$

$2x^2-10x-3x+15=0$

$2x(x-5)-3(x-5)=0$

$(x-5)(2x-3)=0$

Either $x-5=0$ or $2x-3=0$

$x=5$ or $2x=3$

$x=5$ or $x=\frac{3}{2}$

Solution set is $\left\{5,\frac{3}{2}\right\}$

Q.3 Solve the following equations by completing square.

(i) $\mathbf{7x^2+2x-1=0}$

Solution: $7x^2+2x-1=0$

Dividing each term of the equation by 7

$\frac{\cancel{7}}{\cancel{7}}x^2+\frac{2}{7}x-\frac{1}{7}=0$

$x^2+\frac{2}{\cancel{2}}(x)\left(\frac{\cancel{2}}{7}\right)^1-\frac{1}{7}=0$

$(x)^2+2(x)\left(\frac{1}{7}\right)=\frac{1}{7}$

ختم نبوت ﷺ زندہ باد **عظمت صحابہ زندہ باد**

السلام علیکم ورحمۃ اللہ وبرکاتہ:

معزز ممبران: آپ کا وٹس ایپ گروپ ایڈمن "**اردو بکس**" آپ سے مخاطب ہے۔

آپ تمام ممبران سے گزارش ہے کہ:

- گروپ میں صرف PDF کتب پوسٹ کی جاتی ہیں لہذا کتب کے متعلق اپنے کمنٹس / ریویوز ضرور دیں۔ گروپ میں بغیر ایڈمن کی اجازت کے کسی بھی قسم کی (اسلامی وغیر اسلامی، اخلاقی، تحریری) پوسٹ کرنا سختی سے منع ہے۔
- گروپ میں معزز، پڑھے لکھے، سلجھے ہوئے ممبرز موجود ہیں اخلاقیات کی پابندی کریں اور گروپ رولز کو فالو کریں بصورت دیگر معزز ممبرز کی بہتری کی خاطر ریموو کر دیا جائے گا۔
- کوئی بھی ممبر کسی بھی ممبر کو انباکس میں میسیج، مس کال، کال نہیں کرے گا۔ رپورٹ پر فوری ریموو کر کے کاروائی عمل میں لائے جائے گی۔
- ہمارے کسی بھی گروپ میں سیاسی و فرقہ واریت کی بحث کی قطعاً کوئی گنجائش نہیں ہے۔
- اگر کسی کو بھی گروپ کے متعلق کسی قسم کی شکایت یا تجویز کی صورت میں ایڈمن سے رابطہ کیجئے۔
- سب سے اہم بات:

گروپ میں کسی بھی قادیانی، مرزائی، احمدی، گستاخِ رسول، گستاخِ امہات المؤمنین، گستاخِ صحابہ و خلفائے راشدین حضرت ابو بکر صدیق، حضرت عمر فاروق، حضرت عثمان غنی، حضرت علی المرتضیٰ، حضرت حسنین کریمین رضوان اللہ تعالیٰ اجمعین، گستاخ اہلبیت یا ایسے غیر مسلم جو اسلام اور پاکستان کے خلاف پراپیگنڈا میں مصروف ہیں یا ان کے روحانی وذہنی سپورٹرز کے لئے کوئی گنجائش نہیں ہے لہذا ایسے اشخاص بالکل بھی گروپ جوائن کرنے کی زحمت نہ کریں۔ معلوم ہونے پر فوراً ریموو کر دیا جائے گا۔

- تمام کتب انٹرنیٹ سے تلاش / ڈاؤنلوڈ کر کے **فری آف کاسٹ** وٹس ایپ گروپ میں شیئر کی جاتی ہیں۔ جو کتاب نہیں ملتی اس کے لئے معذرت کر لی جاتی ہے۔ جس میں محنت بھی صَرف ہوتی ہے لیکن ہمیں آپ سے صرف دعاؤں کی درخواست ہے۔
- عمران سیریز کے شوقین کیلئے علیحدہ سے عمران سیریز گروپ موجود ہے۔
- **لیڈیز کے لئے الگ گروپ کی سہولت موجود ہے جس کے لئے ویریفکیشن ضروری ہے۔**
- اردو کتب / عمران سیریز یا سٹڈی گروپ میں ایڈ ہونے کے لئے ایڈمن سے وٹس ایپ پر بذریعہ میسیج رابطہ کریں اور جواب کا انتظار فرمائیں۔ برائے مہربانی اخلاقیات کا خیال رکھتے ہوئے موبائل پر کال یا ایم ایس کرنے کی کوشش ہر گز نہ کریں۔ ورنہ گروپس سے تو ریموو کیا ہی جائے گا بلاک بھی کیا جائے گا۔

نوٹ: ہمارے کسی گروپ کی کوئی فیس نہیں ہے۔ سب فی سبیل اللہ ہے

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پاکستان زندہ باد **پاکستان پائندہ باد**

اللہ تبارک تعالیٰ ہم سب کا حامی و ناصر ہو

Adding $\left(\frac{1}{7}\right)^2$ on both sides

$$(x)^2+2(x)\left(\frac{1}{7}\right)+\left(\frac{1}{7}\right)^2=\frac{1}{7}+\left(\frac{1}{7}\right)^2$$

$$\left(x+\frac{1}{7}\right)^2=\frac{1}{7}+\frac{1}{49}$$

$$\left(x+\frac{1}{7}\right)^2=\frac{7+1}{49}$$

$$\left(x+\frac{1}{7}\right)^2=\frac{8}{49}$$

Taking square root of both sides

$$\sqrt{\left(x+\frac{1}{7}\right)^2}=\pm\sqrt{\frac{8}{49}}$$

$$\sqrt{\left(x+\frac{1}{7}\right)^2}=\pm\sqrt{\frac{4\times 2}{49}}$$

$$x+\frac{1}{7}=\pm\frac{2\sqrt{2}}{7}$$

$$x=\frac{-1}{7}\pm\frac{2\sqrt{2}}{7}$$

$$\therefore \quad x=\frac{-1\pm 2\sqrt{2}}{7}$$

Solution set is $\left\{\frac{-1\pm 2\sqrt{2}}{7}\right\}$

(ii) $\mathbf{ax^2+4x-a=0}$, $a\neq 0$

Solution: $ax^2+4x-a=0$, $a\neq 0$

Dividing each term of the equation by 'a'

$$\frac{ax^2}{a}+4\frac{x}{a}-\frac{a}{a}=\frac{0}{a}$$

$$x^2+4\frac{x}{a}-1=0$$

$$x^2+\frac{2}{\cancel{2}}(x)\left(\frac{\overset{2}{\cancel{4}}}{a}\right)=1$$

$$(x)^2+2(x)\left(\frac{2}{a}\right)=1$$

Adding $\left(\frac{2}{a}\right)^2$ on both sides

$$(x)^2+2(x)\left(\frac{2}{a}\right)+\left(\frac{2}{a}\right)^2=1+\left(\frac{2}{a}\right)^2$$

$$\left(x+\frac{2}{a}\right)^2=1+\frac{4}{a^2}$$

$$\left(x+\frac{2}{a}\right)^2=\frac{a^2+4}{a^2}$$

Taking square root of both sides

$$\sqrt{\left(x+\frac{2}{a}\right)^2}=\pm\sqrt{\frac{a^2+4}{a^2}}$$

$$x+\frac{2}{a}=\pm\frac{\sqrt{a^2+4}}{a}$$

$$x=\frac{-2}{a}\pm\frac{\sqrt{a^2+4}}{a}$$

$$x=\frac{-2\pm\sqrt{a^2+4}}{a}$$

Solution set is $\left\{\frac{-2\pm\sqrt{a^2+4}}{a}\right\}$

(iii) $\mathbf{11x^2-34x+3=0}$

Solution: $11x^2-34x+3=0$

Dividing each term of the equation by 11

$$\frac{\cancel{11}}{\cancel{11}}x^2-\frac{34}{11}x+\frac{3}{11}=0$$

$$x^2-\frac{34}{11}x=-\frac{3}{11}$$

$$(x)^2-\frac{2}{\cancel{2}}(x)\left(\frac{\overset{17}{\cancel{34}}}{11}\right)=-\frac{3}{11}$$

$$(x)^2-2(x)\left(\frac{17}{11}\right)=-\frac{3}{11}$$

Adding $\left(\frac{17}{11}\right)^2$ on both sides we get

$$(x)^2-2(x)\left(\frac{17}{11}\right)+\left(\frac{17}{11}\right)^2=-\frac{3}{11}+\left(\frac{17}{11}\right)^2$$

$$\left(x-\frac{17}{11}\right)^2=\frac{-3}{11}+\frac{289}{121}$$

$$\left(x-\frac{17}{11}\right)^2=\frac{-33+289}{121}$$

$$\left(x-\frac{17}{11}\right)^2=\frac{256}{121}$$

Taking square root

$$\sqrt{\left(x-\frac{17}{11}\right)^2}=\pm\sqrt{\frac{256}{121}}$$

$$x-\frac{17}{11}=\pm\frac{16}{11}$$

$$x=\frac{17}{11}\pm\frac{16}{11}$$

$$x=\frac{17\pm16}{11}$$

$$\Rightarrow \quad x=\frac{17-16}{11} \quad \text{or} \quad x=\frac{17+16}{11}$$

$$x=\frac{1}{11} \quad \text{or} \quad x=\frac{33}{11}$$

$$x=3$$

Solution set is $\left\{3,\frac{1}{11}\right\}$

(iv) $\quad \ell x^2+mx+n=0\ ,\ \ell\neq0$

Solution: $\ell x^2+mx+n=0\ ,\ \ell\neq0$

Dividing each term of the equation by ℓ

$$\frac{\cancel{\ell}x^2}{\cancel{\ell}}+(x)\left(\frac{m}{\ell}\right)x=-\frac{n}{\ell}$$

$$x^2+\frac{2}{2}(x)\left(\frac{m}{\ell}\right)=-\frac{n}{\ell}$$

$$x^2+2(x)\left(\frac{m}{2\ell}\right)=-\frac{n}{\ell}$$

Adding $\left(\frac{m}{2\ell}\right)^2$ on both sides

$$(x)^2+2(x)\left(\frac{m}{2\ell}\right)+\left(\frac{m}{2\ell}\right)^2=-\frac{n}{\ell}+\left(\frac{m}{2\ell}\right)^2$$

$$\left(x+\frac{m}{2\ell}\right)^2=-\frac{n}{\ell}+\frac{m^2}{4\ell^2}$$

$$\left(x+\frac{m}{2\ell}\right)^2=\frac{-4\ell n+m^2}{4\ell^2}$$

Taking square root

$$\sqrt{\left(x+\frac{m}{2\ell}\right)^2}=\pm\sqrt{\frac{m^2-4\ell n}{4\ell^2}}$$

$$x+\frac{m}{2\ell}=\pm\frac{\sqrt{m^2-4\ell n}}{2\ell}$$

$$x=-\frac{m}{2\ell}\pm\frac{\sqrt{m^2-4\ell n}}{2\ell}$$

$$x=\frac{-m\pm\sqrt{m^2-4\ell n}}{2\ell}$$

Solution set is $\left\{\frac{-m\pm\sqrt{m^2-4\ell n}}{2\ell}\right\}$

(v) $\quad$ **$3x^2+7x=0$**

Solution: $3x^2+7x=0$

Dividing each term of equation by 3

$$\frac{\cancel{3}x^2}{\cancel{3}}+\frac{7}{3}x=0$$

$$x^2+\frac{2}{2}(x)\left(\frac{7}{3}\right)=0$$

$$(x)^2+2(x)\left(\frac{7}{6}\right)=0$$

Adding $\left(\frac{7}{6}\right)^2$ on both sides

$$(x)^2+2(x)\left(\frac{7}{6}\right)+\left(\frac{7}{6}\right)^2=\left(\frac{7}{6}\right)^2$$

$$\left(x+\frac{7}{6}\right)^2=\frac{49}{36}$$

Taking square root

$$\sqrt{\left(x+\frac{7}{6}\right)^2}=\pm\sqrt{\frac{49}{36}}$$

$$\left(x+\frac{7}{6}\right)=\pm\frac{7}{6}$$

$$x=\frac{-7}{6}\pm\frac{7}{6}$$

$$x=\frac{-7\pm7}{6}$$

$\Rightarrow$ $x=\frac{-7-7}{6}$ or $x=\frac{-7+7}{6}$

$x=\frac{-14}{6}$ or $x=\frac{0}{6}$

$x=\frac{-7}{3}$ or $x=0$

Solution set is $\left\{0, \frac{-7}{3}\right\}$

(vi) $\mathbf{x^2-2x-195=0}$

Solution: $x^2-2x-195=0$

$$x^2-2x=195$$

$$x^2-\frac{2}{2}(x)(2)=195$$

$$(x)^2-2(x)(1)=195$$

Adding $(1)^2$ on both sides

$$(x)^2-2(x)(1)+(1)^2=195+(1)^2$$

$$(x-1)^2=195+1$$

$$(x-1)^2=196$$

Taking square root

$$\sqrt{(x-1)^2}=\pm\sqrt{196}$$

$$x-1=\pm14$$

$$x=1\pm14$$

$\Rightarrow$ $x=1-14$ or $x=1+14$

$x=-13$ or $x=15$

Solution set is $\{-13, 15\}$

(vii) $\mathbf{-x^2+\frac{15}{2}=\frac{7}{2}x}$

Solution: $-x^2+\frac{15}{2}=\frac{7}{2}x$

$$\frac{15}{2}=x^2+\frac{7}{2}x$$

$$\Rightarrow \quad x^2+\frac{7}{2}x=\frac{15}{2}$$

$$x^2+\frac{2}{2}(x)\left(\frac{7}{2}\right)=\frac{15}{2}$$

$$(x)^2+2(x)\left(\frac{7}{4}\right)=\frac{15}{2}$$

Adding $\left(\frac{7}{4}\right)^2$ on both sides

$$(x)^2+2(x)\left(\frac{7}{4}\right)+\left(\frac{7}{4}\right)^2=\frac{15}{2}+\left(\frac{7}{4}\right)^2$$

$$\left(x+\frac{7}{4}\right)^2=\frac{15}{2}+\frac{49}{16}$$

$$\left(x+\frac{7}{4}\right)^2=\frac{120+49}{16}$$

$$\left(x+\frac{7}{4}\right)^2=\frac{169}{16}$$

Taking Square Root

$$\sqrt{\left(x+\frac{7}{4}\right)^2}=\pm\sqrt{\frac{169}{16}}$$

$$x+\frac{7}{4}=\pm\frac{13}{4}$$

$$x=\frac{-7}{4}\pm\frac{13}{4}$$

$$x = \frac{-7 \pm 13}{4}$$

$$\Rightarrow \quad x = \frac{-7-13}{4} \quad \text{or} \quad x = \frac{-7+13}{4}$$

$$x = \frac{-20}{4} \quad \text{or} \quad x = \frac{6}{4}$$

$$x = -5 \quad \text{or} \quad x = \frac{3}{2}$$

Solution set is $\left\{-5, \frac{3}{2}\right\}$

(viii) $x^2 + 17x + \frac{33}{4} = 0$

Solution: $x^2 + 17x + \frac{33}{4} = 0$

$$x^2 + 17x = -\frac{33}{4}$$

$$(x)^2 + 2(x)\left(\frac{17}{2}\right) = -\frac{33}{4}$$

Adding $\left(\frac{17}{2}\right)^2$ on both sides

$$(x)^2 + 2(x)\left(\frac{17}{2}\right) + \left(\frac{17}{2}\right)^2 = \frac{-33}{4} + \left(\frac{17}{2}\right)^2$$

$$\left(x + \frac{17}{2}\right)^2 = \frac{-33}{4} + \frac{289}{4}$$

$$\left(x + \frac{17}{2}\right)^2 = \frac{-33+289}{4}$$

$$\left(x + \frac{17}{2}\right)^2 = \frac{256}{4}$$

Taking square root

$$\sqrt{\left(x + \frac{17}{2}\right)^2} = \pm\sqrt{\frac{256}{4}}$$

$$x + \frac{17}{2} = \pm\frac{16}{2}$$

$$x = \frac{-17}{2} \pm \frac{16}{2}$$

$$x = \frac{-17 \pm 16}{2}$$

$$\Rightarrow \quad x = \frac{-17-16}{2} \quad \text{or} \quad x = \frac{-17+16}{2}$$

$$x = \frac{-33}{2} \quad \text{or} \quad x = \frac{-1}{2}$$

Solution set is $\left\{-\frac{1}{2}, \frac{-33}{2}\right\}$

(ix) $4 - \frac{8}{3x+1} = \frac{3x^2+5}{3x+1}$

Solution: $4 - \frac{8}{3x+1} = \frac{3x^2+5}{3x+1}$

$$4 = \frac{3x^2+5}{3x+1} + \frac{8}{3x+1}$$

$$4 = \frac{3x^2+5+8}{3x+1}$$

$$4(3x+1) = 3x^2 + 13$$

$$12x + 4 = 3x^2 + 13$$

$$3x^2 + 13 - 12x - 4 = 0$$

$$3x^2 - 12x + 9 = 0$$

Dividing each term of the equation by 3

$$\frac{3}{3}x^2 - \frac{{}^{4}12x}{3} + \frac{{}^{3}9}{3} = 0$$

$$x^2 - 4x + 3 = 0$$

$$x^2 - 4x = -3$$

$$(x)^2 - 2(x)(2) = -3$$

Adding $(2)^2$ on both sides

$$(x)^2 - 2(x)(2) + (2)^2 = -3 + (2)^2$$

$$(x-2)^2 = -3 + 4$$

$$(x-2)^2 = 1$$

Taking square root

$$\sqrt{(x-2)^2} = \pm\sqrt{1}$$

$$x - 2 = \pm 1$$

$$\Rightarrow \quad x - 2 = +1 \quad \text{or} \quad x - 2 = -1$$

$$x = 2 + 1 \qquad x = 2 - 1$$

$$x = 3 \quad \text{or} \quad x = 1$$

Solution set is $\{1, 3\}$

(x) $7(x+2a)^2+3a^2=5a(7x+23a)$

Solution: $7(x+2a)^2+3a^2=5a(7x+23a)$

$$7(x^2+4a^2+4ax)+3a^2=35ax+115a^2$$

$$7x^2+28a^2+28ax+3a^2-35ax-115a^2=0$$

$$7x^2-35ax+28ax+28a^2+3a^2-115a^2=0$$

$$7x^2-7ax-84a^2=0$$

Dividing each term of the equation by 7 we get

$$\frac{7x^2}{7}-\frac{7ax}{7}=\frac{84a^2}{7}$$

$$x^2-ax=12a^2$$

$$(x)^2-2(x)\left(\frac{a}{2}\right)=12a^2$$

Adding $\left(\frac{a}{2}\right)^2$ an both sides

$$(x)^2-2(x)\left(\frac{a}{2}\right)+\left(\frac{a}{2}\right)^2=12a^2+\left(\frac{a}{2}\right)^2$$

$$\left(x-\frac{a}{2}\right)^2=12a^2+\frac{a^2}{4}$$

$$\left(x-\frac{a}{2}\right)^2=\frac{48a^2+a^2}{4}$$

$$\left(x-\frac{a}{2}\right)^2=\frac{49a^2}{4}$$

Taking square root

$$\sqrt{\left(x-\frac{a}{2}\right)^2}=\pm\sqrt{\frac{49a^2}{4}}$$

$$x-\frac{a}{2}=\pm\frac{7a}{2}$$

$$x=\frac{a}{2}\pm\frac{7a}{2}$$

$$x=\frac{a\pm7a}{2}$$

$\Rightarrow$ $x=\frac{a+7a}{2}$ or $x=\frac{a-7a}{2}$

$x=\frac{8a}{2}$ or $x=\frac{-6a}{2}$

$x=4a$ or $x=-3a$

Solution set is $\{-3a, 4a\}$

EXERCISE 1.2

Q.1 Solve the following equations using quadratic formula:

(i) $2 - x^2 = 7x$

Solution: $2 - x^2 = 7x$

$1x^2 + 7x - 2 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 1, \quad b = 7, \quad c = -2$

Using Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{49 + 8}}{2}$$

$$\therefore \quad x = \frac{-7 \pm \sqrt{57}}{2}$$

Solution set is $\left\{\frac{-7 \pm \sqrt{57}}{2}\right\}$

(ii) $5x^2 + 8x + 1 = 0$

Solution: $5x^2 + 8x + 1 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 5 \quad , \quad b = 8 \quad , \quad c = 1$

Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(5)(1)}}{2(5)}$$

$$x = \frac{-8 \pm \sqrt{64 - 20}}{10}$$

$$x = \frac{-8 \pm \sqrt{44}}{10}$$

$$x = \frac{-8 \pm \sqrt{4 \times 11}}{10}$$

$$x = \frac{-8 \pm 2\sqrt{11}}{10}$$

$$x = \frac{\cancel{2}\left(-4 \pm \sqrt{11}\right)}{\cancel{10}\ 5}$$

$$x = \frac{-4 \pm \sqrt{11}}{5}$$

Solution set is $\left\{\frac{-4 \pm \sqrt{11}}{5}\right\}$

(iii) $\sqrt{3}x^2 + x = 4\sqrt{3}$

Solution: $\sqrt{3}x^2 + x = 4\sqrt{3}$

$\sqrt{3}\ x^2 + 1x - 4\sqrt{3} = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = \sqrt{3} \quad , \quad b = 1 \quad , \quad c = -4\sqrt{3}$

Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4\left(\sqrt{3}\right)\left(-4\sqrt{3}\right)}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 16\left(\sqrt{3}\right)^2}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 16(3)}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 48}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{49}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm 7}{2\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{-1 - 7}{2\sqrt{3}} \quad \text{or} \quad x = \frac{-1 + 7}{2\sqrt{3}}$$

$$x = \frac{-\cancel{8}^{4}}{\cancel{2}\sqrt{3}} \quad \text{or} \quad x = \frac{\cancel{6}^{3}}{\cancel{2}\sqrt{3}}$$

$$x=\frac{-4}{\sqrt{3}} \quad \text{or} \quad x=\frac{3}{\sqrt{3}}$$

$$x=\frac{-4}{\sqrt{3}} \quad \text{or} \quad x=\sqrt{3} \qquad \left(\because \frac{a}{\sqrt{a}}=\sqrt{a}\right)$$

Solution set is $\left\{\sqrt{3}, -\frac{4}{\sqrt{3}}\right\}$

(iv) $\mathbf{4x^2-14=3}$

Solution: $4x^2-14=3x$

$4x^2-3x-14=0$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=4, b=-3, c=-14$

Using formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(4)(-14)}}{2(4)}$$

$$x=\frac{3\pm\sqrt{9+224}}{8}$$

Solution set is $\left\{\frac{3\pm\sqrt{233}}{8}\right\}$

(v) $\mathbf{6x^2-3-7x=0}$

Solution: $\mathbf{6x^2-3-7x=0}$

$6x^2-7x-3=0$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=6$, $b=-7$, $c=-3$

Using Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(6)(-3)}}{2(6)}$$

$$x=\frac{7\pm\sqrt{49+72}}{12}$$

$$x=\frac{7\pm\sqrt{121}}{12}$$

$$x=\frac{7\pm 11}{12}$$

$$\Rightarrow \quad x=\frac{7-11}{12} \quad \text{or} \quad x=\frac{7+11}{12}$$

$$x=\frac{-4}{12} \quad \text{or} \quad x=\frac{18}{12}$$

$$x=\frac{-1}{3} \quad \text{or} \quad x=\frac{3}{2}$$

Solution set is $\left\{\frac{-1}{3}, \frac{3}{2}\right\}$

(vi) $\mathbf{3x^2+8x+2=0}$

Solution: $3x^2+8x+2=0$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=3$, $b=8$, $c=2$

Using formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-8\pm\sqrt{(8)^2-4(3)(2)}}{2(3)}$$

$$x=\frac{-8\pm\sqrt{64-24}}{6}$$

$$x=\frac{-8\pm\sqrt{40}}{6}$$

$$x=\frac{-8\pm\sqrt{4\times 10}}{6}$$

$$x=\frac{-8\pm 2\sqrt{10}}{6}$$

$$x=\frac{\cancel{2}\left(-4\pm\sqrt{10}\right)}{\cancel{6}3}$$

$$x=\frac{-4\pm\sqrt{10}}{3}$$

Solution set is $\left\{\frac{-4\pm\sqrt{10}}{3}\right\}$

(vii) $\frac{3}{x-6}-\frac{4}{x-5}=1$

Solution: $\frac{3}{x-6}-\frac{4}{x-5}=1$

$$\frac{3(x-5)-4(x-6)}{(x-6)(x-5)}=1$$

$$3(x-5)-4(x-6)=(x-6)(x-5)$$

$$3x-15-4x+24=x^2-5x-6x+30$$

$$-1x+9=x^2-11x+30$$

$$x^2-11x+1x+30-9=0$$

$$1x^2-10x+21=0$$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=1$, $b=-10$, $c=21$

Using formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(21)}}{2(1)}$$

$$x=\frac{10\pm\sqrt{100-84}}{2}$$

$$x=\frac{10\pm\sqrt{16}}{2}$$

$$x=\frac{10\pm4}{2}$$

$$x=\frac{\cancel{2}(5\pm2)}{\cancel{2}}$$

$\Rightarrow$ $x=5+2$ or $x=5-2$

$x=7$ or $x=3$

Solution set is $\{3, 7\}$

(viii) $\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}$

Solution: $\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}$

$$\frac{(x+2)2x-(4-x)(x-1)}{(x-1)(2x)}=\frac{7}{3}$$

$$\frac{(2x^2+4x)-(4x-4-x^2+x)}{2x^2-2x}=\frac{7}{3}$$

$$\frac{2x^2+4x-5x+4+x^2}{2x^2-2x}=\frac{7}{3}$$

$$\frac{3x^2-x+4}{2x^2-2x}=\frac{7}{3}$$

$$3(3x^2-x+4)=7(2x^2-2x)$$

$$9x^2-3x+12=14x^2-14x$$

$\Rightarrow$ $14x^2-9x^2-14x+3x-12=0$

$$5x^2-11x-12=0$$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=5$, $b=-11$, $c=-12$

Using formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-11)\pm\sqrt{(-11)^2-4(5)(-12)}}{2(5)}$$

$$x=\frac{11\pm\sqrt{121+240}}{10}$$

$$x=\frac{11\pm\sqrt{361}}{10}$$

$$x=\frac{11\pm19}{10}$$

$\Rightarrow$ $x=\frac{11-19}{10}$ or $x=\frac{11+19}{10}$

$x=\frac{-8}{10}$ or $x=\frac{30}{10}$

$x=-\frac{4}{5}$ or $x=3$

Solution set is $\left\{3, \frac{-4}{5}\right\}$

(ix) $\dfrac{a}{x-b}+\dfrac{b}{x-a}=2$

Solution: $\dfrac{a}{x-b}+\dfrac{b}{x-a}=2$

$$\frac{a(x-a)+b(x-b)}{(x-b)(x-a)}=2$$

$$\frac{ax-a^2+bx-b^2}{x^2-ax-bx+ab}=2$$

$$ax-a^2+bx-b^2=2(x^2-ax-bx+ab)$$

$$ax-a^2+bx-b^2=2x^2-2ax-2bx+2ab$$

$$2x^2-2ax-2bx+2ab-ax+a^2-bx+b^2=0$$

$$2x^2-2ax-ax-2bx-bx+2ab+a^2+b^2=0$$

$$2x^2-3ax-3bx+2ab+a^2+b^2=0$$

$$2x^2-(3a+3b)x+(a^2+b^2+2ab)=0$$

$$2x^2-(3a+3b)x+(a+b)^2=0$$

As $Ax^2+Bx+C=0$

$\Rightarrow$ $A=2$, $B=-(3a+3b)$, $C=(a+b)^2$

Using Formula

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$x=\frac{-[-(3a+3b)]\pm\sqrt{[-(3a+3b)]^2-4(2)(a+b)^2}}{2(2)}$$

$$x=\frac{+3(a+b)\pm\sqrt{[-3(a+b)]^2-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{\left[(-3)^2(a+b)\right]^2-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{9(a+b)^2-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm(a+b)}{4}$$

$$x=\frac{3(a+b)-(a+b)}{4} \quad \text{or} \quad x=\frac{3(a+b)+(a+b)}{4}$$

$$x=\frac{2(a+b)}{4} \quad \text{or} \quad x=\frac{4(a+b)}{4}$$

$$x=\frac{a+b}{2} \quad \text{or} \quad x=a+b$$

$$x=\frac{1}{2}(a+b) \quad \text{or} \quad x=a+b$$

Solution set is $\left\{(a+b),\ \frac{1}{2}(a+b)\right\}$

(x) $-(\ell+m)-\ell x^2+(2\ell+m)x=0$, $\ell\neq 0$

Solution: $-(\ell+m)-\ell x^2+(2\ell+m)x=0$, $\ell\neq 0$

$\Rightarrow$ $(\ell+m)+\ell x^2-(2\ell+m)x=0$ 01(032)

$\Rightarrow$ $\ell x^2-(2\ell+m)x+(\ell+m)=0$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=\ell$, $b=-(2\ell+m)$, $c=\ell+m$

Using Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-[-(2\ell+m)]\pm\sqrt{[-(2\ell+m)]^2-4(\ell)(\ell+m)}}{2(\ell)}$$

$$x=\frac{(2\ell+m)\pm\sqrt{(-1)^2(2\ell+m)^2-4\ell(\ell+m)}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{(2\ell)^2+(m)^2+2(2\ell)(m)-4\ell^2-4\ell m}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{\cancel{4\ell^2}+m^2+\cancel{4\ell m}-\cancel{4\ell^2}-\cancel{4\ell m}}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{m^2}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm m}{2\ell}$$

$$x=\frac{2\ell+\cancel{m}-\cancel{m}}{2\ell} \quad \text{or} \quad x=\frac{2\ell+m+m}{2\ell}$$

$$x=\frac{\cancel{2\ell}}{\cancel{2\ell}} \quad \text{or} \quad x=\frac{2\ell+2m}{2\ell}$$

$$x=1 \quad \text{or} \quad x=\frac{\cancel{2}(\ell+m)}{\cancel{2}\ell}$$

$$x=\frac{\ell+m}{\ell}$$

Solution set is $\left\{1,\ \frac{\ell+m}{\ell}\right\}$

EXERCISE 1.3

Q.1 $2x^4 - 11x^2 + 5 = 0$

Solution: $2x^4 - 11x^2 + 5 = 0$ .... (i)

Let $x^2 = y$ .... (ii)

$(x^2)^2 = y^2$

$x^4 = y^2$

Put $x^2 = y$ and $x^4 = y^2$ in equation (i)

$2x^4 - 11x^2 + 5 = 0$

$2y^2 - 11y + 5 = 0$

$2y^2 - 10y - y + 5 = 0$

$2y(y-5) - 1(y-5) = 0$

$(y-5)(2y-1) = 0$

$y - 5 = 0$ or $2y - 1 = 0$

$y = 0 + 5$ or $2y = 1$

$y = 5$ or $y = \frac{1}{2}$

From equation (ii) Put $y = x^2$

$x^2 = 5$ or $x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{5}$ or $\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm\sqrt{5}$ or $x = \pm\frac{1}{\sqrt{2}}$

The Solution set is $\left\{\pm\frac{1}{\sqrt{2}},\ \pm\sqrt{5}\right\}$

Q.2 $2x^4 = 9x^2 - 4$

Solution: $2x^4 = 9x^2 - 4$

$2x^4 - 9x^2 + 4 = 0$ ....(i)

Let $x^2 = y$ ....(ii)

$(x^2)^2 = y^2$

$x^4 = y^2$

Put $x^2 = y$ and $x^4 = y^2$ in equation (i)

$2y^2 - 9y + 4 = 0$

$2y^2 - 8y - 1y + 4 = 0$

$2y(y-4) - 1(y-4) = 0$

$(y-4)(2y-1) = 0$

$y - 4 = 0$ or $2y - 1 = 0$

$y = 4$ or $2y = 1$

$\Rightarrow$ $y = 4$ or $y = \frac{1}{2}$

From equation (ii) put $y = x^2$

$x^2 = 4$ or $x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{4}$ or $\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm 2$ or $x = \pm\frac{1}{\sqrt{2}}$

The Solution set is $\left\{\pm\frac{1}{\sqrt{2}},\ \pm 2\right\}$

Q.3 $5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

Solution: $5x^{1/2} = 7x^{1/4} - 2$ ....(i)

Let $x^{\frac{1}{4}} = y$ ....(ii)

Taking square

$\left(x^{\frac{1}{4}}\right)^2 = y^2$

$x^{\frac{1}{2}} = y^2$

Put $x^{\frac{1}{4}} = y$ and $x^{\frac{1}{2}} = y^2$ in equation (i)

$5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

$5y^2 = 7y - 2$

$5y^2 - 7y + 2 = 0$

$5y^2 - 5y - 2y + 2 = 0$

$5y(y-1) - 2(y-1) = 0$

$(y-1)(5y-2) = 0$

$y - 1 = 0$ or $5y - 2 = 0$

$y = 1$ or $5y = 2$

$y = 1$ or $y = \frac{2}{5}$

From equation (ii) put $y = x^{\frac{1}{4}}$

$x^{\frac{1}{4}} = 1$ or $x^{\frac{1}{4}} = \frac{2}{5}$

$\left(x^{\frac{1}{4}}\right)^4 = (1)^4$ or $\left(x^{\frac{1}{4}}\right)^4 = \left(\frac{2}{5}\right)^4$

$x = 1$ or $x = \frac{2^4}{5^4}$

$x = \frac{16}{625}$

Solution set is $\left\{\frac{16}{625}, 1\right\}$

Q.4 $\mathbf{x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}}$

Solution: $x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$ ....(i)

Let $x^{\frac{1}{3}} = y$ ....(ii)

Taking square

$\left(x^{\frac{1}{3}}\right)^2 = y^2$

$x^{\frac{2}{3}} = y^2$

Put $x^{\frac{1}{3}} = y$ and $x^{\frac{2}{3}} = y^2$ in equation (i)

$x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$

$y^2 + 54 = 15y$

$y^2 - 15y + 54 = 0$

$y^2 - 9y - 6y + 54 = 0$

$y(y-9) - 6(y-9) = 0$

$(y-9)(y-6) = 0$

$y - 9 = 0$ or $y - 6 = 0$

$y = 9$ or $y = 6$

From equation (ii) put $y = x^{\frac{1}{3}}$

$x^{\frac{1}{3}} = 9$ or $x^{\frac{1}{3}} = 6$

$\left(x^{\frac{1}{3}}\right)^3 = (9)^3$ or $\left(x^{\frac{1}{3}}\right)^3 = (6)^3$

$x = 9^3$ or $x = 6^3$

$x = 729$ or $x = 216$

Solution set is $\{216, 729\}$

Q.5 $\mathbf{3x^{-2} + 5 = 8x^{-1}}$

Solution: $3x^{-2} + 5 = 8x^{-1}$ ....(i)

Let $x^{-1} = y$ ....(ii)

$(x^{-1})^2 = y^2$

$x^{-2} = y^2$

Put $x^{-1} = y$ and $x^{-2} = y^2$ in equation (i)

$3x^{-2} + 5 = 8x^{-1}$

$3y^2 + 5 = 8y$

$3y^2 - 8y + 5 = 0$

$3y^2 - 5y - 3y + 5 = 0$

$y(3y-5) - 1(3y-5) = 0$

$(3y-5)(y-1) = 0$

$3y - 5 = 0$ or $y - 1 = 0$

$3y = 5$ or $y = 1$

$y = \frac{5}{3}$

From equation (ii) put $y = x^{-1}$

$x^{-1} = \frac{5}{3}$ or $x^{-1} = 1$

$\frac{1}{x} = \frac{5}{3}$ or $\frac{1}{x} = 1$

$x = \frac{3}{5}$ or $x = 1$

Solution set is $\left\{\frac{3}{5}, 1\right\}$

Q.6 $(2x^2+1)+\dfrac{3}{2x^2+1}=4$

Solution: $(2x^2+1)+\dfrac{3}{2x^2+1}=4$ ....(i)

Let $2x^2+1=y$ ....(ii)

Replacing $2x^2+1$ by "y" in equation (i), we get

$$y+\frac{3}{y}=4$$

Multiplying both sides by "y"

$$y^2+3=4y$$
$$y^2-4y+3=0$$
$$y^2-3y-1y+3=0$$
$$y(y-3)-1(y-3)=0$$
$$(y-3)(y-1)=0$$
$$y-3=0 \quad \text{or} \quad y-1=0$$
$$y=3 \quad \text{or} \quad y=1$$

From equation (ii) Put $y=2x^2+1$

$$2x^2+1=3 \quad \text{or} \quad 2x^2+1=1$$
$$2x^2=3-1 \quad \text{or} \quad 2x^2=1-1$$
$$2x^2=2 \quad \text{or} \quad 2x^2=0$$
$$x^2=\frac{2}{2} \quad \text{or} \quad x^2=\frac{0}{2}$$
$$x^2=1 \quad \text{or} \quad x^2=0$$
$$\sqrt{x^2}=\pm\sqrt{1} \quad \text{or} \quad \sqrt{x^2}=\pm\sqrt{0}$$
$$x=\pm1 \quad \text{or} \quad x=0$$

Solution set is $\{-1, 0, 1\}$

Q.7 $\dfrac{x}{x-3}+4\left(\dfrac{x-3}{x}\right)=4$

Solution: $\dfrac{x}{x-3}+4\left(\dfrac{x-3}{x}\right)=4$ ....(i)

Let $\dfrac{x}{x-3}=y$ ....(ii)

$$\frac{x-3}{x}=\frac{1}{y}$$

Equation (i) becomes

$$y+4\left(\frac{1}{y}\right)=4$$

Multiplying both sides by "y"

$$y^2+4=4y$$
$$y^2-4y+4=0$$
$$y^2-2y-2y+4=0$$
$$y(y-2)-2(y-2)=0$$
$$(y-2)(y-2)=0$$
$$(y-2)^2=0$$
$$y-2=0$$
$$y=2$$

From equation (ii) Put $y=\dfrac{x}{x-3}$

$$\frac{x}{x-3}=2$$
$$x=2(x-3)$$
$$x=2x-6$$
$$6=2x-x$$
$$6=x \Rightarrow x=6$$

Solution set is $\{6\}$

Q.8 $\dfrac{4x+1}{4x-1}+\dfrac{4x-1}{4x+1}=2\dfrac{1}{6}$

Solution: $\dfrac{4x+1}{4x-1}+\dfrac{4x-1}{4x+1}=2\dfrac{1}{6}$ .... (i)

Let $\dfrac{4x+1}{4x-1}=y$ .... (ii)

$$\frac{4x-1}{4x+1}=\frac{1}{y}$$

Equation (i) becomes

$$y+\frac{1}{y}=2\frac{1}{6}$$
$$y+\frac{1}{y}=\frac{13}{6}$$

Multiplying both sides by "6y"

$$6y^2+6=13y$$
$$6y^2-13y+6=0$$

$$6y^2-9y-4y+6=0$$
$$3y(2y-3)-2(2y-3)=0$$
$$(2y-3)(3y-2)=0$$
$$2y-3=0 \quad \text{or} \quad 3y-2=0$$
$$2y=3 \quad \text{or} \quad 3y=2$$
$$y=\frac{3}{2} \quad \text{or} \quad y=\frac{2}{3}$$

From equation (ii) Put $y=\frac{4x+1}{4x-1}$

$$\frac{4x+1}{4x-1}=\frac{3}{2} \quad \text{or} \quad \frac{4x+1}{4x-1}=\frac{2}{3}$$
$$2(4x+1)=3(4x-1) \quad \text{or} \quad 3(4x+1)=2(4x-1)$$
$$8x+2=12x-3 \quad \text{or} \quad 12x+3=8x-2$$
$$2+3=12x-8x \quad \text{or} \quad 12x-8x=-2-3$$
$$4x=5 \quad \text{or} \quad 4x=-5$$
$$x=\frac{5}{4} \quad \text{or} \quad x=\frac{-5}{4}$$

Solution set is $\left\{\pm\frac{5}{4}\right\}$

Q.9 $\mathbf{\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}}$

Solution: $\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$ ....(i)

Let $\frac{x-a}{x+a}=y$ ....(ii)

Or $\frac{x+a}{x-a}=\frac{1}{y}$

The equation (i) becomes

$$y-\frac{1}{y}=\frac{7}{12}$$
$$\frac{y^2-1}{y}=\frac{7}{12}$$
$$12(y^2-1)=7y$$
$$12y^2-12=7y$$
$$12y^2-7y-12=0$$
$$12y^2-16y+9y-12=0$$
$$4y(3y-4)+3(3y-4)=0$$
$$(3y-4)(4y+3)=0$$
$$3y-4=0 \quad \text{or} \quad 4y+3=0$$
$$3y=4 \quad \text{or} \quad 4y=-3$$
$$y=\frac{4}{3} \quad \text{or} \quad y=\frac{-3}{4}$$

From equation (ii) Putting $y=\frac{x-a}{x+a}$, we get

$$\frac{x-a}{x+a}=\frac{4}{3} \quad \text{or} \quad \frac{x-a}{x+a}=\frac{-3}{4}$$
$$3(x-a)=4(x+a) \quad \text{or} \quad 4(x-a)=-3(x+a)$$
$$3x-3a=4x+4a \quad \text{or} \quad 4x-4a=-3x-3a$$
$$-3a-4a=4x-3x \quad \text{or} \quad 4x+3x=-3a+4a$$
$$-7a=x \quad \text{or} \quad 7x=a$$
$$\Rightarrow \quad x=-7a \quad \text{or} \quad x=\frac{a}{7}$$

The solution set is $\left\{-7a,\ \frac{a}{7}\right\}$

Q.10 $\mathbf{x^4-2x^3-2x^2+2x+1=0}$

Solution: $x^4-2x^3-2x^2+2x+1=0$

Dividing both sides by "x^2"

$$\frac{x^4}{x^2}-\frac{2x^3}{x^2}-\frac{2x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}=\frac{0}{x^2}$$
$$x^2-2x-2+\frac{2}{x}+\frac{1}{x^2}=0$$
$$\left(x^2+\frac{1}{x^2}\right)-2x+\frac{2}{x}-2=0$$
$$\left(x^2+\frac{1}{x^2}\right)-2\left(x-\frac{1}{x}\right)-2=0 \quad \text{....(i)}$$

Let $x-\frac{1}{x}=y$ ....(ii)

Taking square of both sides

$$\left(x-\frac{1}{x}\right)^2=y^2$$
$$x^2+\frac{1}{x^2}-2=y^2$$
$$x^2+\frac{1}{x^2}=y^2+2$$

Putting these values in equation (i)

$$y^2+\cancel{2}-2(y)-\cancel{2}=0$$
$$y^2-2y=0$$
$$y(y-2)=0$$
$$y=0 \quad \text{or} \quad y-2=0$$
$$y=0 \quad \text{or} \quad y=2$$

From equation (ii) Put $y=x-\frac{1}{x}$

$$x-\frac{1}{x}=0 \quad \text{or} \quad x-\frac{1}{x}=2$$
$$\frac{x^2-1}{x}=0 \quad \text{or} \quad \frac{x^2-1}{x}=2$$
$$x^2-1=0 \quad \text{or} \quad x^2-1=2x$$
$$x^2=1 \quad \text{or} \quad x^2-2x-1=0$$
$$\sqrt{x^2}=\pm\sqrt{1} \quad \text{or} \quad \boxed{1x^2-2x-1=0}$$
$$\boxed{x=\pm 1}$$

Now Solving $1x^2-2x-1=0$ by quadratic formula

$$a=1 \;,\; b=-2 \;,\; c=-1$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$$
$$x=\frac{+2\pm\sqrt{4+4}}{2}$$
$$x=\frac{2\pm\sqrt{8}}{2}$$
$$x=\frac{2\pm\sqrt{4\times 2}}{2}$$
$$x=\frac{2\pm 2\sqrt{2}}{2}$$
$$x=\frac{\cancel{2}\left(1\pm\sqrt{2}\right)}{\cancel{2}}$$
$$\boxed{x=1\pm\sqrt{2}}$$

The solution set is $\left\{\pm 1,\ 1\pm\sqrt{2}\right\}$

Q.11 $2x^4+x^3-6x^2+x+2=0$

Solution: $2x^4+x^3-6x^2+x+2=0$

Dividing each terms by x^2

$$\frac{2x^4}{x^2}+\frac{x^3}{x^2}-\frac{6\cancel{x^2}}{\cancel{x^2}}+\frac{x}{x^2}+\frac{2}{x^2}=\frac{0}{x^2}$$
$$2x^2+x-6+\frac{1}{x}+\frac{2}{x^2}=0$$
$$2x^2+\frac{2}{x^2}+x+\frac{1}{x}-6=0$$
$$2\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)-6=0 \quad \ldots\ldots(i)$$

Let $x+\frac{1}{x}=y$ ……(ii)

Taking square of both sides

$$\left(x+\frac{1}{x}\right)^2=y^2$$
$$x^2+\frac{1}{x^2}+2(\cancel{x})\left(\frac{1}{\cancel{x}}\right)=y^2$$
$$x^2+\frac{1}{x^2}+2=y^2$$
$$x^2+\frac{1}{x^2}=y^2-2$$

Put $x+\frac{1}{x}=y$ and $x^2+\frac{1}{x^2}=y^2-2$ in equation (i)

$$2\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)-6=0$$
$$2(y^2-2)+y-6=0$$
$$2y^2-4+y-6=0$$
$$2y^2+y-10=0$$
$$2y^2+5y-4y-10=0$$
$$y(2y+5)-2(2y+5)=0$$
$$(2y+5)(y-2)=0$$
$$2y+5=0 \quad \text{or} \quad y-2=0$$
$$2y=-5 \quad \text{or} \quad y=2$$
$$y=\frac{-5}{2}$$

From equation (ii) Put $y = x + \frac{1}{x}$

$$x + \frac{1}{x} = \frac{-5}{2}$$
$$\frac{x^2+1}{x} = \frac{-5}{2}$$
$$2(x^2+1) = -5x$$
$$2x^2 + 2 = -5x$$
$$2x^2 + 5x + 2 = 0$$
$$2x^2 + 4x + 1x + 2 = 0$$
$$2x(x+2) + 1(x+2) = 0$$
$$(x+2)(2x+1) = 0$$
$$x+2=0 \text{ or } 2x+1=0$$
$$x=-2 \text{ or } 2x=-1$$
$$\boxed{x=-2} \text{ or } \boxed{x=\frac{-1}{2}}$$

$$x + \frac{1}{x} = 2$$
$$\frac{x^2+1}{x} = 2$$
$$x^2 + 1 = 2x$$
$$x^2 - 2x + 1 = 0$$
$$x^2 - 1x - 1x + 1 = 0$$
$$x(x-1) - 1(x-1) = 0$$
$$(x-1)(x-1) = 0$$
$$(x-1)^2 = 0$$
$$\sqrt{(x-1)^2} = \sqrt{0}$$
$$x - 1 = 0$$
$$x = 1$$

Solution set is $\left\{1, -2, \frac{-1}{2}\right\}$

Q.12 $4.2^{2x+1} - 9.2^x + 1 = 0$

Solution: $4.2^{2x+1} - 9.2^x + 1 = 0$

$$4.2^{2x}2^1 - 9.2^x + 1 = 0$$
$$8(2^x)^2 - 9(2^x) + 1 = 0 \quad \text{........ (i)}$$

Let $2^x = y$ .... (ii)

The equation (i) becomes

$$8y^2 - 9y + 1 = 0$$
$$8y^2 - 8y - 1y + 1 = 0$$
$$8y(y-1) - 1(y-1) = 0$$
$$(y-1)(8y-1) = 0$$
$$y-1=0 \quad \text{or} \quad 8y-1=0$$
$$y=1 \quad \text{or} \quad 8y=1$$
$$\text{or} \quad y=\frac{1}{8}$$

From equation (ii) Put $y = 2^x$

$$2^x = 1 \quad \text{or} \quad 2^x = \frac{1}{8}$$
$$2^x = 2^0 \quad \text{or} \quad 2^x = \frac{1}{2^3}$$
$$\Rightarrow x = 0 \quad \text{or} \quad 2^x = 2^{-3}$$
$$\Rightarrow x = -3$$

Solution set is $\{-3, 0\}$

Q.13 $3^{2x+2} = 12.3^x - 3$

Solution: $3^{2x+2} = 12.3^x - 3$

$$3^{2x}\,3^2 - 12.3^x + 3 = 0$$
$$9(3^x)^2 - 12.(3^x) + 3 = 0 \quad \text{....... (i)}$$

Let $3^x = y$ .... (ii)

Put $3^x = y$ in equation (i)

$$9y^2 - 12y + 3 = 0$$
$$9y^2 - 9y - 3y + 3 = 0$$
$$9y(y-1) - 3(y-1) = 0$$
$$(y-1)(9y-3) = 0$$
$$y-1=0 \quad \text{or} \quad 9y-3=0$$
$$y=1 \quad \text{or} \quad 9y=3$$
$$\boxed{y=1} \quad \text{or} \quad y=\frac{3}{9} \Rightarrow \boxed{y=\frac{1}{3}}$$

From equation (ii) Put $y = 3^x$

$$3^x = 1 \quad \text{or} \quad 3^x = \frac{1}{3}$$
$$3^x = 3^0 \quad \text{or} \quad 3^x = 3^{-1}$$
$$\Rightarrow x = 0 \quad \text{or} \quad x = -1$$

The solution set is $\{0, -1\}$

Q.14 $2^x + 64.2^{-x} - 20 = 0$

Solution: $2^x + 64.2^{-x} - 20 = 0$

$$2^x + \frac{64}{2^x} - 20 = 0 \quad \text{...............(i)}$$

Let $2^x = y$ ....(ii)

Put $2^x = y$ in equation (i)

$$y + \frac{64}{y} - 20 = 0$$

Multiply both sides by "y"

$$y^2 + 64 - 20y = 0$$
$$y^2 - 20y + 64 = 0$$
$$y^2 - 16y - 4y + 64 = 0$$
$$y(y-16) - 4(y-16) = 0$$
$$(y-16)\ (y-4) = 0$$
$$y-16=0 \quad \text{or} \quad y-4=0$$
$$y=16 \quad \text{or} \quad y=4$$

From equation (ii) put $y = 2^x$

$$2^x = 16 \quad \text{or} \quad 2^x = 4$$
$$2^x = 2^4 \quad \text{or} \quad 2^x = 2^2$$
$$\Rightarrow x = 4 \quad \text{or} \quad x = 2$$

The solution set is $\{2, 4\}$

Q.15 $(x+1)(x+3)(x-5)(x-7)=192$

Solution: $(x+1)(x+3)(x-5)(x-7)=192$

$\because \quad a+b=c+d$
$1-5=3-7$
$-4=-4$

$(x+1)(x-5)(x+3)(x-7)=192$

$(x^2-5x+1x-5)(x^2-7x+3x-21)=192$

$(x^2-4x-5)(x^2-4x-21)=192 \quad ....(i)$

Let $\quad x^2-4x=y \quad ....(ii)$

The equation (i) becomes

$(y-5)(y-21)=192$

$y^2-21y-5y+105=192$

$y^2-26y+105-192=0$

$y^2-26y-87=0$

$y^2-29y+3y-87=0$

$y(y-29)+3(y-29)=0$

$(y-29)(y+3)=0$

$y-29=0 \quad$ or $\quad y+3=0$

$y=29 \quad$ or $\quad y=-3$

From equation (ii) Put $y=x^2-4x$

$x^2-4x=29 \quad$ or $\quad x^2-4x=-3$

$x^2-4x-29=0 \quad$ or $\quad x^2-4x+3=0$

$\boxed{x^2-4x-29=0} \quad$ or $\quad \boxed{x^2-4x+3=0}$

First we solve $x^2-4x-29=0$ by quadratic formula

$a=1 \quad , \quad b=-4 \quad , \quad c=-29$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-29)}}{2(1)}$$

$$x=\frac{4\pm\sqrt{16+116}}{2}$$

$$x=\frac{4\pm\sqrt{132}}{2}$$

$$x=\frac{4\pm\sqrt{4\times 33}}{2}$$

$$x=\frac{4\pm 2\sqrt{33}}{2}$$

$$x=\frac{\cancel{2}(2\pm\sqrt{33})}{\cancel{2}}$$

$\boxed{x=2\pm\sqrt{33}}$

Now we solve $x^2-4x+3=0$ by factorization

$x^2-4x+3=0$

$x^2-3x-x+3=0$

$x(x-3)-1(x-3)=0$

$(x-3)(x-1)=0$

$x-3=0 \quad$ or $\quad x-1=0$

$\boxed{x=3} \quad$ or $\quad \boxed{x=1}$

Solution set is $\{1,\ 3,\ 2\pm\sqrt{33}\}$

Q.16 $(x-1)(x-2)(x-8)(x+5)+360=0$

Solution: $(x-1)(x-2)(x-8)(x+5)+360=0$

$[(x-1)(x-2)][(x-8)(x+5)]+360=0$

$[x^2-2x-1x+2][x^2+5x-8x-40]+360=0$

$(x^2-3x+2)(x^2-3x-40)+360=0 \quad ....(i)$

Let $\quad x^2-3x=y \quad ......(ii)$

Put it in equation (i)

$(y+2)(y-40)+360=0$

$y^2-40y+2y-80+360=0$

$y^2-38y+280=0$

$y^2-28y-10y+280=0$

$y(y-28)-10(y-28)=0$

$(y-28)(y-10)=0$

$y-28=0 \quad$ or $\quad y-10=0$

$y=28 \quad$ or $\quad y=10$

From equation (ii) Put $y=x^2-3x$

$x^2-3x=28$	$x^2-3x=10$
$x^2-3x-28=0$	$x^2-3x-10=0$
$x^2-7x+4x-28=0$	$x^2-5x+2x-10=0$
$x(x-7)+4(x-7)=0$	$x(x-5)+2(x-5)=0$
$(x-7)(x+4)=0$	$(x-5)(x+2)=0$
$x-7=0$ or $x+4=0$	$x-5=0$ or $x+2=0$
$x=7$ or $x=-4$	$x=5$ or $x=-2$

The solution set is

$\{5,-2,7,-4\}$

EXERCISE 1.4

Solve the following equations:

Q.1 $\mathbf{2x+5=\sqrt{7x+16}}$

Solution: $2x+5=\sqrt{7x+16}$ ....(i)

Squaring both sides

$$(2x+5)^2=\left(\sqrt{7x+16}\right)^2$$
$$(2x)^2+(5)^2+2(2x)(5)=7x+16$$
$$4x^2+25+20x=7x+16$$
$$4x^2+20x-7x+25-16=0$$
$$4x^2+13x+9=0$$
$$4x^2+4x+9x+9=0$$
$$4x(x+1)+9(x+1)=0$$
$$(x+1)(4x+9)=0$$

$x+1=0$ or $4x+9=0$

$x=-1$ or $4x=-9$

$x=-1$ or $x=\frac{-9}{4}$

Checking

Putting $x=-1$ in the equation (i) we have

$$2x+5=\sqrt{7x+16}$$
$$2(-1)+5=\sqrt{7(-1)+16}$$
$$-2+5=\sqrt{-7+16}$$
$$3=\sqrt{9}$$

$3=3$ which is true

Putting $x=\frac{-9}{4}$ in the Equation (i) we have

$$2x+5=\sqrt{7x+16}$$
$$2\left(\frac{-9}{4}\right)+5=\sqrt{7\left(\frac{-9}{4}\right)+16}$$
$$\frac{-18}{4}+5=\sqrt{\frac{-63}{4}+16}$$
$$\frac{-18+20}{4}=\sqrt{\frac{-63+64}{4}}$$

$\frac{2}{4}=\sqrt{\frac{1}{4}}$ $\Rightarrow \frac{1}{2}=\frac{1}{2}$ which is true

So, the solution set is $\left\{-1,\frac{-9}{4}\right\}$

Q.2 $\mathbf{\sqrt{x+3}=3x-1}$

Solution: $\sqrt{x+3}=3x-1$ ......(i)

Squaring both sides

$$\left(\sqrt{x+3}\right)^2=(3x-1)^2$$
$$x+3=(3x)^2+(1)^2-2(3x)(1)$$
$$x+3=9x^2+1-6x$$
$$9x^2-6x-x+1-3=0$$
$$9x^2-7x-2=0$$
$$9x^2-9x+2x-2=0$$
$$9x(x-1)+2(x-1)=0$$
$$(x-1)(9x+2)=0$$

$x-1=0$ or $9x+2=0$

$x=1$ or $9x=-2$

or $x=\frac{-2}{9}$

Checking:

Putting x = 1 in the equation (i) we have

$$\sqrt{x+3}=3x-1$$
$$\sqrt{1+3}=3(1)-1$$
$$\sqrt{4}=3-1$$

$2=2$ which is true

Putting $x=\frac{-2}{9}$ in the equation (i) we have

$$\sqrt{x+3}=3x-1$$
$$\sqrt{\frac{-2}{9}+3}=3\left(\frac{-2}{9}\right)-1$$
$$\sqrt{\frac{-2+27}{9}}=\frac{-6}{9}-1$$
$$\sqrt{\frac{25}{9}}=\frac{-6-9}{9}$$
$$\frac{5}{3}=\frac{-\cancel{15}^{\,5}}{\cancel{9}_{\,3}}$$

$\frac{5}{3}=\frac{-5}{3}$ which is not true

As $\frac{-2}{9}$ is an extraneous root

So, the solution set is $\{1\}$

Q.3 $4x = \sqrt{13x+14} - 3$

Solution: $4x = \sqrt{13x+14} - 3$ ....(i)

$4x + 3 = \sqrt{13x+14}$

Squaring both sides

$(4x+3)^2 = \left(\sqrt{13x+14}\right)^2$

$(4x)^2 + (3)^2 + 2(4x)(3) = 13x + 14$

$16x^2 + 9 + 24x = 13x + 14$

$16x^2 + 9 + 24x - 13x - 14 = 0$

$16x^2 + 11x - 5 = 0$

$16x^2 + 16x - 5x - 5 = 0$

$16x(x+1) - 5(x+1) = 0$

$(x+1)(16x-5) = 0$

$x + 1 = 0$ or $16x - 5 = 0$

$x = -1$ or $16x = 5$

$\boxed{x = -1}$ or $\boxed{x = \frac{5}{16}}$

Checking

Putting $x = -1$ in the equation (i) we have

$4x = \sqrt{13x+14} - 3$

$4(-1) = \sqrt{13(-1)+14} - 3$

$-4 = \sqrt{-13+14} - 3$

$-4 = \sqrt{1} - 3$

$-4 = 1 - 3$

$-4 \neq -2$

So -1 is an extraneous root

Putting $x = \frac{5}{16}$ in the equation (i) we have

$4x = \sqrt{13x+14} - 3$

$4\left(\frac{5}{16}\right) = \sqrt{13\left(\frac{5}{16}\right)+14} - 3$

$\frac{20}{16} = \sqrt{\frac{65}{16}+14} - 3$

$\frac{5}{4} = \sqrt{\frac{65+224}{16}} - 3$

$\frac{5}{4} = \sqrt{\frac{289}{16}} - 3$

$\frac{5}{4} = \frac{17}{4} - 3$

$\frac{5}{4} = \frac{17-12}{4}$

$\frac{5}{4} = \frac{5}{4}$ which is true

So, the solution set is $\left\{\frac{5}{16}\right\}$

Q.4 $\sqrt{3x+100} - x = 4$

Solution: $\sqrt{3x+100} - x = 4$ ....(i)

$\sqrt{3x+100} = 4 + x$

Squaring both sides

$\left(\sqrt{3x+100}\right)^2 = (4+x)^2$

$3x + 100 = (4)^2 + (x)^2 + 2(4)(x)$

$3x + 100 = 16 + x^2 + 8x$

$x^2 + 8x + 16 - 3x - 100 = 0$

$x^2 + 5x - 84 = 0$

$x^2 + 12x - 7x - 84 = 0$

$x(x+12) - 7(x+12) = 0$

$(x+12)(x-7) = 0$

$x + 12 = 0$ or $x - 7 = 0$

$x = -12$ or $x = 7$

Checking

Putting $x = -12$ in the equation (i) we have

$\sqrt{3x+100} - x = 4$

$\sqrt{3(-12)+100} - (-12) = 4$

$\sqrt{-36+100} + 12 = 4$

$\sqrt{64} + 12 = 4$

$8 + 12 = 4$

$20 \neq 4$

So -12 is an extraneous root

Putting $x = 7$ in the equation (i) we have

$\sqrt{3x+100} - x = 4$

$\sqrt{3(7)+100} - 7 = 4$

$\sqrt{21+100} - 7 = 4$

$\sqrt{121} - 7 = 4$

$11 - 7 = 4$

$4 = 4$ which is true

So, the solution set is $\{7\}$

Q.5 $\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

Solution: $\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$ ...(i)

Squaring on both sides

$$\left(\sqrt{x+5}+\sqrt{x+21}\right)^2=\left(\sqrt{x+60}\right)^2$$

$$\left(\sqrt{x+5}\right)^2+\left(\sqrt{x+21}\right)^2+2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right)=x+60$$

$$x+5+x+21+2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right)=x+60$$

$$2x+26+2\sqrt{x^2+21x+5x+105}=x+60$$

$$2\sqrt{x^2+26x+105}=x+60-2x-26$$

$$2\sqrt{x^2+26x+105}=-x+34$$

$$2\sqrt{x^2+26x+105}=34-x$$

Again squaring both sides

$$\left(2\sqrt{x^2+26x+105}\right)^2=(34-x)^2$$

$$4(x^2+26x+105)=(34)^2+x^2-2(34)(x)$$

$$4x^2+104x+420=1156+x^2-68x$$

$$4x^2+104x+420-1156-x^2+68x=0$$

$$3x^2+172x-736=0$$

$$a=3\ ,\ b=172\ ,\ c=-736$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-172\pm\sqrt{(172)^2-4(3)(-736)}}{2(3)}$$

$$x=\frac{-172\pm\sqrt{29584+8832}}{6}$$

$$x=\frac{-172\pm\sqrt{38416}}{6}$$

$$x=\frac{-172\pm196}{6}$$

$$x=\frac{-172-196}{6}\text{ or }x=\frac{-172+196}{6}$$

$$x=\frac{-368}{6}\quad\text{or}\quad x=\frac{24}{6}$$

$$x=\frac{-184}{3}\quad\text{or}\quad x=4$$

Checking:

Putting $x=4$ in equation (i)

$$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$$

$$\sqrt{4+5}+\sqrt{4+21}=\sqrt{4+60}$$

$$\sqrt{9}+\sqrt{25}=\sqrt{64}$$

$$3+5=8$$

$$8=8 \qquad \text{which is true}$$

Putting $x=\frac{-184}{3}$

$$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$$

$$\sqrt{\frac{-184}{3}+5}+\sqrt{\frac{-184}{3}+21}=\sqrt{\frac{-184}{3}+60}$$

$$\sqrt{\frac{-184+15}{3}}+\sqrt{\frac{-184+63}{3}}=\sqrt{\frac{-184+180}{3}}$$

$$\sqrt{\frac{-169}{3}}+\sqrt{\frac{-121}{3}}=\sqrt{\frac{-4}{3}}$$

$$\sqrt{\frac{-1\times169}{3}}+\sqrt{\frac{-1\times121}{3}}=\sqrt{\frac{-1\times4}{3}}$$

$$\frac{13i}{\sqrt{3}}+\frac{11i}{\sqrt{3}}=\frac{2i}{\sqrt{3}} \qquad \because i=\sqrt{-1}$$

$$\frac{13i+11i}{\sqrt{3}}=\frac{2i}{\sqrt{3}}$$

$$\frac{24i}{\sqrt{3}}=\frac{2i}{\sqrt{3}} \qquad \text{Which is not true}$$

As $x=\frac{-184}{3}$ is extraneous root

so solution set is $\{4\}$

Q.6 $\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$

Solution: $\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$ ......(i)

Squaring both sides

$$\left(\sqrt{x+1}+\sqrt{x-2}\right)^2=\left(\sqrt{x+6}\right)^2$$

$$\left(\sqrt{x+1}\right)^2+\left(\sqrt{x-2}\right)^2+2\left(\sqrt{x+1}\right)\left(\sqrt{x-2}\right)=x+6$$

$$x+1+x-2+2\sqrt{(x+1)(x-2)}=x+6$$

$$2x-1+2\sqrt{x^2-2x+1x-2}=x+6$$

$$2\sqrt{x^2-x-2}=x+6-2x+1$$

$$2\sqrt{x^2-x-2}=7-x$$

Again squaring both sides

$$\left(2\sqrt{x^2-x-2}\right)^2 = (7-x)^2$$
$$4(x^2-x-2) = (7)^2 + (x)^2 - 2(7)(x)$$
$$4x^2-4x-8 = 49+x^2-14x$$
$$4x^2-x^2-4x+14x-8-49=0$$
$$3x^2+10x-57=0$$
$$3x^2+19x-9x-57=0$$
$$x(3x+19)-3(3x+19)=0$$
$$(3x+19)(x-3)=0$$
$$3x+19=0 \quad \text{or} \quad x-3=0$$
$$3x=-19 \quad \text{or} \quad x=3$$
$$x=\frac{-19}{3}$$

Checking

Putting $x=3$ in the equation (i)

$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$$
$$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$$
$$\sqrt{4}+\sqrt{1}=\sqrt{9}$$
$$2+1=3$$
$$3=3 \quad \text{which is true}$$

Putting $x=\frac{-19}{3}$ in the equation (i)

$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$$
$$\sqrt{\frac{-19}{3}+1}+\sqrt{\frac{-19}{3}-2}=\sqrt{\frac{-19}{3}+6}$$
$$\sqrt{\frac{-19+3}{3}}+\sqrt{\frac{-19-6}{3}}=\sqrt{\frac{-19+18}{3}}$$
$$\sqrt{\frac{-1\times16}{3}}+\sqrt{\frac{-1\times25}{3}}=\sqrt{\frac{-1\times1}{3}}$$
$$\frac{4i}{\sqrt{3}}+\frac{5i}{\sqrt{3}}=\frac{1i}{\sqrt{3}} \quad \because i=\sqrt{-1}$$
$$\frac{4i+5i}{\sqrt{3}}=\frac{1i}{\sqrt{3}}$$
$$\frac{9i}{\sqrt{3}}=\frac{1i}{\sqrt{3}} \quad \text{which is not true}$$

As $x=\frac{-19}{3}$ is an extraneous root

So the solution set is $\{3\}$

Q.7 $\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

Solution: $\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$ ......(i)

Squaring both sides

$$\left(\sqrt{11-x}-\sqrt{6-x}\right)^2=\left(\sqrt{27-x}\right)^2$$
$$\left(\sqrt{11-x}\right)^2+\left(\sqrt{6-x}\right)^2-2\left(\sqrt{11-x}\right)\left(\sqrt{6-x}\right)=27-x$$
$$11-x+6-x-2\sqrt{(11-x)(6-x)}=27-x$$
$$17-2x-2\sqrt{66-11x-6x+x^2}=27-x$$
$$-2\sqrt{66-17x+x^2}=27-x-17+2x$$
$$-2\sqrt{66-17x+x^2}=10+x$$

Again squaring both sides

$$\left(-2\sqrt{66-17x+x^2}\right)^2=(10+x)^2$$
$$4(66-17x+x^2)=(10)^2+(x)^2+2(10)(x)$$
$$264-68x+4x^2=100+x^2+20x$$
$$4x^2-x^2-68x-20x+264-100=0$$
$$3x^2-88x+164=0$$

By applying quadratic formula

$$a=3 \quad , \quad b=-88 \quad , \quad c=164$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-88)\pm\sqrt{(-88)^2-4(3)(164)}}{2(3)}$$
$$x=\frac{88\pm\sqrt{7744-1968}}{6}$$
$$x=\frac{88\pm\sqrt{5776}}{6}$$
$$x=\frac{88-76}{6} \quad \text{or} \quad x=\frac{88+76}{6}$$
$$x=\frac{12}{6} \quad \text{or} \quad x=\frac{164}{6}$$
$$x=2 \quad \text{or} \quad x=\frac{82}{3}$$

Checking

Putting $x = 2$ in the equation (i)

$$\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x}$$
$$\sqrt{11-2}+\sqrt{6-2}=\sqrt{27-2}$$
$$\sqrt{9}+\sqrt{4}=\sqrt{25}$$
$$3+2=5$$
$$5=5 \quad \text{which is true}$$

Putting $x=\frac{82}{3}$ in the equation (i)

$$\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x}$$
$$\sqrt{11-\frac{82}{3}}+\sqrt{6-\frac{82}{3}}=\sqrt{27-\frac{82}{3}}$$
$$\sqrt{\frac{33-82}{3}}+\sqrt{\frac{18-82}{3}}=\sqrt{\frac{81-82}{3}}$$
$$\sqrt{\frac{-1\times 49}{3}}+\sqrt{\frac{-1\times 64}{3}}=\sqrt{\frac{-1\times 1}{3}}$$
$$\frac{7i}{\sqrt{3}}+\frac{8i}{\sqrt{3}}=\frac{i}{\sqrt{3}} \qquad \because i=\sqrt{-1}$$
$$\frac{7i+8i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$$
$$\frac{15i}{\sqrt{3}}=\frac{i}{\sqrt{3}} \quad \text{which is not true}$$

As $\frac{82}{3}$ is an extraneous root

So, the solution set is $\{2\}$

Q.8 $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

Solution: $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$ ....(i)

Squaring both sides

$$\left(\sqrt{4a+x}-\sqrt{a-x}\right)^2=\left(\sqrt{a}\right)^2$$
$$\left(\sqrt{4a+x}\right)^2+\left(\sqrt{a-x}\right)^2-2\left(\sqrt{4a+x}\right)\left(\sqrt{a-x}\right)=a$$
$$4a+\cancel{x}+a-\cancel{x}-2\sqrt{(4a+x)(a-x)}=a$$
$$5a-2\sqrt{4a^2-4ax+ax-x^2}=a$$
$$-2\sqrt{4a^2-3ax-x^2}=a-5a$$
$$-2\sqrt{4a^2-3ax-x^2}=-4a$$
$$\sqrt{4a^2-3ax-x^2}=\frac{-\cancel{4}^{2}a}{-\cancel{2}}$$
$$\sqrt{4a^2-3ax-x^2}=2a$$

Again squaring both sides

$$\left(\sqrt{4a^2-3ax-x^2}\right)^2=(2a)^2$$
$$4a^2-3ax-x^2=4a^2$$
$$4a^2-3ax-x^2=4a^2$$
$$4\cancel{a^2}-4\cancel{a^2}=3ax+x^2$$
$$\Rightarrow \quad 3ax+x^2=0$$
$$x(3a+x)=0$$
$$x=0 \quad \text{or} \quad 3a+x=0$$
$$x=0 \quad \text{or} \quad x=-3a$$

Checking:

Putting $x=0$ in the equation (i)

$$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$$
$$\sqrt{4a+0}-\sqrt{a-0}=\sqrt{a}$$
$$\sqrt{4a}-\sqrt{a}=\sqrt{a}$$
$$2\sqrt{a}-\sqrt{a}=\sqrt{a}$$
$$\sqrt{a}=\sqrt{a} \quad \text{which is true}$$

Putting $x=-3a$ in the equation (i)

$$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$$
$$\sqrt{4a-3a}-\sqrt{a+3a}=\sqrt{a}$$
$$\sqrt{a}-\sqrt{4a}=\sqrt{a}$$
$$\sqrt{a}-2\sqrt{a}=\sqrt{a}$$
$$-\sqrt{a}=\sqrt{a} \quad \text{which is not true}$$

As $-3a$ is extraneous root

So, the solution set is $\{0\}$

Q.9 $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$

Solution: $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$ ......(i)

Let $x^2+x=y$ ....(ii)

Put it in equation (i)

$$\sqrt{y+1}-\sqrt{y-1}=1$$

Squaring both sides

$$\left(\sqrt{y+1}-\sqrt{y-1}\right)^2=(1)^2$$

$$\left(\sqrt{y+1}\right)^2+\left(\sqrt{y-1}\right)^2-2\left(\sqrt{y+1}\right)\left(\sqrt{y-1}\right)=1$$

$$y+1+y-1-2\sqrt{(y+1)(y-1)}=1$$

$$2y-2\sqrt{y^2-1}=1$$

$\Rightarrow$ $2y-1=2\sqrt{y^2-1}$

Again squaring both sides

$$(2y-1)^2=\left(2\sqrt{y^2-1}\right)^2$$

$$(2y)^2+(1)^2-2(2y)(1)=4\left(y^2-1\right)$$

$$4y^2+1-4y=4y^2-4$$

$$4y^2+1-4y=4y^2-4$$

$$1-4y=-4$$

$$1+4=4y$$

$$5=4y$$

$\Rightarrow$ $4y=5$

From equation (ii) Put $y=x^2+x$

$$4y=5$$

$$4(x^2+x)=5$$

$$4x^2+4x=5$$

$$4x^2+4x-5=0$$

Solving by quadratic formula

$a=4$, $b=4$, $c=-5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$$

$$x=\frac{-4\pm\sqrt{16+80}}{8}$$

$$x=\frac{-4\pm\sqrt{96}}{8}$$

$$x=\frac{-4\pm\sqrt{16\times6}}{8}$$

$$x=\frac{-4\pm4\sqrt{6}}{8}$$

$$x=\frac{4\left(-1\pm\sqrt{6}\right)}{8_2}$$

$$x=\frac{-1\pm\sqrt{6}}{2}$$

The solution set is $\left\{\frac{-1\pm\sqrt{6}}{2}\right\}$

Q.10 $\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$

Solution: $\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$ ....(i)

Let $x^2+3x=y$ ....(ii)

Put it in equation (i)

$$\sqrt{y+8}+\sqrt{y+2}=3$$

Squaring on both sides

$$\left(\sqrt{y+8}+\sqrt{y+2}\right)^2=(3)^2$$

$$\left(\sqrt{y+8}\right)^2+\left(\sqrt{y+2}\right)^2+2\left(\sqrt{y+8}\right)\left(\sqrt{y+2}\right)=9$$

$$y+8+y+2+2\sqrt{y^2+2y+8y+16}=9$$

$$2y+10+2\sqrt{y^2+10y+16}=9$$

$$2\sqrt{y^2+10y+16}=9-2y-10$$

$$2\sqrt{y^2+10y+16}=-2y-1$$

$$2\sqrt{y^2+10y+16}=-(2y+1)$$

Again squaring on both sides

$$\left(2\sqrt{y^2+10y+16}\right)^2=\left[-(2y+1)\right]^2$$

$$4(y^2+10y+16)=(2y+1)^2$$

$$4y^2+40y+64=(2y)^2+(1)^2+2(2y)(1)$$
$$4\cancel{y^2}+40y+64=4\cancel{y^2}+1+4y$$
$$40y+64=1+4y$$
$$40y-4y=1-64$$
$$36y=-63$$

From equation (ii) Put $y=x^2+3x$

$$36(x^2+3x)=-63$$
$$36x^2+108x+63=0$$

Solving by quadratic formula

$$a=36\quad,\quad b=108\quad,\quad c=63$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-108\pm\sqrt{(108)^2-4(36)(63)}}{2(36)}$$
$$=\frac{-108\pm\sqrt{11664-9072}}{72}$$
$$=\frac{-108\pm\sqrt{2592}}{72}$$
$$x=\frac{-108\pm\sqrt{4\times4\times9\times9\times2}}{72}$$
$$x=\frac{-108\pm\sqrt{4^2\times9^2\times2}}{72}$$
$$x=\frac{-108\pm4\times9\sqrt{2}}{72}$$
$$x=\frac{-108\pm36\sqrt{2}}{72}$$
$$x=\frac{\cancel{36}\left(-3\pm\sqrt{2}\right)}{\cancel{72}_2}$$
$$x=\frac{-3\pm\sqrt{2}}{2}$$

So, the solution set is $\left\{\dfrac{-3\pm\sqrt{2}}{2}\right\}$

Q.11 $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$

Solution: $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$ ....(i)

Let $x^2+3x=y$ ....(ii)

Put it in equation (i)

$$\sqrt{y+9}+\sqrt{y+4}=5$$

Squaring both sides

$$\left(\sqrt{y+9}+\sqrt{y+4}\right)^2=5^2$$
$$\left(\sqrt{y+9}\right)^2+\left(\sqrt{y+4}\right)^2+2\left(\sqrt{y+9}\right)\left(\sqrt{y+4}\right)=25$$
$$y+9+y+4+2\sqrt{(y+9)(y+4)}=25$$
$$2y+13+2\sqrt{y^2+4y+9y+36}=25$$
$$2\sqrt{y^2+13y+36}=25-2y-13$$
$$2\sqrt{y^2+13y+36}=12-2y$$
$$\cancel{2}\sqrt{y^2+13y+36}=\cancel{2}(6-y)$$
$$\sqrt{y^2+13y+36}=6-y$$

Again taking squaring both sides

$$\left(\sqrt{y^2+13y+36}\right)^2=(6-y)^2$$
$$y^2+13y+36=(6)^2+(y)^2-2(6)(y)$$
$$\cancel{y^2}+13y+\cancel{36}=\cancel{36}+\cancel{y^2}-12y$$
$$13y=-12y$$
$$13y+12y=0$$
$$25y=0$$
$$y=\frac{0}{25}$$
$$y=0$$

From equation (ii) Put $x^2+3x=y$

$$x^2+3x=0$$
$$x(x+3)=0$$
$$x=0 \quad\text{or}\quad x+3=0$$
$$x=-3$$

So, the solution set is $\{-3, 0\}$

MISCELLANEOUS EXERCISE – 1

Q. 1 Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

1. An equation, which remains unchanged when x is replaced by $\frac{1}{x}$ is called a/an……
   (a) Exponential equation
   (b) Reciprocal equation
   (c) Radical equation
   (d) None of these

2. The solution set of equation $4x^2-16=0$ is
   (a) $\{\pm4\}$ (b) $\{4\}$
   (c) $\{\pm2\}$ (d) ±2

3. The number of methods to solve a quadratic equation is
   (a) 1 (b) 2
   (c) 3 (d) 4

4. An equation of the form $2^{2x}-3.2^x+5=0$ is called a / an ___ equation.
   (a) Exponential (b) Radical
   (c) Reciprocal (d) None of these

5. Two linear factors of $x^2-15x+56$ are
   (a) $(x-7)$ and $(x+8)$
   (b) $(x+7)$ and $(x-8)$
   (c) $(x-7)$ and $(x-8)$
   (d) $(x+7)$ and $(x+8)$

6. Standard form of quadratic equation is _______
   (a) $bx+c=0, b\neq0$
   (b) $ax^2+bx+c=0, a\neq0$
   (c) $ax^2=bx, a\neq0$
   (d) $ax^2=0, a\neq0$

7. The solution set of the equation $x^2-9=0$ is
   (a) $\{\pm3\}$ (b) $\{3\}$
   (c) $\{-3\}$ (d) $\{9\}$

8. The number of terms in a standard quadratic equation $ax^2+bx+c=0$ is ___
   (a) 1 (b) 2
   (c) 3 (d) 4

9. The solution set of equation $2+9x=5x^2$ is…….
   (a) $\left\{\frac{-1}{5},2\right\}$ (b) $\left\{\frac{+1}{5},2\right\}$
   (c) $\left\{\frac{1}{5},-2\right\}$ (d) $\left\{\frac{-1}{5},-2\right\}$

10. The solution set of equation $x^2-16=0$ is……..
   (a) $\{\pm4\}$ (b) $\{+4\}$
   (c) $\{-4\}$ (d) None of these

11. The quadratic formula is
   (a) $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ (b) $\frac{b\pm\sqrt{b^2-4ac}}{2a}$
   (c) $\frac{-b\pm\sqrt{b^2+4ac}}{2a}$ (d) $\frac{b\pm\sqrt{b^2+4ac}}{2a}$

12. An equation of the type $3^x+3^{2-x}+6=0$ is a/an…..
   (a) Exponential equation
   (b) Reciprocal equation
   (c) Radical equation
   (d) None of these

13. If a=0, in $ax^2+bx+c=0$, then it reduces to…………
   (a) pure quadratic equation
   (b) linear equation
   (c) quadratic equations
   (d) exponential equation

14. How many linear factors a quadratic equation has?
   (a) 1 (b) 2
   (c) 3 (d) 4

15. An equation of the form $2x^4-3x^3+7x^2-3x+2=0$ is called a/an
   (a) Reciprocal equation
   (b) Radical equation
   (c) Exponential equation
   (d) None of these

16. The solution set of equation $5x^2=30x$ is…….
   (a) $\{5, 30\}$ (b) $\{0,6\}$
   (c) $\{0,-6\}$ (d) $\{5,0\}$

17. **The standard form of the quadratic equation is $ax^2+bx+c=0$ where a, b, c are ....**
(a) Irrational numbers
(b) Rational numbers
(c) Real numbers
(d) Whole numbers

18. **The solution set of $25x^2-1=0$ is**
(a) $\left\{\pm\frac{1}{5}\right\}$ (b) $\left\{-\frac{1}{5}\right\}$
(c) $\left\{+\frac{1}{5}\right\}$ (d) None of these

19. **Cancellation of x on both sides of $5x^2=30x$ means**
(a) the loss of one root
(b) no loss of any root
(c) gain of one root
(d) undefined solution

20. **If $2^x=1$, then x = ...**
(a) 0 (b) 1
(c) 2 (d) $\frac{1}{2}$

21. **What should be done to make the co-efficient of x^2 equal to 1 in $3x^2+7x=0$?**
(a) multiply the equation by $\frac{1}{3}$
(b) divide the equation by $\frac{1}{3}$
(c) add $\frac{1}{3}$ in both sides
(d) subtract $\frac{1}{3}$ from both sides

22. **Sentences involving the Sign........ between two algebraic expressions are called equations**
(a) < (b) ≥
(c) = (d) < or >

23. **The solution set of equation $x^2-x-2=0$ is......**
(a) {2, 1} (b) {−2, 1}
(c) {2, −1} (d) {−2, −1}

24. **An equation of type $x^{\cdot}+x^{-}+x^{-}+x+1=0$ is called a/ an ......equation.**
(a) Radical (b) Reciprocal
(c) Exponential (d) None of these

25. **Solve the equation $5^{1+x}+5^{1-x}=26$**
(a) {1} (b) {±1}
(c) {2} (d) {±2}

26. **The value of variable of an equation not satisfying the equation is called.**
(a) root (b) extraneous root
(c) exponent (d) solution set

27. **The number of roots of a quadratic equation is:**
(a) 1 (b) 2
(c) 3 (d) 4

28. **The solution set of equation $x^2-7x+6=0$ is......**
(a) {1, 6} (b) {−1, −6}
(c) {−1, 6} (d) { 1, −6}

29. **What should be done to make the co-efficient of x^2 equal to 1, in $7x^2+2x-1=0$?**
(a) multiply the equation by 7
(b) divide the equation by 7
(c) add 7 in both sides
(d) subtract 7 from both sides

30. **If b=0 in a quadratic equation $ax^2+bx+c=0$, then it is called.**
(a) Pure quadratic equation
(b) Linear equation
(c) Quadratic equation
(d) Exponential equation

31. **What is the degree of quadratic equation?**
(a) 1 (b) 2
(c) 3 (d) 4

32. **The cancellation of x on both sides of the equation of the type $ax^2=bx$ means the loss of one root. That root is always equal to.....**
(a) 0 (b) 1
(c) a (d) b

33. **If $y=2^x$ and 8y =1, then, x =...**
(a) 8 (b) $\frac{1}{8}$

(c) 3 (d) −3

34. **The solution set of equation $3x^2 + 4x = 5$ is……..**

(a) $\left\{\frac{-2\pm\sqrt{19}}{3}\right\}$ (b) $\left\{\frac{2\pm\sqrt{19}}{3}\right\}$

(c) $\left\{\frac{4\pm\sqrt{19}}{3}\right\}$ (d) None of these

35. **If $y=x^{-1}$ and $3y=5$, the value of x is ….**

(a) $\frac{5}{3}$ (b) $\frac{-5}{3}$

(c) $\frac{-3}{5}$ (d) $\frac{3}{5}$

ANSWER KEY

1.	b	2.	c	3.	c	4.	a	5.	c
6.	b	7.	a	8.	c	9.	a	10.	a
11.	a	12.	a	13.	b	14.	b	15.	a
16.	b	17.	c	18.	a	19.	a	20.	a
21.	a	22.	c	23.	c	24.	b	25.	b
26.	b	27.	b	28.	a	29.	b	30.	a
31.	b	32.	a	33.	d	34.	a	35.	d

Q.2. Write short answers of the following questions.

(i) Solve $x^2 + 2x - 2 = 0$

Solution: $x^2 + 2x - 2 = 0$

$ax^2 + bx + c = 0$

⇒ $a = 1$, $b = 2$, $c = -2$

We know that

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-2\pm\sqrt{(2)^2-4(1)(-2)}}{2(1)}$$

$$x=\frac{-2\pm\sqrt{4+8}}{2}$$

$$x=\frac{-2\pm\sqrt{12}}{2}$$

$$x=\frac{-2\pm\sqrt{4\times3}}{2}$$

$$x=\frac{-2\pm2\sqrt{3}}{2}$$

$$x=\frac{\cancel{2}(-1\pm\sqrt{3})}{\cancel{2}}$$

$$x=-1\pm\sqrt{3}$$

$$\text{S.S} = \{-1\pm\sqrt{3}\}$$

(ii) Solve by factorization $5x^2 = 15x$

Solution:

$5x^2 = 15x$

$5x^2 - 15x = 0$

$5x(x - 3) = 0$

$5x(x - 3) = 0$

$5x = 0$ or $x - 3 = 0$

$x = \frac{0}{5}$ or $x = 3$

$x = 0$ or $x = 3$

Solution set is $\{0, 3\}$

(iii) Write in standard form $\frac{1}{x+4}+\frac{1}{x-4}=3$

Solution: $\frac{1}{x+4}+\frac{1}{x-4}=3$

$$\frac{(x-4)+(x+4)}{(x+4)(x-4)}=3$$

$$\frac{x-\cancel{4}+x+\cancel{4}}{(x)^2-(4)^2}=3$$

$$\frac{2x}{x^2-16}=3$$

$2x = 3(x^2 - 16)$

$2x = 3x^2 - 48$

$0 = 3x^2 - 2x - 48$

$3x^2 - 2x - 48 = 0$

(iv) Write the name of methods for solving a quadratic equation.

Ans. Following three methods are used for solving the quadratic equation.

(i) Factorization method
(ii) Completing square method
(iii) Quadratic formula method

(v) Solve $\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$

Solution:

$$\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$$

Taking square root

$$\sqrt{\left(2x-\frac{1}{2}\right)^2} \pm\sqrt{\frac{9}{4}}$$

$$2x-\frac{1}{2}=\pm\frac{3}{2}$$

$$2x=\pm\frac{3}{2}+\frac{1}{2}$$

$$2x=\frac{\pm3+1}{2}$$

$$x=\frac{\pm3+1}{2\times2}$$

$$x=\frac{\pm3+1}{4}$$

$$\Rightarrow \quad x=\frac{3+1}{4} \quad \text{or} \quad x=\frac{-3+1}{4}$$

$$x=\frac{4}{4} \quad \text{or} \quad x=\frac{-2}{4}$$

$$x=1 \quad \text{or} \quad x=\frac{-1}{2}$$

The Solution set is $\left\{1,\frac{-1}{2}\right\}$

(vi) Solve $\sqrt{3x+18}=x$

Solution:

$$\sqrt{3x+18}=x \ \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Taking square of both sides

$$(\sqrt{3x+18})^2=(x)^2$$

$$3x+18=x^2$$

$$\Rightarrow \quad x^2-3x-18=0$$

$$x^2-6x+3x-18=0$$

$$x(x-6)+3(x-6)=0$$

$$(x-6)(x+3)=0$$

$$x-6=0 \quad \text{or} \quad x+3=0$$

$$x=6 \quad \text{or} \quad x=-3$$

Checking

Put x = 6 in equation (i)

$$\sqrt{3x+18}=x$$

$$\sqrt{3(6)+18}=6$$

$$\sqrt{18+18}=6$$

$$\sqrt{36}=6$$

6 = 6 which is true

Put x = −3 in given equation

$$\sqrt{3x+18}=x$$

$$\sqrt{3(-3)+18}=-3$$

$$\sqrt{-9+18}=-3$$

$$\sqrt{9}=-3$$

3 = −3 which is not true

As −3 is an extraneous root

So, the solution set is {6}

(vii) Define quadratic equation

Quadratic Equation

An equation which contains the square of the unknown (variable) quantity, but no higher power, is called a quadratic equation. Standard form of quadratic equation in one variable is $ax^2+bx+c=0$, where $a, b, c \in R$ and $a \neq 0$.

(viii) Define reciprocal equation

Reciprocal Equation

An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\frac{1}{x}$.

Example

(i) $ax^4+bx^3+cx^2+bx+a=0$ OR

(ii) $a\left(x^2+\frac{1}{x^2}\right)+b\left(x+\frac{1}{x}\right)+c=0$

(ix) Define exponential equation:

Exponential Equation:

An equation in which variable occurs in exponent is called exponential equation.

Example: $2^x-8=0$

(x) **Define radical equation**

Radical Equation

An equation involving expression of variable under the radical sign is called radical equation.

Examples: (i) $\sqrt{x+3}=1$

(ii) $\sqrt{x-1}=\sqrt{x-2}$

Q.3 Fill in the blanks:

i. The standard form of the quadratic equation is__________.

ii. The number of methods to solve a quadratic equation are……

iii. The name of the method to derive a quadratic formula is ……

iv. The solution of the equation $ax^2+bx+c=0$, $a \neq 0$___________.

v. The solution set of $25x^2-1=0$ is_______.

vi. An equation of the form $2^{2x}-3.2^x+5=0$ is called a/an _______equation.

vii. The solution set of the equation $x^2-9=0$ is_________.

viii. An equation of the type $x^4+x^3+x^2+x+1=0$ is called an_____ equation.

ix. A root of an equation, which do not satisfy the equation is called _______root.

x. An equation involving expression of the variable under_______ is called radical equation.

Answers:

i. $ax^2+bx+c=0, \quad a \neq 0$

ii. Three

iii. Completing square

iv. $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

v. $\left\{\pm\dfrac{1}{5}\right\}$

vi. Exponential

vii. $\{\pm 3\}$

viii. Reciprocal

ix. Extraneous

x. Radical Sign

EXERCISE 2.1

Q.1 Find the discriminant of the following given quadratic equation.

Solutions:

(i) $2x^2+3x-1=0$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=2$, $b=3$, $c=-1$

Disc. $=b^2-4ac$

$=(3)^2-4(2)(-1)$

$=9+8$

$=17$

(ii) $6x^2-8x+3=0$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=6$, $b=-8$, $c=3$

Disc. $=b^2-4ac$

$=(-8)^2-4(6)(3)$

$=64-72$

$=-8$

(iii) $9x^2-30x+25=0$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=9$, $b=-30$, $c=25$

Dis $=b^2-4ac$

$=(-30)^2-4(9)(25)$

$=900-900$

$=0$

(iv) $4x^2-7x-2=0$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=4$, $b=-7$, $c=-2$

Disc $=b^2-4ac$

$=(-7)^2-4(4)(-2)$

$=49+32$

$=81$

Q.2 Find the nature of the roots of the following given quadratic equations and verify the result by solving the equations

(i) $x^2-23x+120=0$

Solutions:

$x^2-23x+120=0$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=1$, $b=-23$, $c=120$

Disc $=b^2-4ac$

$=(-23)^2-4(1)(120)$

$=529-480$

$=49$

$=(7)^2$

As Disc. is positive and perfect square, therefore roots of equation are real, rational and un equal.

Verification.

Now solving the Equation

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-23)\pm\sqrt{(-23)^2-4(1)(120)}}{2(1)}$$

$$x=\frac{23\pm\sqrt{529-480}}{2}$$

$$x=\frac{23\pm\sqrt{49}}{2}$$

$$x=\frac{23\pm7}{2}$$

$x=\frac{23+7}{2}$ or $x=\frac{23-7}{2}$

$x=\frac{30}{2}$ or $x=\frac{16}{2}$

$x=15$ or $x=8$

Thus roots are real, rational and unequal.

(ii) $\mathbf{2x^2 + 3x + 7 = 0}$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 2 \quad , b = 3 \quad , c = 7$

$$\text{Disc} = b^2 - 4ac$$
$$= (3)^2 - 4(2)(7)$$
$$= 9 - 56$$
$$= -47 < 0$$

As Disc. is negative, therefore roots of the equation are imaginary(complex conjugates)

Verification

We verify the results by solving the equation.

$$2x^2 + 3x + 7 = 0$$

$$a = 2 \ , \ b = 3 \ , \ c = 7$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(7)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{9 - 56}}{4}$$

$$x = \frac{-3 \pm \sqrt{-47}}{4}$$

Thus roots are imaginary.

(iii) $\mathbf{16x^2 - 24x + 9 = 0}$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 16 \quad , \quad b = -24 \quad , \quad c = 9$

$$\text{Disc} = b^2 - 4ac$$
$$= (-24)^2 - 4(16)(9)$$
$$= 576 - 576$$
$$= 0$$

As Disc. = 0 therefore roots of equation are rational (real) and equal.

Verification:

We verify the result by solving the equation. Here

$$a = 16 \ , \ b = -24 \ , \ c = 9$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(16)(9)}}{2(16)}$$

$$x = \frac{24 \pm \sqrt{576 - 576}}{32}$$

$$x = \frac{24 \pm \sqrt{0}}{32}$$

$$x = \frac{24 \pm 0}{32}$$

$$x = \frac{24 + 0}{32} \quad \text{or} \quad x = \frac{24 - 0}{32}$$

$$x = \frac{24}{32} \quad \text{or} \quad x = \frac{24}{32}$$

$$x = \frac{3}{4} \quad \text{or} \quad x = \frac{3}{4}$$

Thus roots are rational (real) and equal.

(iv) $\mathbf{3x^2 + 7x - 13 = 0}$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 3 \ , \ b = 7 \ , \ c = -13$

$$\text{Disc} = b^2 - 4ac$$
$$= (7)^2 - 4\,(3)\,(-13)$$
$$= 49 + 156$$
$$= 205$$

Therefore roots of equation are real, irrational, and unequal.

Verification:

We verify the result by solving the equation.

$$\mathbf{3x^2 + 7x - 13 = 0}$$

$$a=3 \ , \ b=7 \ , \ c = -13$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(3)(-13)}}{2(3)}$$

$$x = \frac{-7 \pm \sqrt{49 + 156}}{6}$$

$$x = \frac{-7 \pm \sqrt{205}}{6}$$

Thus roots are real, irrational and unequal.

Q.3 For what value of k, the expression $k^2x^2 + 2(k+1)x + 4$ is perfect square.

Solution:

$a = k^2$, $b = 2(k+1)$, $c = 4$

$$b^2 - 4ac = [2(k+1)]^2 - 4(k^2)(4)$$
$$= 2^2(k+1)^2 - 16k^2$$
$$= 4(k^2 + 2k + 1) - 16k^2$$
$$= 4k^2 + 8k + 4 - 16k^2$$
$$= -12k^2 + 8k + 4$$

As given that expression is perfect square therefore

$$Disc = 0$$
$$-12k^2 + 8k + 4 = 0$$
$$-4(3k^2 - 2k - 1) = 0$$
$$\Rightarrow \quad 3k^2 - 2k - 1 = 0 \quad (\because -4 \neq 0)$$
$$3k^2 - 3k + k - 1 = 0$$
$$3k(k-1) + 1(k-1) = 0$$
$$(k-1)(3k+1) = 0$$

$k - 1 = 0$ or $3k + 1 = 0$

$k = 0 + 1$ or $3k = 0 - 1$

$k = 1$ or $k = \frac{-1}{3}$

So values of k are 1 and $\frac{-1}{3}$

Q.4 Find the value of K, if the roots of the following equations are equal.

Solution:

(i) $(2k-1)x^2 + 3k\,x + 3 = 0$

As $ax^2 + b\,x + c = 0$

$a = 2k - 1$, $b = 3k$, $c = 3$

$$\text{Disc.} = b^2 - 4ac$$
$$= (3k)^2 - 4(2k-1)(3)$$
$$= 9k^2 - 12(2k-1)$$
$$= 9k^2 - 24k + 12$$

As the roots of given equation are equal, So

$$\text{Disc.} = 0$$
$$9k^2 - 24k + 12 = 0$$
$$3(3k^2 - 8k + 4) = 0$$
$$3k^2 - 8k + 4 = 0 \quad (\because 3 \neq 0)$$
$$3k^2 - 6k - 2k + 4 = 0$$
$$3k(k-2) - 2(k-2) = 0$$
$$(k-2)(3k-2) = 0$$

Either $k - 2 = 0$ or $3k - 2 = 0$

$k = 2$ or $3k = 2$

or $k = \frac{2}{3}$

So values of k are 2 and $\frac{2}{3}$

(ii) $x^2 + 2(k+2)x + (3k+4) = 0$

Solution:

As $ax^2 + bx + c = 0$

$a = 1$, $b = 2(k+2)$, $c = 3k + 4$

$$\text{Disc.} = b^2 - 4ac$$
$$= [2(k+2)]^2 - 4(1)(3k+4)$$
$$= 4(k+2)^2 - 4(3k+4)$$
$$= 4(k^2 + 4k + 4) - 12k - 16$$
$$= 4k^2 + 16k + \cancel{16} - 12k - \cancel{16}$$
$$= 4k^2 + 4k$$
$$= 4k(k+1)$$

As roots of given equation are equal, so

$$\text{Disc.} = 0$$
$$4k(k+1) = 0$$

Either $4k = 0$ or $k + 1 = 0$

As $4 \neq 0$ or $k = -1$

$\Rightarrow k = 0$

So values of k are 0 and -1

(iii) $(3k+2)x^2 - 5(k+1)x + (2k+3) = 0$

Solution:

As $ax^2 + bx + c = 0$

$a = (3k+2)$ $b = -5(k+1)$, $c = 2k + 3$

$$\text{Disc} = b^2 - 4ac$$
$$= [-5(k+1)]^2 - 4(3k+2)(2k+3)$$
$$= (-5)^2(k+1)^2 - 4(6k^2 + 9k + 4k + 6)$$
$$= 25\left[k^2 + 2(k)(1) + 1^2\right] - 4(6k^2 + 13k + 6)$$

$= 25k^2 + 50k + 25 - 24k^2 - 52k - 24$
$= k^2 - 2k + 1$
$= k^2 - k - k + 1$
$= k(k-1) - 1(k-1)$
$= (k-1)(k-1)$

As root of given equation are equal
Disc = 0

$(k-1)(k-1) = 0$
$k - 1 = 0$ or $k - 1 = 0$
$k = 1$ or $k = 1$

So values of k is 1

Q.5 Show that the equation $x^2 + (mx + c)^2 = a^2$ has equal roots if $c^2 = a^2(1 + m^2)$

Solution: We have given

$x^2 + (mx + c)^2 = a^2$
$x^2 + (mx)^2 + (c)^2 + 2(mx)(c) = a^2$
$x^2 + m^2x^2 + c^2 + 2mcx - a^2 = 0$
$(1 + m^2)x^2 + 2mc\,x + c^2 - a^2 = 0$
$Ax^2 + Bx + C = 0$
$A = (1 + m^2), B = 2mc, C = c^2 - a^2$

Disc $= B^2 - 4AC$
$= (2mc)^2 - 4(1 + m^2)(c^2 - a^2)$
$= 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - a^2m^2)$
$= \cancel{4m^2c^2} - 4c^2 + 4a^2 - \cancel{4m^2c^2} + 4a^2m^2$
$= -4c^2 + 4a^2 + 4a^2m^2$

As roots of the equation are equal,
So Disc = 0

$-4c^2 + 4a^2 + 4a^2m^2 = 0$
$-4\left[c^2 - a^2 - a^2m^2\right] = 0$
As $-4 \neq 0$, so $c^2 - a^2 - a^2m^2 = 0$
$c^2 = a^2 + a^2m^2$
$c^2 = a^2(1 + m^2)$

Hence proved

Q.6 Find the condition that the roots of the equation $(mx + c)^2 - 4ax = 0$ are equal

Solution: We have given

$(mx + c)^2 - 4ax = 0$
$(mx^2) + (c)^2 + 2(mx)(c) - 4ax = 0$
$m^2x^2 + c^2 + 2mcx - 4ax = 0$
$m^2x^2 + (2mc - 4a)x + c^2 = 0$
$Ax^2 + Bx + C = 0$
$A = m^2, B = 2mc - 4a, C = c^2$

Disc. $= B^2 - 4AC$
$= (2mc - 4a)^2 - 4(m^2)(c^2)$
$= (2mc)^2 + (4a)^2 - 2(2mc)(4a) - 4m^2c^2$
$= \cancel{4m^2c^2} + 16a^2 - 16acm - \cancel{4m^2c^2}$
$= 16a^2 - 16acm$
$= 16a(a - mc)$

As root of the equation are equal,
So Disc = 0

$16a(a - mc) = 0$
Either $16a = 0$ or $a - mc = 0$
As $16 \neq 0$ So $a = 0$ or $a = mc$

Thus required condition is a=0 or a=mc

Q.7 If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are equal, then a = 0 or $a^3 + b^3 + c^3 = 3abc$

Solution: We have given

$(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$
$Ax^2 + Bx + C = 0$
$A = (c^2 - ab), B = -2(a^2 - bc), C = (b^2 - ac)$

Disc. $= B^2 - 4AC$
$= \left[-2(a^2 - bc)\right]^2 - 4(c^2 - ab)(b^2 - ac)$
$= (-2)^2(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac)$
$= 4\left[(a^2)^2 + (bc)^2 - 2(a^2)(bc)\right] - 4(b^2c^2 - ac^3 - ab^3 + a^2bc)$
$= 4(a^4 + b^2c^2 - 2a^2bc) - 4(b^2c^2 - ac^3 - ab^3 + a^2bc)$
$= 4a^4 + \cancel{4b^2c^2} - 8a^2bc - \cancel{4b^2c^2} + 4ac^3 + 4ab^3 - 4a^2bc$

$= 4a^4 + 4ac^3 + 4ab^3 - 12a^2bc$

$= 4a(a^3 + b^3 + c^3 - 3abc)$

If roots of given equation are equal, then

Disc. = 0

$4a(a^3 + b^3 + c^3 - 3abc) = 0$

$4a = 0$ or $a^3 + b^3 + c^3 - 3abc = 0$

As $4 \neq 0$, or $\boxed{a^3 + b^3 + c^3 = 3abc}$

So, $a = 0$

Q.8 Show that the roots of the following equations are rational

Solution:

(i) $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$

$Ax^2 + Bx + C = 0$

$A = a(b-c),\ B = b(c-a),\ C = c(a-b)$

Disc. $= B^2 - 4AC$

$= [b(c-a)]^2 - 4[a(b-c)c(a-b)]$

$= b^2(c-a)^2 - 4[ac(b-c)(a-b)]$

$= b^2(c^2 + a^2 - 2ac) - 4ac(ab - b^2 - ca + bc)$

$= b^2c^2 + a^2b^2 - 2ab^2c - 4a^2bc + 4ab^2c + 4c^2a^2 - 4abc^2$

$= a^2b^2 + b^2c^2 + 4c^2a^2 + 2ab^2c - 4abc^2 - 4a^2bc$

$= (ab)^2 + (bc)^2 + (-2ca)^2 + 2(ab)(bc)$
$+ 2(bc)(-2ca) + 2(-2ca)(ab)$

By using formula

$(A+B+C)^2 = A^2 + B^2 + C^2 + 2AB + 2BC + 2CA$

So, $= [(ab) + (bc) + (-2ca)]^2$

$= (ab + bc - 2ca)^2 > 0$

As Disc. is perfect square so the roots of equations are rational.

(ii) $(a+2b)x^2 + 2(a+b+c)x + (a+2c) = 0$

Solution:

$A = a+2b$, $B = 2(a+b+c)$, $C = (a+2c)$

Disc. $= B^2 - 4AC$

$= [2(a+b+c)]^2 - 4(a+2b)(a+2c)$

$= 4[a^2+b^2+c^2+2ab+2bc+2ca] - 4(a^2+2ac+2ab+4bc)$

$= \cancel{4a^2} + 4b^2 + 4c^2 + \cancel{8ab} + 8bc + \cancel{8ca} - \cancel{4a^2} - \cancel{8ac} - \cancel{8ab} - 16bc$

$= 4b^2 + 4c^2 + 8bc - 16bc$

$= 4b^2 + 4c^2 - 8bc$

$= (2b)^2 + (2c)^2 - 2(2b)(2c)$

$= (2b - 2c)^2 > 0$

As Disc. is perfect square therefore roots, are rational (real and unequal)

Q.9. For all values of K, Prove that the Roots, of the Equation.

$x^2 - 2\left(k + \frac{1}{k}\right)x + 3 = 0,\ k \neq 0$ are real

Solution. $a = 1$, $b - 2\left(k + \frac{1}{k}\right)$, $c = 3$

Disc. $= b^2 - 4ac$

$= \left[-2\left(k + \frac{1}{k}\right)\right]^2 - 4(1)(3)$

$= 4\left(k + \frac{1}{k}\right)^2 - 12$

$= 4\left[\left(k + \frac{1}{k}\right)^2 - 3\right]$

$= 4\left[k^2 + \frac{1}{k^2} + 2(k)\left(\frac{1}{k}\right) - 3\right]$

$= 4\left[k^2 + \frac{1}{k^2} + 2 - 3\right]$

$= 4\left[k^2 + \frac{1}{k^2} - 1\right]$

$= 4\left[(k)^2 + \left(\frac{1}{k}\right)^2 - 2 + 1\right]$

$= 4\left[(k)^2 + \left(\frac{1}{k}\right)^2 - 2(k)\left(\frac{1}{k}\right) + 1\right]$

$= 4\left[\left(k - \frac{1}{k}\right)^2 + 1\right] > 0$

As Disc. is positive so roots of the given equation are real.

Q.10. Show that the roots of the equation $(b-c)x^2+(c-a)x+(a-b)=0$ are real.

Solution: $(b-c)x^2+(c-a)x+(a-b)=0$

$Ax^2 + Bx + C = 0$

$A = (b-c)$, $B = (c-a)$, $C = (a-b)$

Disc $= B^2 - 4AC$

$= (c-a)^2 - 4(b-c)(a-b)$

$= (c^2+a^2-2ca) - 4(ab-b^2-ca+bc)$

$= c^2+a^2-2ca-4ab+4b^2+4ca-4bc$

$= c^2+a^2+4b^2+2ca-4bc-4ab$

$= a^2 + 4b^2 + c^2 - 4ab - 4bc + 2ca$

$= (a)^2+(-2b)^2+(c)^2+2(a)(-2b)+2(-2b)(c)+2(c)(a)$

$= (a-2b+c)^2$

Perfect square shows that the roots of the given equation are real.

EXERCISE 2.2

Q.1 Find the cube roots of $-1, 8, -27, 64$.

(i) Cube roots of -1

Solution:

Let $x = (-1)^{\frac{1}{3}}$

$x^3 = -1$

$x^3 + 1 = 0$

$x^3 + (1)^3 = 0$

$\because (a^3 + b^3) = (a+b)(a^2 - ab + b^2)$

$(x+1)\left[x^2 - (x)(1) + 1^2\right] = 0$

$(x+1)(x^2 - x + 1) = 0$

$x + 1 = 0$ or

$\boxed{x = -1}$ or $\boxed{x^2 - x + 1 = 0}$

Now we solve $x^2 - x + 1 = 0$ by formula

$ax^2 + bx + c = 0$

$a = 1,\ b = -1,\ c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2\times 1}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2\times 1}$$

$$= \frac{1 \pm \sqrt{1-4}}{2}$$

$$= \frac{1 \pm \sqrt{-3}}{2}$$

$$= -1\left(\frac{-1 \pm \sqrt{-3}}{2}\right)$$

$$x = -1\left(\frac{-1+\sqrt{-3}}{2}\right)\ ;\ x = -1\left(\frac{-1-\sqrt{-3}}{2}\right)$$

$x = -1\omega$ or $x = -1(\omega^2)$

$x = -\omega$ or $x = -\omega^2$

So, cube roots of -1 are $-1, -\omega$ and $-\omega^2$

(ii) Cube roots of 8

Solution:

Let $x = (8)^{\frac{1}{3}}$

$x^3 = 8$

$x^3 - 8 = 0$

$x^3 - 2^3 = 0$

$\because (a^3 - b^3) = (a-b)(a^2 + ab + b^2)$

$(x-2)\left[x^2 + (x)(2) + 2^2\right] = 0$

$(x-2)(x^2 + 2x + 4) = 0$

$x - 2 = 0$ or $x^2 + 2x + 4 = 0$

$x = 2$

Now we solve $x^2 + 2x + 4 = 0$ by formula

$a = 1\ ,\ b = 2\ ,\ c = 4$

$$x = \frac{-2 \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(4)}}{2\times 1}$$

$$= \frac{-2 \pm \sqrt{4-16}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm \sqrt{4\times(-3)}}{2}$$

$$= \frac{-2 \pm \sqrt{4}\sqrt{-3}}{2}$$

$$= \frac{-2 \pm 2\sqrt{-3}}{2}$$

$$x = \frac{2\left(-1 \pm \sqrt{-3}\right)}{2}$$

$$x = 2\left(\frac{-1+\sqrt{-3}}{2}\right)\ ,\ x = 2\left(\frac{-1-\sqrt{3}}{2}\right)$$

$x = 2\omega\ ,\ x = 2\omega^2$

So cube roots of 8 are $2,\ 2\omega,\ 2\omega^2$

(iii) Cube roots of –27

Solution:

Let $x=(-27)^{\frac{1}{3}}$

$x^3=-27$

$x^3+27=0$

$x^3+3^3=0$

$\because \quad (a^3+b^3)=(a+b)(a^2-ab+b^2)$

$(x+3)\left[x^2-(x)(3)+3^2\right]=0$

Either $(x+3)(x^2-3x+9)=0$

$x+3=0$ or $x^2-3x+9=0$

Now we solve $x^2-3x+9=0$ by formula

$a=1,\ b=-3,\ c=19$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(19)}}{2\times1}$$

$$x=\frac{3\pm\sqrt{9-36}}{2}$$

$$x=\frac{3\pm\sqrt{-27}}{2}$$

$$x=\frac{3\pm\sqrt{(9)(-3)}}{2}$$

$$x=\frac{3\pm3\sqrt{-3}}{2}$$

$$x=-3\left(\frac{-1\pm\sqrt{-3}}{2}\right)$$

$x=-3\left(\frac{-1+\sqrt{-3}}{2}\right)$ or $x=-3\left(\frac{-1-\sqrt{-3}}{2}\right)$

$x=-3\omega$ or $x=-3\omega^2$

So cube roots of –27 are –3 , -3ω and $-3\omega^2$

(iv) Cube roots of 64

Solution:

Let $x=(64)^{\frac{1}{3}}$

$x^3=64$

$x^3-64=0$

$x^3-4^3=0$

$\because \quad a^3-b^3=(a-b)(a^2+ab+b^2)$

$(x-4)\left[x^2+(x)(4)+4^2\right]=0$

$(x-4)(x^2+4x+16)=0$

Either $x-4=0$ or $x^2+4x+16=0$

$x=4$

Now we solve $x^2+4x+16=0$ by formula

$a=1$, $b=4$, $c=16$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{4^2-4\times1\times16}}{2(1)}$$

$$x=\frac{-4\pm\sqrt{16-64}}{2}$$

$$x=\frac{-4\pm\sqrt{-48}}{2}$$

$$x=\frac{-4\pm\sqrt{16(-3)}}{2}$$

$$x=\frac{-4\pm4\sqrt{-3}}{2}$$

$$x=\frac{4\left(-1\pm\sqrt{-3}\right)}{2}$$

Either

$x=4\left(\frac{-1+\sqrt{-3}}{2}\right)$ or $x=4\left(\frac{-1-\sqrt{-3}}{2}\right)$

Here $\omega=\frac{-1+\sqrt{-3}}{2}$, $\omega^2=\frac{-1-\sqrt{-3}}{2}$

Therefore,

$x=4\omega$, or $x=4\omega^2$

So cube roots of 64 are 4 , 4ω *and* $4\omega^2$

Q.2 Evaluate

(i) $(1-\omega-\omega^2)^7$

Solution: $(1-\omega-\omega^2)^7$

$=\left[1-(\omega+\omega^2)\right]^7$

$\because 1+\omega+\omega^2=0$
$\omega+\omega^2=-1$

$=\left[1-(-1)\right]^7$

$=(1+1)^7$

$=2^7$

$=128$

(ii) $(1-3\omega-3\omega^2)^5$

Solution: $(1-3\omega-3\omega^2)^5$

$=\left[1-3(\omega+\omega^2)\right]^5$

$\because 1+\omega+\omega^2=0$
$\omega+\omega^2=-1$

$=\left[1-3(-1)\right]^5$

$=(1+3)^5$

$=4^5=1024$

(iii) $(9+4\omega+4\omega^2)^3$

Solution: $(9+4\omega+4\omega^2)^3$

$=\left[9+4(\omega+\omega^2)\right]^3$

$=\left[9+4(-1)\right]^3 \quad (\because \omega+\omega^2=-1)$

$=\left[9-4\right]^3$

$=5^3$

$=125$

(iv) $(2+2\omega-2\omega^2)(3-3\omega+3\omega^2)$

Solution: $(2+2\omega-2\omega^2)(3-3\omega+3\omega^2)$

$=[2(1+\omega)-2\omega^2][3+3\omega^2-3\omega]$

$=[2(1+\omega)-2\omega^2][3(1+\omega^2)-3\omega]$

$\because 1+\omega+\omega^2=0$
$1+\omega=-\omega^2 \qquad 1+\omega^2=-\omega$

$\therefore \quad =[2(-\omega^2)-2\omega^2][3(-\omega)-3\omega]$

$=(-2\omega^2-2\omega^2)(-3\omega-3\omega)$

$=(-4\omega^2)(-6\omega)$

$=24\omega^3 \qquad$ As $\omega^3=1$

$=24(1)$

$=24$

(v) $\left(-1+\sqrt{-3}\right)^6+\left(-1-\sqrt{-3}\right)^6$

Solution:

$\left(-1+\sqrt{-3}\right)^6+\left(-1-\sqrt{-3}\right)^6 \quad ....(i)$

As $\dfrac{-1+\sqrt{-3}}{2}=\omega$ *and* $\dfrac{-1-\sqrt{-3}}{2}=\omega^2$

$-1+\sqrt{-3}=2\omega$ *and* $-1-\sqrt{-3}=2\omega^2$

Now equation (i) becomes

$=(2\omega)^6+(2\omega^2)^6$

$=2^6\omega^6+2^6\omega^{12}$

$=2^6\left[(\omega^3)^2+(\omega^3)^4\right] \qquad$ As $\omega^3=1$

$=2^6\left[(1)^2+(1)^4\right]$

$=64(1+1)$

$=64(2)$

$=128$

(vi) $\left(\dfrac{-1+\sqrt{-3}}{2}\right)^9+\left(\dfrac{-1-\sqrt{-3}}{2}\right)^9$

Solution:

$\left(\dfrac{-1+\sqrt{-3}}{2}\right)^9+\left(\dfrac{-1-\sqrt{-3}}{2}\right)^9 \quad ......(i)$

As $\dfrac{-1+\sqrt{-3}}{2}=\omega$ *and* $\dfrac{-1-\sqrt{-3}}{2}=\omega^2$

Now equation (i) becomes

$=(\omega)^9+(\omega^2)^9$

$=\omega^9+\omega^{18}$

$=(\omega^3)^3+(\omega^3)^6$

$=(1)^3+(1)^6 \qquad$ As $\omega^3=1$

$=1+1$

$=2$

(vii) $\omega^{37}+\omega^{38}-5$

Solution:

$\omega^{37}+\omega^{38}-5$

$=\omega^{36}\omega+\omega^{36}\omega^2-5$

$=(\omega^3)^{12}\omega+(\omega^3)^{12}\omega^2-5$

$=(1)^{12}\omega+(1)^{12}\omega^2-5 \qquad$ As $\omega^3=1$

$=1\omega+1\omega^2-5$

$=(\omega+\omega^2)-5$

$=(-1)-5 \qquad$ As $1+\omega+\omega^2=0$

$=-1-5=-6 \qquad \omega+\omega^2=-1$

(viii) $\omega^{-13} + \omega^{-17}$

Solution:

$$\omega^{-13} + \omega^{-17}$$

$$= \frac{1}{\omega^{13}} + \frac{1}{\omega^{17}}$$

$$= \frac{1}{\omega^{12}\omega} + \frac{1}{\omega^{15}\omega^2}$$

$$= \frac{1}{(\omega^3)^4\omega} + \frac{1}{(\omega^3)^5\omega^2}$$

$$= \frac{1}{(1)^4\omega} + \frac{1}{(1)^5\omega^2} \qquad \because \omega^3 = 1$$

$$= \frac{1}{\omega} + \frac{1}{\omega^2}$$

$$= \frac{\omega^2 + \omega}{(\omega)(\omega^2)}$$

$$= \frac{-1}{\omega^3} \qquad \because 1 + \omega + \omega^2 = 0, \quad \omega + \omega^2 = -1$$

$$= \frac{-1}{1}$$

$$= -1$$

Q.3. Prove that,

$$x^3 + y^3 = (x + y)(x + \omega y)(x + \omega^2 y) \quad \textbf{02(032)}$$

Solution: Let,

R.H.S. $= (x + y)(x + \omega y)(x + \omega^2 y)$

$$= (x + y)(x^2 + \omega^2 xy + \omega xy + \omega^3 y^2)$$

$$= (x + y)[x^2 + (\omega^2 + \omega)xy + \omega^3 y^2]$$

$\because 1+\omega+\omega^2 = 0 \Rightarrow \omega+\omega^2 = -1$ and $\omega^3 = 1$

$$= (x + y)(x^2 + (-1)xy + 1y^2)$$

$$= (x + y)(x^2 - xy + y^2)$$

Using Formula: $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

$$= x^3 + y^3 = \text{L.H.S.}$$

Q.4. Prove that

$$\mathbf{x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x + \omega y + \omega^2 z)(x + \omega^2 y + \omega z)}$$

Solution:

Let: R.H.S

$$= (x + y + z)(x + \omega y + \omega^2 z)(x + \omega^2 y + \omega z)$$

$$= (x + y + z)(x^2 + \omega^2 xy + \omega xz + \omega yx + \omega^3 y^2 + \omega^2 yz + \omega^2 xz + \omega^4 zy + \omega^3 z^2)$$

$$= (x + y + z)[(x^2 + \omega^3 y^2 + \omega^3 z^2 + (\omega^2 + \omega)xy + (\omega^2 + \omega^4)yz + (\omega + \omega^2)zx]$$

$\because 1+\omega+\omega^2 = 0 \Rightarrow \omega+\omega^2 = -1$ and $\omega^3 = 1$

$$= (x+y+z)\ [x^2 + 1y^2 + 1z^2 + (-1)xy + (\omega^2+\omega^3\omega)yz + (-1)zx]$$

$$= (x+y+z)\ [x^2 + 1y^2 + 1z^2 + (-1)xy + (\omega^2+1\omega)yz + (-1)zx]$$

$$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= x^3 + y^3 + z^3 - 3xyz = \text{L.H.S}$$

Using Formula

$(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = a^3 + b^3 + c^3 - 3abc$

Q.5. Prove that **02(034)**

$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$...........2n factors = 1

Solution:

Let L.H.S.

$=(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$ .... 2n factors

$=(1+\omega)(1+\omega^2)(1+\omega\omega^3)(1+\omega^6\omega^2)$ .... 2n factors

$\because \omega^3 = 1 \Rightarrow \omega^6 = (\omega^3)^2 = (1)^2 = 1$

$=(1+\omega)(1+\omega^2)(1+\omega)(1+\omega^2)$ .... 2n factors

$=\left[(1+\omega)(1+\omega^2)\right]\left[(1+\omega)(1+\omega^2)\right]$..........n factors

$$=\left[(1+\omega)(1+\omega^2)\right]^n$$

$$=\left[1+\omega^2+\omega+\omega^3\right]^n$$

$$=\left[1+\omega+\omega^2+\omega^3\right]^n$$

$\because 1+\omega+\omega^2 = 0$, $\omega^3 = 1$

$$=[0+1]^n$$

$$=[1]^n$$

$= 1 = \text{R. H. S} \Rightarrow \text{L.H.S} = \text{R.H.S}$

EXERCISE 2.3

Q.1 Without solving, Find the sum and product of the Roots of following Quadratic Equations.

(i) $x^2-5x+3=0$

Solution: $x^2-5x+3=0$

$ax^2+bx+c=0$

$\Rightarrow$ $a=1$, $b=-5$, $c=3$

Sum of roots $= S = \dfrac{-b}{a} = -\left(\dfrac{-5}{1}\right) = 5$

Product of roots $= P = \dfrac{c}{a} = \dfrac{3}{1} = 3$

(ii) $3x^2+7x-11=0$

Solution: $3x^2+7x-11=0$

$ax^2+bx+c=0$

$\Rightarrow$ $a=3$, $b=7$, $c=-11$

Sum of roots $= S = \dfrac{-b}{a} = \dfrac{-7}{3}$

Product of roots $= P = \dfrac{c}{a} = \dfrac{-11}{3}$

(iii) $px^2-qx+r=0$

Solution: $px^2-qx+r=0$

$ax^2+bx+c=0$

$\Rightarrow$ $a=p$, $b=-q$, $c=r$

Sum of roots $= S = \dfrac{-b}{a} = -\left(\dfrac{-q}{p}\right) = \dfrac{q}{p}$

Product of roots $= P = \dfrac{c}{a} = \dfrac{r}{p}$

(iv) $(a+b)x^2-ax+b=0$

Solution: $(a+b)x^2-ax+b=0$

$Ax^2+Bx+C=0$

$\Rightarrow$ $A=a+b$, $B=-a$, $C=b$

Sum of roots $= S = \dfrac{-B}{A} = -\left(\dfrac{-a}{a+b}\right) = \dfrac{a}{a+b}$

Product of roots $= P = \dfrac{C}{A} = \dfrac{b}{a+b}$

(v) $(\ell+m)x^2+(m+n)x+n-\ell=0$

Solution: $(\ell+m)x^2+(m+n)x+n-\ell=0$

$ax^2+bx+c=0$

$\Rightarrow$ $a=\ell+m$, $b=m+n$, $c=n-\ell$

Sum of roots $= S = \dfrac{-b}{a} = \dfrac{-(m+n)}{\ell+m}$

Product of roots $= P = \dfrac{c}{a} = \dfrac{n-\ell}{\ell+m}$

(vi) $7x^2-5mx+9n=0$

Solution: $7x^2-5mx+9n=0$

$ax^2+bx+c=0$

$\Rightarrow$ $a=7$, $b=-5m$, $c=9n$

Sum of roots $= \dfrac{-b}{a} = -\left(\dfrac{-5m}{7}\right) = \dfrac{5m}{7}$

Product of roots $= \dfrac{c}{a} = \dfrac{9n}{7}$

Q. 2 Find the Value of k if.

(i) Sum of the roots of the equation $2kx^2-3x+4k=0$ is twice the product of the roots.

Solution:

$2kx^2-3x+4k=0$

$ax^2+bx+c=0$

$a=2k$, $b=-3$, $c=4k$

Let α, β be the Roots of Equation

Sum of the roots $= \alpha+\beta = \dfrac{-b}{a}$

$S = \alpha+\beta = \dfrac{-(-3)}{2k} = \dfrac{3}{2k}$

product of the Roots $= \alpha\beta = \dfrac{c}{a}$

$P = \alpha\beta = \dfrac{4k}{2k}$

$= 2$

Given condition

$S = 2p$

$\frac{3}{2k} = 2(2)$

$\frac{3}{2k} = 4$

$3 = 4(2k)$

$3 = 8k$

$\frac{3}{8} = k \quad \Rightarrow \quad \boxed{k = \frac{3}{8}}$

(ii). Sum of the roots of the equation $x^2 + (3k-7)x + 5k = 0$ is $\frac{3}{2}$ times the product of roots.

Solution:

$1x^2 + (3k-7)x + 5k = 0$

$ax^2 + bx + c = 0$

$a = 1, \; b = 3k-7, \; c = 5k$

Let α, β be the roots of the given equation.

Sum of the roots

$S = \alpha + \beta = \frac{-b}{a}$

$S = \frac{-(3K-7)}{1}$

$S = -3K + 7$

Product of the roots $= P = \alpha\beta = \frac{c}{a}$

$P = \frac{5k}{1}$

$P = 5k$

Given condition

$S = \frac{3}{2}P$

$-3k + 7 = \frac{3}{2}(5k)$

$2(-3k+7) = 3(5k)$

$-6k + 14 = 15k$

$14 = 15k + 6k$

$14 = 21k$

$k = \frac{14}{21} = \frac{2}{3}$

$\boxed{k = \frac{2}{3}}$

Q. 3 Find k if,

(i) Sum of the squares of the roots of the equation. $4kx^2 + 3kx - 8 = 0$ is 2

Solution:

$4kx^2 + 3kx - 8 = 0 \text{ is } 2$

$ax^2 + bx + c = 0$

$a = 4k, \; b = 3k, \; c = -8$

Sum of roots

$S = \alpha + \beta = \frac{-b}{a}$

$\alpha + \beta = \frac{-3k}{4k}$

$\alpha + \beta = \frac{-3}{4}$

Product of roots $= P = \alpha\beta = \frac{c}{a}$

$\alpha\beta = \frac{-8}{4k}$

$\alpha\beta = \frac{-2}{k}$

Given that sum of square of roots is 2 i.e.

$\alpha^2 + \beta^2 = 2$

$\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta = 2$

$(\alpha + \beta)^2 - 2\alpha\beta = 2$

Put $\alpha + \beta = \frac{-3}{4}$ and $\alpha\beta = \frac{-2}{k}$

$\left(\frac{-3}{4}\right)^2 - 2\left(\frac{-2}{k}\right) = 2$

$\frac{9}{16} + \frac{4}{k} = 2$

$\frac{9k + 64}{16k} = 2$

$9k + 64 = 2(16k)$

$9k + 64 = 32k$

$64 = 32k - 9k$

$\Rightarrow \quad 23k = 64$

$k = \frac{64}{23}$

Required value of $k = \frac{64}{23}$

(ii) Sum of square of the roots of the equation $x^2 - 2kx + (2k+1) = 0$ is 6.

Solution: $1x^2 - 2kx + (2k+1) = 0$

$ax^2 + bx + c = 0$

a = 1 , b= −2k , c = 2k+1

α, β be the Roots of given equation.

$$S = \alpha + \beta = -\frac{b}{a}$$

$$\alpha + \beta = \frac{-(-2k)}{1} = 2k$$

$$\alpha\beta = \frac{c}{a}$$

$$P = \alpha\beta = \frac{2k+1}{1} = 2k+1$$

Given condition

$$\alpha^2 + \beta^2 = 6$$

$$\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta = 6$$

$$(\alpha+\beta)^2 - 2(\alpha\beta) = 6$$

$$(-2k)^2 - 2(2k+1) = 6$$

$$4k^2 - 4k - 2 = 6$$

$$4k^2 - 4k - 2 - 6 = 0$$

$$4k^2 - 4k - 8 = 0$$

$$4(k^2 - k - 2) = 0$$

$\because$ $k^2 - k - 2 = 0$ $(\because 4 \neq 0)$

$$k^2 - 2k + k - 2 = 0$$

$$k(k-2) + 1(k-2) = 0$$

$$(k-2)(k+1) = 0$$

k−2 = 0 or k + 1 = 0

k = 2 or k = −1

k = − 1 , 2

Q.4 Find p if

(i) The roots of the equation $x^2 - x + p^2 = 0$ differ by unity. 02(045)

Solution: $x^2 - x + p^2 = 0$

$ax^2 + bx + c = 0$

a = 1 , b= −1 , c = p^2

Let the roots are 'α and $\alpha - 1$'

Sum of roots = S $= \alpha + \alpha - 1 = \frac{-b}{a} = -\left(\frac{-1}{1}\right) = 1$

$$2\alpha - 1 = 1$$

$$\Rightarrow 2\alpha = 1 + 1$$

$$\Rightarrow 2\alpha = 2 \Rightarrow \boxed{\alpha = 1}$$

Product of roots = P $= \alpha(\alpha - 1) = \frac{c}{a} = \frac{p^2}{1} = p^2$

$$\alpha(\alpha - 1) = p^2$$

Putting the value of $\alpha = 1$

$$1(1-1) = p^2$$

$\Rightarrow$ $p^2 = 0$

$\Rightarrow$ $\boxed{p = 0}$

(ii)Find p if the roots of the equation $x^2 + 3x + p - 2 = 0$ differ by 2.

Solution: $x^2 + 3x + p - 2 = 0$

a = 1, b = 3 , c = p − 2

Let 'α' and $\alpha - 2$ are the roots

Sum of roots

$$S = \alpha + \alpha - 2 = \frac{-b}{a} = \frac{-3}{1} = -3$$

$$2\alpha - 2 = -3$$

$$2\alpha = -3 + 2$$

$\Rightarrow 2\alpha = -1 \quad \Rightarrow \boxed{\alpha = \frac{-1}{2}}$ .....(i)

Product of roots

$$P = \alpha(\alpha - 2) = \frac{c}{a} = \frac{p-2}{1} = p - 2$$

$$\alpha(\alpha - 2) = p - 2$$

Putting the value of α from equation (i)

$$-\frac{1}{2}\left(\frac{-1}{2} - 2\right) = p - 2$$

$$\frac{-1}{2}\left(\frac{-5}{2}\right) = p - 2$$

$$\frac{5}{4} = p - 2$$

$\Rightarrow$ $p = \frac{5}{4} + 2$

$p = \frac{5+8}{4} \quad \Rightarrow \boxed{p = \frac{13}{4}}$

Q.5 Find m if

(i) The roots of the equation $x^2 - 7x + 3m - 5 = 0$
Satisfy the relation $3\alpha + 2\beta = 4$

Solution:

Let α, β be the roots of given equation.

$1x^2 - 7x + 3m - 5 = 0$

$a = 1$, $b = -7$, $c = 3m - 5$

Sum of roots

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-7)}{1} = 7 \ldots\ldots\ldots (i)$$

Product of roots

$$\alpha\beta = \frac{c}{a} = \frac{3m-5}{1} = 3m - 5 \ldots\ldots (ii)$$

Since $3\alpha + 2\beta = 4\,(given) \ldots\ldots (iii)$

From equation (i)

$\alpha + \beta = 7$

$\beta = 7 - \alpha$

Put $\beta = 7 - \alpha$ in equation (iii)

$3\alpha + 2(7 - \alpha) = 4$

$3\alpha + 14 - 2\alpha = 4$

$\alpha + 14 = 4$

$\alpha = 4 - 14$

$\boxed{\alpha = -10}$

Put $\alpha = -10$ *in* $\beta = 7 - \alpha$

$\beta = 7 - (-10)$

$\beta = 7 + 10$

$\boxed{\beta = 17}$

Put $\alpha = -10$ *and* $\beta = 17$ in equation (ii)

$\alpha\beta = 3m - 5$

$(-10)(17) = 3m - 5$

$-170 = 3m - 5$

$-170 + 5 = 3m$

$-165 = 3m$

$\frac{-165}{3} = m$

$-55 = m \quad \Rightarrow \boxed{m = -55}$

(ii) Find m if the roots of the equation $x^2 + 7x + 3m - 5 = 0$ satisfy the relation $3\alpha - 2\beta = 4$

Solution:

Let α, β be the roots of the equation.

$1x^2 + 7x + 3m - 5 = 0$

$a = 1$, $b = 7$, $c = 3m - 5$

Sum of roots

$$\alpha + \beta = \frac{-b}{a}$$

$$\alpha + \beta = \frac{-7}{1}$$

$\alpha + \beta = -7 \ldots\ldots\ldots (i)$

Product of roots

$$\alpha\beta = \frac{c}{a}$$

$$\alpha\beta = \frac{3m-5}{1}$$

$\alpha\beta = 3m - 5 \ldots\ldots\ldots\ldots (ii)$

$3\alpha - 2\beta = 4 \ldots\ldots\ldots\ldots (iii)$ (Given)

From equation (i)

$\alpha + \beta = -7$

$\beta = -7 - \alpha$

Putting the value of β in equation (iii)

$3\alpha - 2\beta = 4$

$3\alpha - 2(-7 - \alpha) = 4$

$3\alpha + 14 + 2\alpha = 4$

$5\alpha = 4 - 14$

$5\alpha = -10$

$$\alpha = \frac{-10}{5}$$

$\boxed{\alpha = -2}$

Put $\alpha = -2$ in equation (i)

$\alpha + \beta = -7$

$-2 + \beta = -7$

$\beta = -7 + 2$

$\boxed{\beta = -5}$

Now Put $\alpha = -2$ and $\beta = -5$ in equation (ii)

$\alpha\beta = 3m - 5$

$(-2)(-5) = 3m - 5$

$10 = 3m - 5$

$10 + 5 = 3m$

$15 = 3m$

$m = \frac{15}{3} \quad \Rightarrow \boxed{m = 5}$

(iii) Find m if the roots of the equation $3x^2 - 2x + 7m + 2 = 0$ satisfy the relation $7\alpha - 3\beta = 18$

Solution: Let α, β be the roots of the equation

$3x^2 - 2x + 7m + 2 = 0$

$ax^2 + bx + c = 0$

$a = 3, b = -2, c = 7m + 2$

Sum of roots

$$\alpha + \beta = -\frac{b}{a} = \frac{-(-2)}{3} = \frac{2}{3} \quad \text{.........(i)}$$

Product of roots

$$\alpha\beta = \frac{c}{a} = \frac{7m+2}{3} \quad \text{.................. (ii)}$$

$$7\alpha - 3\beta = 18 \quad \text{.................. (iii)}$$

From equation (i)

$$\alpha + \beta = \frac{2}{3}$$

$$\beta = \frac{2}{3} - \alpha$$

Putting this value in eq. (iii)

$$7\alpha - 3\beta = 18$$

$$7\alpha - 3\left(\frac{2}{3} - \alpha\right) = 18$$

$$7\alpha - \frac{\cancel{6}^{2}}{\cancel{3}} + 3\alpha = 18$$

$$7\alpha - 2 + 3\alpha = 18$$

$$10\alpha = 18 + 2$$

$$10\alpha = 20$$

$$\alpha = \frac{20}{10}$$

$$\boxed{\alpha = 2}$$

Putting $\alpha = 2$ in equation (i)

$$\alpha + \beta = \frac{2}{3}$$

$$2 + \beta = \frac{2}{3}$$

$$\beta = \frac{2}{3} - 2$$

$$\beta = \frac{2-6}{3} = \frac{-4}{3}$$

$$\boxed{\beta = \frac{-4}{3}}$$

Now putting the value of α and β in equation (ii)

$$\alpha\beta = \frac{7m+2}{3}$$

$$2\left(-\frac{4}{3}\right) = \frac{7m+2}{3}$$

$$-\frac{8}{\cancel{3}} \times \cancel{3} = 7m + 2$$

$$\Rightarrow 7m + 2 = -8$$

$$7m = -8 - 2$$

$$\Rightarrow m = \frac{-10}{7}$$

Q.6 Find m, if sum and product of the roots of the following equations is equal to given number λ.

(i) $(2m+3)x^2 + (7m-5)x + (3m-10) = 0$

Solution:

$(2m+3)x^2 + (7m-5)x + (3m-10) = 0$

$ax^2 + bx + c = 0$

$a = 2m + 3, \quad b = 7m - 5, \quad c = 3m - 10$

Let α, β are the roots of the given equation, then

Sum of Roots

$$\alpha + \beta = -\frac{b}{a} = -\frac{(7m-5)}{2m+3} = \frac{5-7m}{2m+3}$$

Product of Roots

$$\alpha\beta = \frac{c}{a} = \frac{3m-10}{2m+3}$$

As given that

$\alpha + \beta = \lambda$............(i) , $\alpha\beta = \lambda$............(ii)

From (i) and (ii)

$$\alpha + \beta = \alpha\beta$$

$$\frac{5-7m}{\cancel{2m+3}} = \frac{3m-10}{\cancel{2m+3}}$$

$$5 - 7m = 3m - 10$$

$$5 + 10 = 3m + 7m$$

$$15 = 10m$$

$$\frac{\cancel{15}^{3}}{\cancel{10}_{2}} = m$$

$$\boxed{m = \frac{3}{2}}$$

(ii) $4x^2-(3+5m)x-(9m-17)=0$

Solution:

Let α, β be the roots of the equation.

$$4x^2-(3+5m)x-(9m-17)=0$$

$\because$ $ax^2+bx+c=0$

$\therefore$ $a=4$, $b=-(3+5m)$, $c=-(9m-17)$

Sum of roots

$$\alpha+\beta=\frac{-b}{a}=-\left(-\frac{(3+5m)}{4}\right)=\frac{3+5m}{4}$$

Product of roots

$$\alpha\beta=\frac{c}{a}=-\left(\frac{9m-17}{4}\right)$$

Let $\alpha+\beta=\lambda.....(i)$ and $\alpha\beta=\lambda.....(ii)$

from (i) and (ii)

$$\alpha+\beta=\alpha\beta$$

$$\frac{3+5m}{4}=\frac{-(9m-17)}{4}$$

$$3+5m=-9m+17$$

$$9m+5m=17-3$$

$$14m=14$$

$$\Rightarrow m=\frac{14}{14} \Rightarrow \boxed{m=1}$$

EXERCISE 2.4

Q.1 If α, β are the roots of the equations $x^2 + px + q = 0$ then evaluate

(i) $\alpha^2 + \beta^2$ (ii) $\alpha^3\beta + \alpha\beta^3$ (iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Solution:

$a = 1, b = p, c = q$

Sum of roots,

$$\alpha + \beta = \frac{-b}{a} = \frac{-p}{1} = -p$$

Product of roots

$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q$$

(i) $\alpha^2 + \beta^2$

As

$$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$$

or $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$$= (\alpha+\beta)^2 - 2\alpha\beta$$

$$= (-p)^2 - 2(q)$$

$$= p^2 - 2q$$

(ii) $\alpha^3\beta + \alpha\beta^3$

$$= \alpha\beta(\alpha^2 + \beta^2)$$

$$= \alpha\beta\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$$

$$= q\left[(-p)^2 - 2q\right]$$

$$= q(p^2 - 2q)$$

(iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{1}{\alpha\beta}(\alpha^2 + \beta^2)$$

$$= \frac{1}{\alpha\beta}\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$$

$$= \frac{1}{q}\left[(-p)^2 - 2\times q\right]$$

$$= \frac{1}{q}(p^2 - 2q)$$

Q.2 If α, β are the roots of the equation $4x^2-5x+6=0$, the find the value of

(i) $\frac{1}{\alpha}+\frac{1}{\beta}$ (ii) $\alpha^2\beta^2$ (iii) $\frac{1}{\alpha^2\beta}+\frac{1}{\alpha\beta^2}$ (iv) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

Solution: $4x^2-5x+6=0$

$ax^2+bx+c=0$

$a=4,\ b=-5,\ c=6$

Sum of roots,

$$\alpha+\beta=\frac{-b}{a}=-\frac{(-5)}{4}=\frac{5}{4}$$

Product of roots,

$$\alpha\beta=\frac{c}{a}=\frac{6}{4}$$

(i) $$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{5/4}{6/4}=5/6$$

(ii) $$\alpha^2\beta^2=(\alpha\beta)^2=\left(\frac{6}{4}\right)^2=\left(\frac{3}{2}\right)^2=\frac{9}{4}$$

(iii) $$\frac{1}{\alpha^2\beta}+\frac{1}{\alpha\beta^2}=\frac{\beta+\alpha}{\alpha^2\beta^2}=\frac{\alpha+\beta}{(\alpha\beta)^2}$$

$$=\frac{\frac{5}{4}}{\left(\frac{6}{4}\right)^2}=\frac{\frac{5}{4}}{\frac{36}{16}}=\frac{5}{4}\times\frac{16}{36}=\frac{5}{9}$$

(iv) $$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$$

Using Formula: $(\alpha+\beta)^3=\alpha^3+\beta^3+3(\alpha\beta)(\alpha+\beta)$

$\Rightarrow\ \alpha^3+\beta^3=(\alpha+\beta)^3-3(\alpha\beta)(\alpha+\beta)$

$$=\frac{(\alpha+\beta)^3-3(\alpha\beta)(\alpha+\beta)}{\alpha\beta}$$

$$=\frac{\left(\frac{5}{4}\right)^3-3\left(\frac{6}{4}\right)\left(\frac{5}{4}\right)}{\left(\frac{6}{4}\right)}=\left(\frac{125}{64}-\frac{90}{16}\right)\frac{4}{6}$$

$$=\left(\frac{125-360}{64}\right)\frac{4}{6}=\frac{-235}{96}$$

Q.3 If α, β are the roots of the equation $\ell x^2+mx+n=0$ $(\ell\neq 0)$ then find the value of

(i) $\alpha^3\beta^2+\alpha^2\beta^3$ (ii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

Solution: $lx^2+mx+n=0$

$ax^2+bx+c=0$

$a=l,\ b=m,\ c=n$

If α,β be the Roots of given equation

Sum of roots $=\frac{-b}{a}$

$$\alpha+\beta=\frac{-m}{l}$$

Product of Roots $=\frac{c}{a}$

$$\alpha\beta=\frac{n}{l}$$

(i) $\alpha^3\beta^2+\alpha^2\beta^3$

$=\alpha^2\beta^2(\alpha+\beta)$

$=(\alpha\beta)^2(\alpha+\beta)$

$=\left(\frac{n}{l}\right)^2\left(\frac{-m}{l}\right)$

$=\left(\frac{n^2}{l^2}\right)\left(\frac{-m}{l}\right)$

$=\frac{-mn^2}{l^3}$

(ii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$

$=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$

$$=\frac{\left(\frac{-m}{l}\right)^2-2\left(\frac{n}{l}\right)}{\left(\frac{n}{l}\right)^2}=\frac{\frac{m^2}{l^2}-\frac{2n}{l}}{\frac{n^2}{l^2}}$$

$=\left(\frac{m^2-2nl}{l^2}\right)\cdot\frac{l^2}{n^2}$

$=\left(\frac{m^2-2nl}{n^2}\right)$

$=\frac{1}{n^2}\left(m^2-2nl\right)$

EXERCISE 2.5

Q.1 Write the quadratic equation having following roots.

(a) 1, 5 (b) 4, 9 (c) −2, 3
(d) 0,−3 (e) 2,−6 (f). −1, −7
(g). 1+i, 1−i (h) $3+\sqrt{2}$, $3-\sqrt{2}$

(a) 1, 5

Solution: Since 1 and 5 are the roots of the required quadratic equation, therefore

Sum of roots = S = 1 + 5 = 6

Product of roots = P = 1×5 = 5

As $x^2 - Sx + P = 0$ so the required equation is

$x^2 - 6x + 5 = 0$

(b) 4, 9

Solution: Since 4 and 9 are the roots of the required quadratic equation, therefore

Sum of roots = S = $4+9=13$

Product of roots = P = $4\times 9=36$

As $x^2 - Sx + P = 0$

So the required equation is

$x^2 - 13x + 36 = 0$

(c) −2, 3

Since −2, 3 are the roots of required quadratic equation, therefore

Sum of roots = S = − 2 + 3 = 1

Product of roots = P = −2 ×3 = −6

As $x^2 - Sx + P = 0$

Therefore the required quadratic equation is

$x^2 - x - 6 = 0$

(d) 0, −3

Since 0, −3 are the roots of required quadratic equation therefore

Sum of roots = S = 0 + (−3) = −3

Product of roots = P = $0\times(-3) = 0$

As $x^2 - Sx + P = 0$

Therefore the required quadratic equation is

$x^2 + 3x + 0 = 0 \Rightarrow$

$\Rightarrow \quad x^2 + 3x = 0$

(e) 2, −6

Solution: Since 2 and −6 are the roots of the required quadratic equation therefore

Sum of roots = S = 2 + (−6) = 2 − 6 = − 4

Product of roots = P = 2 × (−6) = −12

As $x^2 - Sx + P = 0$ so the required equation is

$x^2 + 4x - 12 = 0$

(f) −1, −7

Solution: Since −1 and −7 are the roots of the required quadratic equation therefore

Sum of roots = S = (−1) + (−7)

$= -1 - 7 = -8$

Product of roots = P = $(-1)(-7) = 7$

As $x^2 - Sx + P = 0$ so the required equation is

$x^2 + 8x + 7 = 0$

(g) $1+i, 1-i$

Solution: Since 1 + *i* and 1 − *i* are the roots of the required quadratic equation therefore

Sum of roots = S = $1+\cancel{i}+1-\cancel{i}=2$

Product of roots = P = $(1+i)(1-i)$

$P = (1)^2 - (i)^2$

$P = 1 - (-1)$

$P = 1 + 1 = 2$

As $x^2 - Sx + P = 0$ so the required equation is

$x^2 - 2x + 2 = 0$

(h) $3+\sqrt{2}, 3-\sqrt{2}$

Solution: $3+\sqrt{2}$ and $3-\sqrt{2}$ are the roots of the required quadratic equation therefore

Sum of roots = S = $3+\sqrt{2}+3-\sqrt{2}=6$

Product of roots

$P = \left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)$

$P = (3)^2 - \left(\sqrt{2}\right)^2$

$P = 9 - 2 = 7$

As $x^2 - Sx + P = 0$, so the required equation is

$x^2 - 6x + 7 = 0$

Q.2 If α, β are the roots of the equation $x^2 - 3x + 6 = 0$. Form equation whose roots are (a) $2\alpha+1, 2\beta+1$ (b) α^2, β^2

(c) $\frac{1}{\alpha}, \frac{1}{\beta}$ (d) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

(e) $\alpha+\beta, \frac{1}{\alpha}+\frac{1}{\beta}$

Solution: As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

a = 1, b = −3, c = 6

Therefore,

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3 \Rightarrow \boxed{\alpha+\beta=3}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6 \Rightarrow \boxed{\alpha\beta=6}$$

(a) $2\alpha+1, 2\beta+1$

Sum of roots

$$S = 2\alpha+1+2\beta+1$$
$$S = 2\alpha+2\beta+2$$
$$S = 2(\alpha+\beta)+2$$
$$S = 2(3)+2 = 6+2 = 8$$
$$\boxed{S=8}$$

Product of roots

$$P = (2\alpha+1)(2\beta+1)$$
$$P = 4\alpha\beta+2\alpha+2\beta+1$$
$$P = 4\alpha\beta+2(\alpha+\beta)+1$$
$$P = 4(6)+2(3)+1$$
$$P = 24+6+1 = 31$$
$$\boxed{P=31}$$

Using $x^2 - Sx + P = 0$, we have

$$x^2 - 8x + 31 = 0$$

(b) α^2, β^2

Solution: As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

a = 1, b = −3, c = 6

Therefore,

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3 \Rightarrow \boxed{\alpha+\beta=3}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6 \Rightarrow \boxed{\alpha\beta=6}$$

Sum of roots = $S = \alpha^2+\beta^2$

$$S = (\alpha+\beta)^2 - 2\alpha\beta$$
$$S = (3)^2 - 2(6)$$
$$S = 9 - 12 = -3$$
$$\boxed{S=-3}$$

Product of roots = P = $\alpha^2\beta^2$

$$P = (\alpha\beta)^2$$
$$P = (6)^2 = 36$$
$$\boxed{P=36}$$

Using $x^2 - Sx + P = 0$, we have

$$x^2 + 3x + 36 = 0$$

(c) $\frac{1}{\alpha}, \frac{1}{\beta}$

Solution: As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

a = 1, b = −3, c = 6

Therefore,

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3 \Rightarrow \boxed{\alpha+\beta=3}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6 \Rightarrow \boxed{\alpha\beta=6}$$

Sum of roots = S $= \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta}$

$$S = (\alpha+\beta).\frac{1}{\alpha\beta}$$
$$S = 3\times\frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$
$$\boxed{S=\frac{1}{2}}$$

Product of roots = P = $\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)$

$$P = \frac{1}{\alpha\beta} = \frac{1}{6} \quad \boxed{P=\frac{1}{6}}$$

Using $x^2 - Sx + P = 0$, we have

$$x^2 - \frac{1}{2}x + \frac{1}{6} = 0$$

Multiplying by '6' on both side, we have

$$6x^2 - 3x + 1 = 0$$

(d) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution: As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

$a = 1, \; b = -3, \; c = 6$

Therefore,

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3 \Rightarrow \boxed{\alpha + \beta = 3}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6 \Rightarrow \boxed{\alpha\beta = 6}$$

Sum of roots = S $= \frac{\alpha}{\beta} + \frac{\beta}{a}$

$$S = \frac{\alpha^2 + \beta^2}{\beta\alpha}$$

$$S = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$S = \frac{(3)^2 - 2(6)}{6}$$

$$S = \frac{9 - 12}{6}$$

$$S = \frac{-3}{6}$$

$$\boxed{S = \frac{-1}{2}}$$

Product of roots = P $= \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = 1$

Using $x^2 - Sx + P = 0$, we have

$$x^2 + \frac{1}{2}x + 1 = 0$$

Multiplying both sides by '2', we have

$$2x^2 + x + 2 = 0$$

(e) $\alpha + \beta, \frac{1}{\alpha} + \frac{1}{\beta}$

Solution: As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

$a = 1, \; b = -3, \; c = 6$

Therefore,

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3 \Rightarrow \boxed{\alpha + \beta = 3}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6 \Rightarrow \boxed{\alpha\beta = 6}$$

Sum of roots = S $= (\alpha + \beta) + \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

$$S = (\alpha + \beta) + \frac{\beta + \alpha}{\alpha\beta}$$

$$S = (\alpha + \beta) + \frac{(\alpha + \beta)}{a\beta}$$

$$S = 3 + \frac{3}{6}$$

$$S = 3 + \frac{1}{2}$$

$$S = \frac{6 + 1}{2}$$

$$\boxed{S = \frac{7}{2}}$$

Product of roots = P $= (\alpha + \beta)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

$$P = (\alpha + \beta)\left(\frac{\beta + \alpha}{\alpha\beta}\right)$$

$$P = (\alpha + \beta)\left(\frac{\alpha + \beta}{\alpha\beta}\right)$$

$$P = 3\left(\frac{3}{6}\right)$$

$$\boxed{P = \frac{3}{2}}$$

Using $x^2 - Sx + P = 0$, we have

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

Multiplying both sides by '2' we have

$$2x^2 - 7x + 3 = 0$$

Q.3 If α, β are the roots of the equation $x^2+px+q=0$. From equation whose roots are

(a). α^2, β^2 (b) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution:

Since α, β are the roots of the equation

$x^2+px+q=0$

$ax^2+bx+c=0$

By comparing the coefficients of these equations, we have

$a=1,\ b=p,\ c=q$

$\alpha+\beta=\frac{-b}{a}=\frac{-p}{1}=-p \Rightarrow \boxed{\alpha+\beta=-p}$

$\alpha\beta=\frac{c}{a}=\frac{q}{1}=q \Rightarrow \boxed{\alpha\beta=q}$

(a) α^2, β^2

Sum of roots = S = $\alpha^2+\beta^2$

$S=(\alpha+\beta)^2-2\alpha\beta$

$S=(-p)^2-2q$

$S=p^2-2q$

Product of roots = P = $\alpha^2\beta^2$

$P=(\alpha\beta)^2$

$P=q^2$

Using $x^2-Sx+P=0$, we have

$x^2-(p^2-2q)x+q^2=0$

(b) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution:

Since α, β are the roots of the equation

$x^2+px+q=0$

$ax^2+bx+c=0$

By comparing the coefficients of these equations, we have

$a=1,\ b=p,\ c=q$

$\alpha+\beta=\frac{-b}{a}=\frac{-p}{1}=-p \Rightarrow \boxed{\alpha+\beta=-p}$

$\alpha\beta=\frac{c}{a}=\frac{q}{1}=q \Rightarrow \boxed{\alpha\beta=q}$

Sum of roots = S = $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

$S=\frac{\alpha^2+\beta^2}{\alpha\beta}$

$S=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$S=\frac{(-p)^2-2(q)}{q}$

$S=\frac{p^2-2q}{q}$

Product of roots = P = $\left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right)=1$

Using $x^2-Sx+P=0$, we have

$x^2-\left(\frac{p^2-2q}{q}\right)x+1=0$

Multiplying by q

$qx^2-(p^2-2q)x+q=0$

EXERCISE 2.6

Q.1 Use synthetic division to find the quotient and the remainder, when

(i) $(x^2+7x-1)\div(x+1)$

As $x+1=x-(-1)$ So $a=-1$

Now write the co–efficient of dividend in a row and a = –1 on the left side

	1	7	–1
–1	↓	–1	–6
	1	6	–7

Quotient $Q(x)=x+6$

Remainder R = –7

(ii) $(4x^3-5x+15)\div(x+3)$

$\left(4x^3+0x^2-5x+15\right)\div(x+3)$

or As $x+3=x-(-3)$, So $a=-3$

Now write the co–efficient of dividend in a row and a = –3 on the left side

	4	0	–5	15
–3	↓	–12	36	–93
	4	–12	31	–78

Quotient = $Q(x)=4x^2-12x+31$

Remainder = $R=-78$

(iii) $(x^3+x^2-3x+2)\div(x-2)$

As $(x-2)$ So $a=2$

Now write the co–efficient of dividend in a row and a = 2 on the left side.

	1	1	–3	2
2	↓	2	6	6
	1	3	3	8

Quotient $Q(x)=x^2+3x+3$

Remainder R = 8

Q.2 Find the value of *h* using synthetic division, if 3 is the zero of the polynomial $2x^3-3hx^2+9$

(i) Solution: Let $P(x)=2x^3-3hx^2+0x+9$ and its zero is 3. Then by synthetic division.

	2	–3h	0	9
3	↓	6	18–9h	54–27h
	2	6–3h	18–9h	63–27h

Remainder = 63 – 27h

Since 3 is the zero of the polynomial, therefore Remainder = 0

$63-27h=0$

$63=27h$

$\Rightarrow h=\dfrac{63}{27}$ $\Rightarrow \boxed{h=\dfrac{7}{3}}$

(ii) Find the value of *h* using synthetic division, if 1 is the zero of the polynomial x^3-2hx^2+11

Solution:

Let $P(x)=x^3-2hx^2+0x+11$ and its zero is 1. Then by synthetic division

	1	–2h	0	11
1	↓	1	1–2h	1–2h
	1	1–2h	1–2h	12–2h

Remainder = 12 – 2h

Since 1 is the zero of the polynomial

So, Remainder = 0 that is

$12-2h=0$

$12=2h$

$\Rightarrow h=\dfrac{12}{2}$

$\boxed{h=6}$

(iii) Find the value of *h* using synthetic division, if –1 is the zero of the Polynomial $2x^3+5hx-23$

Solution:

Let $P(x)=2x^3+5hx-23$

$P(x)=2x^3+0x^2+5hx-23$

If–1 is zero of p(x) then by Synthetic division

	2	0	5h	–23
–1	↓	–2	2	–5h – 2
	2	–2	5h+2	–5h – 25

Remainder = –5h – 25

Since –1 is the zero of the Polynomial

So Remainder = 0

$-5h-25=0$

$-5h=25$

$h=\frac{25}{-5}$ ⇒ $\boxed{h=-5}$

Q.3 Use synthetic division to find the values of *l* and *m*,

(i). if (x+3) and (x–2) are the factors of the polynomial $x^3+4x^2+2lx+m$

Solution: Since $(x+3)$ and $(x-2)$ are the factors of $P(x)=x^3+4x^2+2\ell x+m$

Therefore –3 and 2 are the zeros of polynomial P(x). Now by synthetic division.

	1	4	2ℓ	m
–3	↓	–3	–3	$-6\ell+9$
	1	1	$2\ell-3$	$m+(-6\ell+9)$

Since –3 is the zero of polynomial, therefore remainder is zero that is

$m-6\ell+9=0$

⇒ $m-6\ell=-9$ ....(i)

And

	1	4	2ℓ	m
2	↓	2	12	$4\ell+24$
	1	6	$2\ell+12$	$m+4\ell+24$

Since 2 is the zero of polynomial, therefore remainder is zero that is

$m+4\ell+24=0$

$m+4\ell=-24$ ....(ii)

Subtracting equations (ii) from (i)

$$\begin{array}{r} \cancel{m}-6\ell=-9 \\ \pm\cancel{m}\pm4\ell=\mp24 \\ \hline -10\ell=15 \end{array}$$

$\ell=\frac{15}{-10}=\frac{3}{-2}$

⇒ $\boxed{\ell=-\frac{3}{2}}$

Put it in equations (i), we get

$m-6\left(\frac{-3}{2}\right)=-9$

$m+\frac{18}{2}=-9$

$m+9=-9$

$m=-9-9$

$\boxed{m=-18}$

(ii). Find the values of *l* and *m if* (x–1) and (x+1) are the factors of the polynomial $x^3-3lx^2+2mx+6$

Solution: Since $(x-1)$ and $(x+1)$ are the factors of $P(x)=x^3-3\ell x^2+2mx+6$

Therefore 1 and –1 are zeros of polynomial P(x). Now by synthetic division

	1	-3ℓ	$2m$	6
1	↓	1	$1-3\ell$	$1-3\ell+2m$
	1	$1-3\ell$	$1-3\ell+2m$	$7-3\ell+2m$

Since 1 is the zero of polynomial, therefore remainder is zero that is

$7-3\ell+2m=0$

$2m-3\ell=-7$ .... (i)

And

	1	-3ℓ	$2m$	6
–1	↓	–1	$1+3\ell$	$-1-3\ell-2m$
	1	$-1-3\ell$	$1+3\ell+2m$	$5-3\ell-2m$

Since –1 is the zero of polynomial therefore remainder is zero that is

$$5-3\ell-2m=0$$

$\Rightarrow \quad 2m+3\ell=5$ ....(ii)

Adding equations (i) and (ii)

$$2m-3\ell=-7$$
$$2m+3\ell=5$$
$$4m=-2$$
$$m=\frac{-2}{4}$$
$$\boxed{m=\frac{-1}{2}}$$

Put it in equation (i),

$$2\left(-\frac{1}{2}\right)-3\ell=-7$$
$$-1-3\ell=-7$$
$$-3\ell=-7+1$$
$$-3\ell=-6$$
$$\Rightarrow \quad \ell=\frac{-6}{-3}$$
$$\Rightarrow \quad \boxed{\ell=2}$$

Q.4 Solve by using synthetic division,

(i) If 2 is the root of the equation $x^3-28x+48=0$

Solution: Let $P(x) = x^3 + 0x^2 - 28x + 48$

Since 2 is the root of the equation $x^3-28x+48=0$ then by synthetic division.

	1	0	−28	48
2	↓	2	4	−48
	1	2	−24	0

The depressed equation is

$$x^2+2x-24=0$$
$$x^2+6x-4x-24=0$$
$$x(x+6)-4(x+6)=0$$
$$(x+6)(x-4)=0$$

Either $x+6=0$ or $x-4=0$

$x=-6$ or $x=4$

Thus 2, −6 and 4 are the roots of the given equation

(ii). If 3 is the root of the equation $2x^3-3x^2-11x+6=0$

Solution: Since 3 is the root of the equation

$$2x^3-3x^2-11x+6=0$$

Then by synthetic division

	2	−3	−11	6
3	↓	6	9	−6
	2	3	−2	0

The depressed equation is

$$2x^2+3x-2=0$$
$$2x^2+4x-x-2=0$$
$$2x(x+2)-1(x+2)=0$$
$$(x+2)(2x-1)=0$$

Either $x+2=0$ or $2x-1=0$

$\boxed{x=-2}$ or $2x=1$

$$\boxed{x=\frac{1}{2}}$$

Thus, 3, −2 and $\frac{1}{2}$ are the roots of the given equation.

(iii). If −1 is the root of the equation $4x^3-x^2-11x-6=0$

Solution: Since −1 is the root of the equation.

$$4x^3-x^2-11x-6=0$$

Then by synthetic division

	4	−1	−11	−6
−1	↓	−4	5	6
	4	−5	−6	0

The depressed equation is

$$4x^2-5x-6=0$$
$$4x^2-8x+3x-6=0$$
$$4x(x-2)+3(x-2)=0$$
$$(x-2)(4x+3)=0$$

Either $x-2=0$ or $4x+3=0$

$\boxed{x=2}$ or $4x=-3$

$$\boxed{x=\frac{-3}{4}}$$

Thus −1, 2 and $\frac{-3}{4}$ are the roots of the given equation

Q.5

(i) Solve by using synthetic division, if 1 and 3 are the roots of the equation

$$x^4 - 10x^2 + 9 = 0$$

Solution: Since 1 and 3 are the roots of the equation $x^4 - 10x^2 + 9 = 0$

Then by synthetic division, we get

	1	0	−10	0	9
1	↓	1	1	−9	−9
	1	1	−9	−9	0
3	↓	3	12	9	
	1	4	3	0	

Thus the depressed equation is

$$x^2 + 4x + 3 = 0$$
$$x + 3x + x + 3 = 0$$
$$x(x+3) + 1(x+3) = 0$$
$$(x+3)(x+1) = 0$$

Either $x + 3 = 0$ or $x + 1 = 0$

$\boxed{x = -3}$ $\boxed{x = -1}$

Hence 1, 3, −3 and −1 are the roots of the given equation

(ii) Solve by using synthetic division, if 3 and −4 are the roots of the equation

$$x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$$ **02(084)**

Solution: Since 3 and −4 are the roots of the given equation $x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$ then by synthetic division, we get

	1	2	−13	−14	24
3	↓	3	15	6	−24
	1	5	2	−8	0
−4	↓	−4	−4	8	
	1	1	−2	0	

The depressed equation is

$$x^2 + x - 2 = 0$$
$$x^2 + 2x - x - 2 = 0$$
$$x(x+2) - 1(x+2) = 0$$
$$(x+2)(x-1) = 0$$

Either $x + 2 = 0$ or $x - 1 = 0$

$\boxed{x = -2}$ $\boxed{x = 1}$

Thus 3, −4, −2 and 1 are the roots of the given equation.

EXERCISE 2.7

Solve the following simultaneous equations.

Q. 1 $x + y = 5$

$x^2 - 2y - 14 = 0$

Solution:

$x + y = 5$ .... (i)

$x^2 - 2y - 14 = 0$ .... (ii)

From equation (i)

$x + y = 5$

$x = 5 - y$

Put it in equation (ii)

$(5 - y)^2 - 2y - 14 = 0$

$25 + y^2 - 10y - 2y - 14 = 0$

$y^2 - 12y + 11 = 0$

$y^2 - 11y - y + 11 = 0$

$y(y - 11) - 1(y - 11) = 0$

$(y - 11)(y - 1) = 0$

Either $y - 11 = 0$ or $y - 1 = 0$

$y = 11$ or $y = 1$

Putting the values of y in eq .....(i)

When $y = 11$	When $y = 1$
$x + y = 5$	$x + y = 5$
$x + 11 = 5$	$x + 1 = 5$
$x = 5 - 11$	$x = 5 - 1$
$x = -6$	$x = 4$

Solution Set is $\{(-6, 11), (4, 1)\}$

Q. 2 $3x - 2y = 1$

$x^2 + xy - y^2 = 1$

Solution:

$3x - 2y = 1$ .... (i)

$x^2 + xy - y^2 = 1$ ....(ii)

From equation (i)

$3x = 1 + 2y$

$x = \dfrac{1+2y}{3}$ ....(iii)

Put it in equation (ii)

$$\left(\frac{1+2y}{3}\right)^2 + \left(\frac{1+2y}{3}\right)y - y^2 = 1$$

$$\frac{1+4y^2+4y}{9} + \frac{y+2y^2}{3} - y^2 = 1$$

Multiplying by '9' on both sides we have

$$\frac{\cancel{9}(1+4y^2+4y)}{\cancel{9}} + \frac{{}^{3}\cancel{9}(y+2y^2)}{\cancel{3}} - 9(y^2) = 9\times1$$

$1 + 4y^2 + 4y + 3y + 6y^2 - 9y^2 = 9$

$y^2 + 7y - 8 = 0$

$y^2 + 8y - y - 8 = 0$

$y(y + 8) - 1(y + 8) = 0$

$(y + 8)(y - 1) = 0$

Either

$y + 8 = 0$ or $y - 1 = 0$

$y = -8$ or $y = 1$

Putting these values in equation (iii)

when $y = -8$	when $y = 1$
$x = \dfrac{1+2y}{3}$	$x = \dfrac{1+2y}{3}$
$x = \dfrac{1+2(-8)}{3}$	$x = \dfrac{1+2(1)}{3}$
$x = \dfrac{1-16}{3}$	$x = \dfrac{1+2}{3}$
$x = \dfrac{-15}{3} = -5$	$x = \dfrac{3}{3} = 1$

Solution Set is $\{(-5,-8),(1,1)\}$

Q. 3 $x - y = 7$

$\dfrac{2}{x} - \dfrac{5}{y} = 2$

Solution:

$x - y = 7$ .... (i)

$\dfrac{2}{x} - \dfrac{5}{y} = 2$ .... (ii)

Multiplying equation (ii) by "xy" we have

$2y - 5x = 2xy$........ (iii)

From equation (i)

$x = 7 + y$

Put it in equation (iii)

$2y - 5(7 + y) = 2(7 + y)y$

$2y - 35 - 5y = 14y + 2y^2$

$\Rightarrow$ $2y^2 + 17y + 35 = 0$

$2y^2 + 10y + 7y + 35 = 0$

$2y(y + 5) + 7(y + 5) = 0$

$(y + 5)(2y + 7) = 0$

Either

$y + 5 = 0$ or $2y + 7 = 0$

$y = -5$ or $y = \dfrac{-7}{2}$

Now putting these values of y in equation (i)

When $y = -5$

$x = 7 + y$

$x = 7 + (-5)$

$x = 7 - 5$

$x = 2$

When $y = \frac{-7}{2}$

$x = 7 + y$

$x = 7 + \left(\frac{-7}{2}\right)$

$x = 7 - \frac{7}{2}$

$x = \frac{14-7}{2}$

$x = \frac{7}{2}$

Solution Set is $\left\{(2,-5),\left(\frac{7}{2},\frac{-7}{2}\right)\right\}$

Q. 4 $x + y = a - b$

$\frac{a}{x} - \frac{b}{y} = 2$

Solution:

$x + y = a - b$ ....(i)

$\frac{a}{x} - \frac{b}{y} = 2 \Rightarrow \frac{ay - bx}{xy} = 2$

$ay - bx = 2xy$ ....(ii)

From equation (i)

$x = a - b - y$ ....(iii)

Put it in equation (ii)

$ay - bx = 2xy$

$ay - b(a - b - y) = 2(a - b - y)y$

$ay - ba + b^2 + by = 2ay - 2by - 2y^2$

$2y^2 - 2ay + ay + 2by + by + b^2 - ab = 0$

$2y^2 - ay + 3by + b^2 - ab = 0$

$2y^2 - y(a - 3b) + (b^2 - ab) = 0$

By using quadratic formula

$a = 2, \quad b = -(a - 3b), \; c = (b^2 - ab)$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-[-(a-3b)] \pm \sqrt{\left[-(a-3b)\right]^2 - 4(2)(b^2 - ab)}}{2(2)}$$

$$y = \frac{(a-3b) \pm \sqrt{a^2 + 9b^2 - 6ab - 8b^2 + 8ab}}{4}$$

$$y = \frac{(a-3b) \pm \sqrt{a^2 + b^2 + 2ab}}{4}$$

$$y = \frac{(a-3b) \pm \sqrt{(a+b)^2}}{4}$$

$$y = \frac{(a-3b) \pm (a+b)}{4}$$

$y = \frac{\not{a} - 3b - \not{a} - b}{4}$ or $y = \frac{a - 3b + a + b}{4}$

$y = \frac{-4b}{4}$ or $y = \frac{2a - 2b}{4}$

$y = \frac{2(a-b)}{4}$

$y = -b$ or $y = \frac{a-b}{2}$

Putting these values of y in equation (iii)

when $y = -b$

$x = a - b - y$

$x = a - b - (-b)$

$x = a - \not{b} + \not{b}$

$x = a$

when $y = \frac{a-b}{2}$

$x = a - b - y$

$x = a - b - \frac{a-b}{2}$

$x = \frac{2a - 2b - a + b}{2}$

$x = \frac{a-b}{2}$

Solution Set is $\left\{(a,-b),\left(\frac{a-b}{2},\frac{a-b}{2}\right)\right\}$

Q. 5 $x^2 + (y-1)^2 = 10$
$x^2 + y^2 + 4x = 1$

Solution:

$x^2 + (y-1)^2 = 10$ .... (i)

$x^2 + y^2 + 4x = 1$ ....(ii)

Subtracting equation (ii) from (i)

$$\cancel{x^2} + \cancel{y^2} + 1 - 2y = 10$$
$$\pm \cancel{x^2} \pm \cancel{y^2} \quad \pm 4x = \pm 1$$

$$-4x - 2y + 1 = 9$$
$$-4x - 2y = 9 - 1$$
$$-4x - 2y = 8$$
$$-2(2x + y) = 8$$
$$\Rightarrow \quad 2x + y = \frac{8}{-2}$$
$$2x + y = -4$$
$$y = -4 - 2x \quad ....(iii)$$

Put in equation (ii)

$$x^2 + (-4 - 2x)^2 + 4x = 1$$
$$x^2 + [-(4 + 2x)]^2 + 4x = 1$$
$$x^2 + [16 + 4x^2 + 16x] + 4x = 1$$
$$5x^2 + 20x + 16 - 1 = 0$$
$$5x^2 + 20x + 15 = 0$$
$$5(x^2 + 4x + 3) = 0$$
$$\Rightarrow \quad x^2 + 4x + 3 = 0 \qquad (\because 5 \neq 0)$$
$$x^2 + 3x + x + 3 = 0$$
$$x(x + 3) + 1(x + 3) = 0$$
$$(x + 3)(x + 1) = 0$$

Either $x + 3 = 0$ or $x + 1 = 0$

$x = -3$ or $x = -1$

Putting these values of x in equation (iii)

when $x = -3$	when $x = -1$
$y = -4 - 2x$	$y = -4 - 2x$
$y = -4 - 2(-3)$	$y = -4 - 2(-1)$
$y = -4 + 6$	$y = -4 + 2$
$y = 2$	$y = -2$

So, the solution Set is $\{(-3, 2), (-1, -2)\}$

Q.6 $(x+1)^2 + (y+1)^2 = 5, (x+2)^2 + y^2 = 5$

Solution: $(x+1)^2 + (y+1)^2 = 5$ ....(i)

$(x+2)^2 + y^2 = 5$ ....(ii)

From equation (i)

$$x^2 + 1 + 2x + y^2 + 1 + 2y = 5$$
$$x^2 + y^2 + 2x + 2y + 2 = 5$$
$$x^2 + y^2 + 2x + 2y = 3 \quad .... (iii)$$

From equation (ii)

$$(x+2)^2 + y^2 = 5$$
$$x^2 + 4 + 4x + y^2 = 5$$
$$x^2 + y^2 + 4x = 5 - 4$$
$$x^2 + y^2 + 4x = 1 \quad .... (iv)$$

Subtracting equation (iv) from (iii)

$$\cancel{x^2} + \cancel{y^2} + 2x + 2y = 3$$
$$\pm \cancel{x^2} \pm \cancel{y^2} \pm 4x \quad = \pm 1$$

$$-2x + 2y = 2$$
$$-2(x - y) = 2$$
$$x - y = \frac{2}{-2}$$
$$x - y = -1$$
$$x = y - 1 \quad ....(v)$$

Put it in equation (iv)

$$(y-1)^2 + y^2 + 4(y-1) = 1$$
$$y^2 + 1 - 2y + y^2 + 4y - 4 = 1$$
$$2y^2 + 2y - 4 + \cancel{1} - \cancel{1} = 0$$
$$2y^2 + 2y - 4 = 0$$
$$2(y^2 + y - 2) = 0$$
$$\Rightarrow \quad y^2 + y - 2 = 0 \qquad (\because 2 \neq 0)$$
$$y^2 + y - 2 = 0$$
$$y^2 + 2y - y - 2 = 0$$
$$y(y+2) - 1(y+2) = 0$$
$$(y+2)(y-1) = 0$$

Either $y + 2 = 0$ or $y - 1 = 0$

$y = -2$ $y = 1$

Putting these values of y in equation (v)

when $y = -2$	when $y = 1$
$x = y - 1$	$x = y - 1$
$x = -2 - 1$	$x = 1 - 1$
$x = -3$	$x = 0$

So, the solution Set is $\{(-3, -2), (0, 1)\}$

Q. 7 $x^2 + 2y^2 = 22$

$5x^2 + y^2 = 29$

Solution:

$x^2 + 2y^2 = 22$ .... (i)

$5x^2 + y^2 = 29$ .... (ii)

Multiplying equation (ii) by "2" we have

$10x^2 + 2y^2 = 58$ .... (iii)

Subtracting equation (i) from (iii)

$10x^2 + \cancel{2y^2} = 58$

$\pm x^2 \quad \pm \cancel{2y^2} = \pm 22$

$9x^2 = 36$

$x^2 = \frac{36}{9}$

$x^2 = 4$

Taking square root, we have

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

$\Rightarrow x = -2$ or $x = 2$

Now putting these values of x in equation (i)

When $x = -2$	When $x = 2$
$x^2 + 2y^2 = 22$	$x^2 + 2y^2 = 22$
$(-2)^2 + 2y^2 = 22$	$(2)^2 + 2y^2 = 22$
$4 + 2y^2 = 22$	$4 + 2y^2 = 22$
$2y^2 = 22 - 4$	$2y^2 = 22 - 4$
$2y^2 = 18$	$2y^2 = 18$
$y^2 = \frac{18}{2}$	$y^2 = \frac{18}{2}$
$y^2 = 9$	$y^2 = 9$
$\Rightarrow y = \pm 3$	$\Rightarrow y = \pm 3$

So, the solution set is $\{(\pm 2, \pm 3)\}$

Q.8 $4x^2 - 5y^2 = 6$

$3x^2 + y^2 = 14$

Solution:

$4x^2 - 5y^2 = 6$ .... (i)

$3x^2 + y^2 = 14$ .... (ii)

Multiplying equation (ii) by 5 and add in equation (i)

$4x^2 - \cancel{5y^2} = 6$

$15x^2 + \cancel{5y^2} = 70$

$19x^2 = 76$

$x = \frac{76}{19}$

$x^2 = 4$

$\Rightarrow x = \pm 2$

Either $x = 2$ or $x = -2$

Putting these values of x in equation (ii)

When x = 2,	When x = −2
$3(2)^2 + y^2 = 14$	$3(-2)^2 + y^2 = 14$
$3(4) + y^2 = 14$	$3(4) + y^2 = 14$
$12 + y^2 = 14$	$12 + y^2 = 14$
$y^2 = 14 - 12$	$y^2 = 14 - 12$
$y^2 = 2$	$y^2 = 2$
$y = \pm\sqrt{2}$	$y = \pm\sqrt{2}$

So the solution set is $\{(\pm 2, \pm\sqrt{2})\}$

Q. 9 $7x^2 - 3y^2 = 4$

$2x^2 + 5y^2 = 7$

Solution:

$7x^2 - 3y^2 = 4$ ———— (i)

$2x^2 + 5y^2 = 7$ ———— (ii)

Multiply equation (i) by 5 and equation (ii) by 3 and add them

$35x^2 - \cancel{15y^2} = 20$

$6x^2 + \cancel{15y^2} = 21$

$41x^2 = 41$

$x^2 = \frac{41}{41}$

$x^2 = 1$

$x = \pm\sqrt{1}$

$x = \pm 1$

Either x = 1 or x = − 1

Putting these values of x in equation (i)

When x = 1	When x = −1
$7(1)^2-3y^2=4$	$7(-1)^2-3y^2=4$
$7-3y^2=4$	$7(1)-3y^2=4$
$-3y^2=4-7$	$7-3y^2=4$
$-3y^2=-3$	$-3y^2=4-7$
$y^2=\frac{-3}{-3}$	$-3y^2=-3$
$y^2=1$	$y^2=\frac{-3}{-3}$
$y=\pm\sqrt{1}$	$y^2=1$
$y=\pm 1$	$y=\pm 1$

So the solution set is $\{(\pm 1,\pm 1)\}$

Q. 10 $\mathbf{x^2+2y^2=3}$

$\mathbf{x^2+4xy-5y^2=0}$

Solution:

$x^2+2y^2=3$ ——— (i)

$x^2+4xy-5y^2=0$ ——— (ii)

Factorizing equation (ii) we get

$x^2+4xy-5y^2=0$

$x^2+5xy-xy-5y^2=0$

$x(x+5y)-y(x+5y)=0$

$(x+5y)(x-y)=0$

Either $x+5y=0$ or $x-y=0$

$x=-5y$ —— (iii) $x=y$ —— (iv)

Putting these values of x in equation (i)

When x = −5y	When x = y
$(-5y)^2+2y^2=3$	$y^2+2(y^2)=3$
$25y^2+2y^2=3$	$3y^2=3$
$27y^2=3$	$y^2=\frac{3}{3}$
$y^2=\frac{3}{27}$	$y^2=1$
$y^2=\frac{1}{9}$	$y=\pm\sqrt{1}$
$y=\pm\sqrt{\frac{1}{9}}$	$y=-1$, or $y=1$
$y=\pm\frac{1}{3}$	
$y=\frac{1}{3}$ or $y=\frac{-1}{3}$	

Putting the value of $y=\pm\frac{1}{3}$ in equation (iii)

When $y=\frac{1}{3}$	When $y=\frac{-1}{3}$
$x=-5y$	$x=-5y$
$x=-5\left(\frac{1}{3}\right)$	$x=-5\left(\frac{-1}{3}\right)$
$x=\frac{-5}{3}$	$x=\frac{5}{3}$

Now putting the values of y=± 1in equation (iv)

$x=y$

When y = 1 then x = 1

When y = −1 then x = −1

Solution Set is $\left\{(-1,-1),(1,1),\left(\frac{5}{3},\frac{-1}{3}\right),\left(\frac{-5}{3},\frac{1}{3}\right)\right\}$

Q. 11 $\mathbf{3x^2-y^2=26}$

$\mathbf{3x^2-5xy-12y^2=0}$

Solution:

$3x^2-y^2=26$ ——— (i)

$3x^2-5xy-12y^2=0$ ——— (ii)

Factorizing equation (ii)

$3x^2-5xy-12y^2=0$

$3x^2-9xy+4xy-12y^2=0$

$3x(x-3y)+4y(x-3y)=0$

$(x-3y)(3x+4y)=0$

Either

$x-3y=0$ or $3x+4y=0$

$x=3y$ ——— (iii) $3x=(-4y)$

$x=\frac{-4y}{3}$ ...(iv)

From equation (iii) putting the value of x in equation (i)

$3(3y)^2-y^2=26$

$3(9y)^2-y^2=26$

$27y^2-y^2=26$

$26y^2=26$

$y^2=1$

$\Rightarrow y=\pm 1$

$y=1$ *or* $y=-1$

Putting these value of y in equation (iii)

When y = 1	When y = −1
$x=3y$	$x=3y$
$x=3(1)$	$x=3(-1)$
$x=3$	$x=-3$
$(x,y)=(3,1)$	$(x,y)=(-3,-1)$

From equation (iv) putting the values of x in equation (i)

$$3\left(\frac{-4y}{3}\right)^2-y^2=26$$

$$3\times\frac{16y^2}{9}-y^2=26$$

$$\frac{48y^2-9y^2}{9}=26$$

$$39y^2=26\times 9$$

$$y^2=\frac{234}{39}$$

$$y^2=6$$

$$\Rightarrow \quad y=\pm\sqrt{6}$$

$$y=\sqrt{6} \quad or \quad y=-\sqrt{6}$$

Putting these values of y in equation (iv)

When y = $\sqrt{6}$	When y = $-\sqrt{6}$
$x=\frac{-4y}{3}$	$x=\frac{-4y}{3}$
$x=\frac{-4\sqrt{6}}{3}$	$x=\frac{-4(-\sqrt{6})}{3}$
$(x,y)=\left(\frac{-4\sqrt{6}}{3},\sqrt{6}\right)$	$x=\frac{4\sqrt{6}}{3}$
	$(x,y)=\left(\frac{4\sqrt{6}}{3},-\sqrt{6}\right)$

So, the Solution set is

$$\left\{(3,1),(-3,-1),\left(\frac{-4\sqrt{6}}{3},\sqrt{6}\right),\left(\frac{4\sqrt{6}}{3},-\sqrt{6}\right)\right\}$$

Q. 12 $x^2+xy=5$ ——— (i)

$y^2+xy=3$ ——— (ii)

Multiply equation (i) by 3 and equation (ii) by 5 and subtract them

$$3x^2+3xy=15$$

$$\pm 5xy \pm 5y^2=-15$$

$$3x^2-2xy-5y^2=0$$

$$3x^2-5xy+3xy-5y^2=0$$

$$x(3x-5y)+y(3x-5y)=0$$

Either $(3x-5y)(x+y)=0$

3x−5y=0 or x + y = 0

3x=5y or $\boxed{x=-y}$ …(iv)

$\boxed{x=\frac{5y}{3}}$ ——(iii)

From equation (iv) put y = −x in equation (i)

$$(-y)^2+(-y)y=5$$

$$y^2-y^2=5$$

$$0\neq 5$$

Impossible

Now from equation (iii) put $x=\frac{5y}{3}$ in equation (i)

$$\left(\frac{5y}{3}\right)^2+\frac{5y}{3}\times y=5$$

$$\frac{25y^2}{9}+\frac{5y^2}{3}=5$$

Multiply by 9

$$9\times\frac{25y^2}{9}+9\times\frac{5y^2}{3}=9\times 5$$

$$25y^2+15y^2=45$$

$$40y^2=45$$

$$y^2=\frac{45}{40}$$

$$y^2=\frac{9}{8}$$

$$y=\pm\sqrt{\frac{9}{8}}$$

$= \pm\sqrt{\dfrac{3^2}{4\times 2}}$

$y = \pm\dfrac{3}{2\sqrt{2}}$

$y = \dfrac{3}{2\sqrt{2}}$ or $y = \dfrac{-3}{2\sqrt{2}}$

Now putting the value of y in equation (iii)

When $y = \dfrac{3}{2\sqrt{2}}$

Then $x = \dfrac{5}{3}\times\dfrac{3}{2\sqrt{2}}$

$x = \dfrac{5}{2\sqrt{2}}$

$\left(\dfrac{5}{2\sqrt{2}}, \dfrac{3}{2\sqrt{2}}\right)$

When $y = \dfrac{-3}{2\sqrt{2}}$

Then $x = \dfrac{5}{3}\times\left(\dfrac{-3}{2\sqrt{2}}\right)$

$x = \dfrac{-5}{2\sqrt{2}}$

$\left(\dfrac{-5}{2\sqrt{2}}, \dfrac{-3}{2\sqrt{2}}\right)$

Solution set is $\left\{\left(\dfrac{5}{2\sqrt{2}}, \dfrac{3}{2\sqrt{2}}\right), \left(\dfrac{-5}{2\sqrt{2}}, \dfrac{-3}{2\sqrt{2}}\right)\right\}$

Q. 13 $x^2 - 2xy = 7$

$xy + 3y^2 = 2$

Solution:

$x^2 - 2xy = 7$ ——— (i)

$xy + 3y^2 = 2$ ——— (ii)

Multiplying equation (i) by 2 and equation (ii) by 7 and subtracting them, we get

$2x^2 - 4xy \quad = 14$

$\pm 7xy \pm 21y^2 = 14$

$2x^2 - 11xy - 21y^2 = 0$

$2x^2 - 14xy + 3xy - 21y^2 = 0$

$2x(x - 7y) + 3y(x - 7y) = 0$

$(x - 7y)(2x + 3y) = 0$

Either $x - 7y = 0$ or $2x + 3y = 0$

$x = 7y$ ——— (iii) or $2x = -3y$

or $x = \dfrac{-3}{2}y$ ——(iv)

From equation (iii) Put $x = 7y$ in equation (i)

$(7y)^2 - 2(7y)y = 7$

$49y^2 - 14y^2 = 7$

$35y^2 = 7$

$y^2 = \dfrac{7}{35}$

$y^2 = \dfrac{1}{5}$

$y = \pm\dfrac{1}{\sqrt{5}}$

Either $y = \dfrac{1}{\sqrt{5}}$ or $y = \dfrac{-1}{\sqrt{5}}$

Putting these values of y in equation (iii)

When $y = \dfrac{1}{\sqrt{5}}$

$x = 7y$

Then $x = 7\left(\dfrac{1}{\sqrt{5}}\right)$

$x = \dfrac{7}{\sqrt{5}}$

$(x, y) = \left(\dfrac{7}{\sqrt{5}}, \dfrac{1}{\sqrt{5}}\right)$

When $y = \dfrac{-1}{\sqrt{5}}$

$x = 7y$

Then $x = 7\left(\dfrac{-1}{\sqrt{5}}\right)$

$x = \dfrac{-7}{\sqrt{5}}$

$(x, y) = \left(\dfrac{-7}{\sqrt{5}}, \dfrac{-1}{\sqrt{5}}\right)$

From equation (iv) putting the value of x in equation (i)

$\left(\dfrac{-3}{2}y\right)^2 - 2\left(\dfrac{-3}{2}y\right)y = 7$

$\dfrac{9}{4}y^2 + 3y^2 = 7$

$9y^2 + 12y^2 = 28$

$21y^2 = 28$

$y^2 = \dfrac{28}{21}$

$y^2 = \dfrac{4}{3}$

$\sqrt{y^2} = \pm\sqrt{\dfrac{4}{3}}$

$y = \pm\dfrac{2}{\sqrt{3}}$

Either $y = \dfrac{2}{\sqrt{3}}$ or $y = \dfrac{-2}{\sqrt{3}}$

Putting these values of y in equation (iv)

When $y=\frac{2}{\sqrt{3}}$

Then $x=\frac{-3}{2}\left(\frac{2}{\sqrt{3}}\right)$

$x=-\sqrt{3}$

$(x,y)=\left(-\sqrt{3},\frac{2}{\sqrt{3}}\right)$

When $y=\frac{-2}{\sqrt{3}}$

Then $x=-\frac{3}{2}\left(\frac{-2}{\sqrt{3}}\right)$

$x=\sqrt{3}$

$(x,y)=\left(\sqrt{3},\frac{-2}{\sqrt{3}}\right)$

So, the Solution set is

$$\left\{\left(\frac{7}{\sqrt{5}},\frac{1}{\sqrt{5}}\right),\left(\frac{-7}{\sqrt{5}},\frac{-1}{\sqrt{5}}\right),\left(-\sqrt{3},\frac{2}{\sqrt{3}}\right),\left(\sqrt{3},\frac{-2}{\sqrt{3}}\right)\right\}$$

EXERCISE 2.8

Q. 1 The product of two positive consecutive numbers is 182. Find the numbers.

Solution:

Suppose first positive number = x

Very next positive number = x + 1

By given condition

$$x(x+1)=182$$
$$x^2+x-182=0$$
$$x^2+14x-13x-182=0$$
$$x(x+14)-13(x+14)=0$$
$$(x+14)(x-13)=0$$

Either $x+14=0$ or $x-13=0$

$x=-14$ or $x=13$

As x is positive number therefore we neglect the negative value, So x = 13

Thus first positive number = x = 13

Very next positive number = x + 1

= 13+1 = 14

Thus 13 and 14 are two required consecutive positive numbers.

Q. 2 The sum of the squares of three positive consecutive numbers is 77. Find them.

Solution:

Let $x,(x+1)$ and $(x+2)$ be the three consecutive positive number

By Give condition

$$x^2+(x+1)^2+(x+2)^2=77$$
$$x^2+[x^2+(1)^2+2(1)(x)]+[(x)^2+(2)^2+2(x)(2)]=77$$
$$x^2+x^2+1+2x+x^2+4+4x=77$$
$$3x^2+6x+5-77=0$$
$$3x^2+6x-72=0$$
$$3\left[x^2+2x-24\right]=0$$
$$\therefore x^2+2x-24=0 \qquad (\because 3\neq 0)$$
$$x^2+6x-4x-24=0$$
$$x(x+6)-4(x+6)=0$$
$$(x+6)(x-4)=0$$

Either $x+6=0$ or $x-4=0$

$x=-6$ or $x=4$

As x is a positive number therefore we neglect the negative value and we take positive value of x i.e. x = 4

1st Number $= x = 4$

2nd Number $= x+1=4+1=5$

3rd Number $= x+2=4+2=6$

Thus 4, 5 and 6 are three required positive numbers.

Q. 3 The sum of five times a number and the square of the number is 204. Find the number.

Solution:

Let required number = x

Five times the number = 5x

Square of the number = x^2

By given condition

$$x^2+5x=204$$
$$x^2+5x-204=0$$
$$x^2+17x-12x-204=0$$
$$x(x+17)-12(x+17)=0$$
$$(x+17)(x-12)=0$$

Either

$x+17=0$ or $x-12=0$

$\boxed{x=-17}$ or $\boxed{x=12}$

Thus required number is –17 or 12.

Q. 4 The product of five less than three times a certain number and one less than four times the number is 7. Find the number.

Solution:

Let required number = x

Five less than three times the number = 3x–5

One less than four times the number = 4x–1

By given condition

$$(3x-5)(4x-1)=7$$
$$12x^2-3x-20x+5-7=0$$
$$12x^2-23x-2=0$$
$$12x^2-24x+x-2=0$$
$$12x(x-2)+1(x-2)=0$$
$$(x-2)(12x+1)=0$$

$x-2=0$ or $12x+1=0$
$x=2$ or $12x=-1$
$x=2$ or $x=\frac{-1}{12}$

Thus required number is 2 or $\frac{-1}{12}$

Q. 5 The difference of a number and its reciprocal is $\frac{15}{4}$. Find the number.

Solution:

Let required number = x

Reciprocal of the number = $\frac{1}{x}$

By given condition

$$x-\frac{1}{x}=\frac{15}{4}$$
$$\frac{x^2-1}{x}=\frac{15}{4}$$
$$4(x^2-1)=15x$$
$$4x^2-4-15x=0$$
$$4x^2-15x-4=0$$
$$4x^2-16x+1x-4=0$$
$$4x(x-4)+1(x-4)=0$$
$$(x-4)(4x+1)=0$$

Either $x-4=0$ or $4x+1=0$
$x=0+4$ or $4x=-1$
$\boxed{x=4}$ or $\boxed{x=\frac{-1}{4}}$

Thus required numbers is 4 or $\frac{-1}{4}$.

Q. 6 The sum of the squares of two digits of a positive integral number is 65 and the number is 9 times the sum of its digits. Find the number.

Solution:

Let

Digits at unit's place of a number = x

Digit at ten's place of a number = y

Required number = 10y + x

By 1st condition

$$x^2+y^2=65 \quad \text{.................(i)}$$

By 2nd condition

$$10y+x=9(x+y)$$
$$10y+x=9x+9y$$
$$10y-9y=9x-x$$
$$y=8x \quad \text{.......................(ii)}$$

Put value of y in equation (i)

$$x^2+(8x)^2=65$$
$$x^2+64x^2=65$$
$$65x^2=65$$
$$x^2=1$$
$$\sqrt{x^2}=\pm\sqrt{1}$$
$$x=\pm1$$
$$x=1 \text{ or } x=-1$$

As x is a digit at unit's place which is always positive therefore we neglect the negative value and take the positive value i.e. $x=1$

Put x = 1 in equation (ii)

$$y=8(1)$$
$$y=8$$

So, required number $=10y+x$
$=10(8)+1$
$=80+1$
$=81$

Q. 7 The sum of the co–ordinates of a point is 9 and sum of their squares is 45. Find the co–ordinates of the point.

Solution:

Let (x, y) are co–ordinates of required point.

By given conditions

$$x+y=9 \quad \text{...............(i)}$$
$$x^2+y^2=45 \quad \text{...........(ii)}$$

From equation (i)

$$x+y=9$$
$$x=9-y \quad \text{.............. (iii)}$$

Putting this in equation (ii) , we get

$$(9-y)^2+y^2=45$$
$$(9)^2-2(9)(y)+(y)^2+(y)^2=45$$
$$81-18y+y^2+y^2=45$$
$$2y^2-18y+81-45=0$$

$$2y^2 - 18y + 36 = 0$$
$$2(y^2 - 9y + 8) = 0$$
$$\therefore \quad y^2 - 9y^2 + 18 = 0 \qquad (\because 2 \neq 0)$$
$$y^2 - 6y - 3y + 18 = 0$$
$$y(y-6) - 3(y-6) = 0$$
$$(y-6)(y-3) = 0$$

Either $y - 6 = 0$ or $y - 3 = 0$

$y = 6$ or $y = 3$

Putting the values of y in equation (iii), we get

When y = 6	When y = 3
$x = 9 - 6$	x = 9 – 3
$x = 3$	x = 6

Thus co–ordinates of the point are either (3, 6) or (6, 3)

Q. 8 Find two integers whose sum is 9 and the difference of their squares is also 9.

Solution: **02(105)**

Suppose x and y are two integer

By given conditions

$$x + y = 9 \text{ .... (i)}$$
$$x^2 - y^2 = 9 \text{ .... (ii)}$$

From equation (i)

$$x + y = 9$$
$$x = 9 - y \text{ .... (iii)}$$

Putting the value of x in equation (ii), we get

$$(9-y)^2 - y^2 = 9$$
$$(9)^2 + (y)^2 - 2(9)(y) - y^2 = 9$$
$$81 + \cancel{y^2} - 18y - \cancel{y^2} - 9 = 0$$
$$72 - 18y = 0$$
$$-18y = -72$$
$$y = \frac{-72}{-18}$$
$$y = 4$$

Putting the value of y in equation (iii), we get

$$x = 9 - y$$
$$x = 9 - 4$$
$$x = 5$$

So 4 and 5 are required integers.

Q. 9 Find two integers whose difference is 4 and whose squares differ by 72.

Solution:

Let x and y are two integers

By given conditions

$$x - y = 4 \text{ ....(i)}$$
$$x^2 - y^2 = 72 \text{ .... (ii)}$$

From equation (i)

$$x = 4 + y \text{ .... (iii)}$$

Putting the value of x in equation (ii), we get

$$(4+y)^2 - y^2 = 72$$
$$[(4)^2 + (y)^2 + 2(4)(y)] - y^2 = 72$$
$$16 + \cancel{y^2} + 8y - \cancel{y^2} = 72$$
$$16 + 8y = 72$$
$$8y = 72 - 16$$
$$8y = 56$$
$$y = \frac{56}{8}$$
$$y = 7$$

Putting the value of y in equation (iii)

$$x = 4 + y$$
$$x = 4 + 7$$
$$x = 11$$

So 7 and 11 are required integers

Q. 10 Find the dimensions of a rectangle, whose perimeter is 80cm and its area is $375cm^2$.

Solution:

Let width of rectangle = x cm

Length of rectangle = y cm

Perimeter of rectangle = 80cm

Area of rectangle = 375 cm^2

We know that

$$2(L + W) = P$$
$$2(x + y) = 80$$
$$x + y = \frac{80}{2}$$
$$x + y = 40 \text{ .... (i)}$$

Area = Length × Width

$375 = x \times y$

$\Rightarrow \quad xy = 375$ .... (ii)

From equation (i)

$$x + y = 40$$
$$y = 40 - x$$

Put it in equation (ii)

$$x(40-x)=375$$
$$40x-x^2=375$$
$$0=x^2-40x+375$$
$$\Rightarrow \quad x^2-40x+375=0$$
$$x^2-25x-15x+375=0$$
$$x(x-25)-15(x-25)=0$$
$$(x-25)(x-15)=0$$

Either $x-25=0$ or $x-15=0$

$x=25$ or $x=15$

Putting these values of x in equation (i), we get

When $x=25$	When $x=15$
Then $25+y=40$	Then $15+y=40$
$y=40-25$	$y=40-15$
$y=15$	$y=25$
If x = 25 then y = 15	If x = 15 then y = 25

Thus dimensions of rectangle is either 25cm by 15cm or 15cm by 25cm.

MISCELLANEOUS EXERCISE – 2

Q.1 Multiply Choice Questions.
Four possible answers are given for the following questions. Tick (✓) the correct answer.

1. $\frac{1}{\alpha}+\frac{1}{\beta}$ **is equal to**
(a) $\frac{1}{\alpha}$ (b) $\frac{1}{\alpha}-\frac{1}{\beta}$
(c) $\frac{\alpha-\beta}{\alpha\beta}$ (d) $\frac{\alpha+\beta}{\alpha\beta}$

2. Product of cube roots of unity is
(a) 0 (b) 1
(c) −1 (d) 3

3. Roots of the equation $4x^2-5x+2 = 0$ are
(a) irrational (b) imaginary
(c) rational (d) none of these

4. Two square roots of unity are
(a) $1, -1$ (b) $1, \omega$
(c) $1, -\omega$ (d) ω, ω^2

5. Roots of the equation $4x^2-4x+1=0$ are
(a) real, equal (b) real, unequal
(c) imaginary (d) irrational

6. If α, β are the roots of $7x^2-x+4=0$ then $\alpha\beta$ is……..
(a) $\frac{-1}{7}$ (b) $\frac{4}{7}$
(c) $\frac{7}{4}$ (d) $\frac{-4}{7}$

7. If $b^2 - 4ac < 0$ then the roots of $ax^2+bx+c= 0$ are
(a) irrational (b) rational
(c) imaginary (d) None of these

8. If α, β are the roots of $px^2+qx+r = 0$, then sum of the roots 2α and 2β is
(a) $\frac{-q}{p}$ (b) $\frac{r}{p}$
(c) $\frac{-2q}{p}$ (d) $-\frac{q}{2p}$

9. If α, β are the roots of $3x^2+5x-2=0$ then $\alpha+\beta$ is……
(a) $\frac{5}{3}$ (b) $\frac{3}{5}$
(c) $\frac{-5}{3}$ (d) $\frac{-2}{3}$

10. If $b^2 - 4ac > 0$ and is a perfect square, then roots of $ax^2 + bx + c = 0$ are
(a) irrational, equal
(b) rational, equal
(c) rational, unequal
(d) irrational, unequal

11. Cube roots of −1 are
(a) $-1, -\omega, -\omega^2$ (b) $-1, \omega, -\omega^2$
(c) $-1, -\omega, \omega^2$ (d) $1, -\omega, -\omega^2$

12. Sum of the cube roots of unity is
(a) 0 (b) 1
(c) −1 (d) 3

13. If $b^2 - 4ac > 0$, but not a perfect square then roots of $ax^2+bx+c=0$ are
(a) imaginary (b) rational
(c) irrational (d) None of these

14. If α, β are the roots of $x^2-x-1=0$, then product of the roots 2α and 2β is
(a) −2 (b) 2
(c) 4 (d) −4

15. Disc. of $x^2 - 3x + 3 = 0$ is ......
(a) 6 (b) 12
(c) 21 (d) −3

16. The discriminant of $ax^2+bx+c=0$ is
(a) b^2-4ac (b) b^2+4ac
(c) $-b^2+4ac$ (d) $-b^2-4ac$

17. $\alpha^2 + \beta^2$ is equal to
(a) $\alpha^2 - \beta^2$ (b) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$
(c) $(a+\beta)^2 - 2\alpha\beta$ (d) $\alpha + \beta$

18. If roots of a quadratic equation are irrational and distinct, then Disc. is.......
(a) perfect square
(b) not perfect square
(c) zero
(d) negative

19. Disc. of $2x^2 - 7x + 1 = 0$ is .....
(a) 47 (b) 41
(c) 40 (d) 51

20. The nature of the roots of equation $ax^2+bx+c = 0$ is determined by.
(a) Sum of the roots
(b) Product of the roots
(c) Synthetic division
(d) Discriminant

21. If for a quadratic equation $b^2 - 4ac = 205$, then roots are...
(a) complex (b) irrational
(c) rational (d) equal

22. If roots of a quadratic equation are equal, then Disc. is............
(a) positive (b) negative
(c) zero (d) irrational

23. If roots of a quadratic equation are real, rational and unequal then possible value of Disc. is...........
(a) 0 (b) 36
(c) 40 (d) −25

24. If roots of an quadratic equation are real and distinct then Disc. is ....
(a) positive (b) negative
(c) zero (d) imaginary

25. If ω and ω^2 are complex cube root of unity, then $\omega.\omega^2$=..........
(a) 1 (b) −1
(c) 0 (d) 2

26. If $b^2 - 4ac = 0$, then roots of $ax^2 + bx + c = 0$ are
(a) irrational, equal
(b) rational, equal
(c) rational, unequal
(d) irrational, unequal

27. If for a quadratic equation $b^2 - 4ac = 49$, then roots are real and ....
(a) equal (b) unequal
(c) irrational (d) imaginary

28. If roots of a quadratic equation are imaginary, and unequal, the possible value of Disc. is..........
(a) 0 (b) 9
(c) 8 (d) −9

29. If for a quadratic equation $b^2 - 4ac = 0$, then roots are .
(a) complex (b) irrational
(c) repeated (d) distinct

30. The roots of $x^2 + 8x + 16 = 0$ are...
(a) imaginary (b) equal
(c) unequal (d) irrational

31. If $\omega = \frac{-1-\sqrt{-3}}{2}$, then ω^2=.....
(a) $\frac{-1\pm\sqrt{3}}{2}$ (b) $\frac{-1+\sqrt{3}}{2}$
(c) $\frac{-1+\sqrt{-3}}{2}$ (d) $\frac{-1\pm\sqrt{-3}}{2}$

32. If 1, ω, ω^2 are cube root of unity, then $1+\omega+\omega^2=$ ....
(a) 0 (b) ω^3
(c) 1 (d) -1

33. If roots of a quadratic equation are imaginary, then Disc. is......
(a) positive (b) negative
(c) zero (d) irrational

34. Cube root of 64 are.............
(a) $-4,-4\omega,-4\omega^2$ (b) $4,16\omega$
(c) $4,4\omega,4\omega^2$ (d) $(4)^3$

35. If for a quadratic equation $b^2-4ac=-47$, then roots are.....
(a) real (b) rational
(c) irrational (d) complex

36. Which of the following is true description of nature of roots of a quadratic equation?
(a) real, irrational, equal
(b) real, imaginary, unequal
(c) real, irrational, unequal
(d) complex, repeated, rational

37. If roots of a quadratic equation are rational and distinct, then Disc. is.......
(a) perfect square
(b) not perfect square
(c) zero
(d) negative

38. If ω is complex cube root of unity, then $\omega^{63}=$ ....
(a) ω (b) 1
(c) $-\omega$ (d) $-\omega^2$

39. If roots of a quadratic equation are real, rational and equal, then possible value of Disc. is............
(a) 0 (b) 36
(c) 40 (d) -49

40. If ω is complex cube root of unity, then $\omega^{-16}=$ ....
(a) ω (b) $-\omega$
(c) $-\omega^2$ (d) ω^2

41. $\left(-1+\sqrt{-3}\right)^3=$ ....
(a) 8 (b) 1
(c) -4 (d) -28

42. Cube roots of -27 are..........
(a) $3,-3\omega,3\omega^2$
(b) $-3,-3\omega,-3\omega^2$
(c) $-3,3\omega,3\omega^2$
(d) $3,3\omega,-3\omega^2$

43. If ω is complex cube root of unity, then $\omega^{-27}=$ ....
(a) 1 (b) -1
(c) ω (d) ω^2

44. $\left(9+4\omega+4\omega^2\right)^3=$ ....
(a) 15 (b) 25
(c) 125 (d) $(17)^3$

45. $\omega^4=$ ....
(a) ω^2 (b) ω
(c) 1 (d) 0

46. Which of the following shows "the product of two consecutive positive numbers."
(a) $x(x+1)$ (b) $x(x+2)$
(c) $x(x+3)$ (d) $x(x+4)$

47. If 1, ω, ω^2 are cube root of unity, then $1+\omega=$ ....
(a) 0 (b) ω
(c) ω^2 (d) $-\omega^2$

48. If ω is complex cube root of unity, then $\omega^7=$ ....
(a) ω (b) $-\omega$
(c) ω^2 (d) $-\omega^2$

49. If 1, ω, ω^2 are cube root of unity, then $1+\omega^2=$ ....
(a) $-\omega$ (b) ω
(c) ω^2 (d) $-\omega^2$

50. "Five less than three times a certain number" is
(a) $3x-5$ (b) $3x+5$
(c) $5x+3$ (d) $5x-3$

51. If 1, ω, ω^2 are cube root of unity, then $\omega+\omega^2=$ ....
(a) 1 (b) -1
(c) ω^3 (d) $2\omega^2$

52. If roots of a quadratic equation are real, irrational and unequal then possible value of Disc. is…………

(a) 0 (b) 9
(c) 5 (d) –7

53. $(1-\omega-\omega^2)^5 =$ ………

(a) 6 (b) 16
(c) 32 (d) 64

54. If length and width of a rectangle are x and y respectively then which of the following shows perimeter?

(a) $(x+y)^2$ (b) $2x-2y$
(c) $2xy$ (d) $2(x+y)$

55. If ω is complex cube root of unity, then $\omega^{23} =$ ………..

(a) ω (b) $-\omega$
(c) ω^2 (d) $-\omega^2$

56. Which of the following are symmetric functions of the roots of a quadratic equation?

(a) $\alpha^2+\beta^2$ (b) $\alpha^3+\beta^3$
(c) $\frac{1}{\alpha}+\frac{1}{\beta}$ (d) all of these

57. If ω is complex cube root of unity, then $\omega^{-5} =$ ……………

(a) ω (b) 1
(c) $-\omega$ (d) $-\omega^2$

58. The equation $x^4-49x^2+36x+252=0$ is called ………. equation.

(a) quadratic (b) quartic
(c) linear (d) cubic

59. $(1-3\omega-3\omega^2)^3 =$……………..

(a) 12 (b) 16
(c) –125 (d) 64

60. Cube roots of 8 are…………

(a) $2, 2\omega, 2\omega^2$
(b) $-2, -2\omega, -2\omega^2$
(c) $2, -2\omega, -2\omega^2$
(d) $2, -2\omega, 2\omega^2$

61. The sum of five times a number and the square of the number is ……

(a) $5x^2+x$ (b) $5x+x^2$
(c) $(5x+x)^2$ (d) $5(x+x^2)$

ANSWER KEY

1.	d	2.	b	3.	b	4.	a	5.	a
6.	b	7.	c	8.	c	9.	c	10.	c
11.	a	12.	a	13.	c	14.	d	15.	d
16.	a	17.	c	18.	b	19.	b	20.	d
21.	b	22.	c	23.	b	24.	a	25.	a
26.	b	27.	b	28.	d	29.	c	30.	b
31.	c	32.	a	33.	b	34.	c	35.	d
36.	c	37.	a	38.	b	39.	a	40.	d
41.	a	42.	b	43.	a	44.	c	45.	b
46.	a	47.	d	48.	a	49.	a	50.	a
51.	b	52.	c	53.	c	54.	d	55.	c
56.	d	57.	a	58.	b	59.	d	60.	a
61.	b								

Q.2. Write short answers of the following questions.

(i) Discuss the nature of roots of the following equations.

(a) $x^2+3x+5=0$

Solution:

$x^2+3x+5=0$

$ax^2+bx+c=0$

⇒ a=1, b=3, c=5

Disc. $= b^2-4ac$

$= (3)^2-4(1)(5)$

$= 9-20$

$= -11 < 0$

So roots of equation are imaginary

(b) $2x^2-7x+3=0$

Solution:

$2x^2-7x+3=0$

$ax^2+bx+c=0$

⇒ a=2 , b = –7 , c=3

Disc. $= b^2-4ac$

$= (-7)^2 - 4(2)(3)$
$= 49 - 24$
$= 25$
$= (5)^2 > 0$

As Disc. is positive and perfect square so roots of equation are real, rational and distinct.

(c) $\mathbf{x^2 + 6x - 1 = 0}$

Solution:

$x^2 + 6x - 1 = 0$
$ax^2 + bx + c = 0$
$\Rightarrow$ $a = 1$, $b = 6$, $c = -1$
Disc. $= b^2 - 4ac$
$= (6)^2 - 4(1)(-1)$
$= 36 + 4$
$= 40 > 0$

As Disc. is positive and not perfect square, so roots of equations are real, irrational and unequal.

(d) $\mathbf{16x^2 - 8x + 1 = 0}$

Solution: $16x^2 - 8x + 1 = 0$
$ax^2 + bx + c = 0$
$\Rightarrow$ $a = 16$, $b = -8$, $c = 1$
Disc. $= b^2 - 4ac$
$= (-8)^2 - 4(16)(1)$
$= 64 - 64$
$= 0$

So roots of equation are real rational and equal.

(ii) Find ω^2, if $\omega = \dfrac{-1+\sqrt{-3}}{2}$

Solution: $\omega = \dfrac{-1+\sqrt{-3}}{2}$

Taking square of both sides, we get

$$\omega^2 = \left(\frac{-1+\sqrt{-3}}{2}\right)^2$$

$$\omega^2 = \frac{\left(-1+\sqrt{-3}\right)^2}{(2)^2}$$

$$\omega^2 = \frac{(-1)^2 + \left(\sqrt{-3}\right)^2 + 2(-1)\sqrt{-3})}{4}$$

$$\omega^2 = \frac{1 + (-3) - 2\sqrt{-3}}{4}$$

$$\omega^2 = \frac{-2 - 2\sqrt{-3}}{4}$$

$$\omega^2 = \frac{\cancel{2}(-1-\sqrt{-3})}{\cancel{4}2}$$

$$\omega^2 = \frac{-1-\sqrt{-3}}{2}$$

(iii) Prove that the sum of all the cube roots of unity is zero.

Solution: We have cube roots of unity

$$1, \omega = \frac{-1+\sqrt{3}}{2}, \omega^2 = \frac{-1-\sqrt{-3}}{2}$$

Taking sum, we get

$$1+\omega+\omega^2 = 1 + \frac{\left(-1+\sqrt{-3}\right)}{2} + \frac{\left(-1-\sqrt{-3}\right)}{2}$$

$$1+\omega+\omega^2 = 1 + \frac{(-1+\sqrt{-3})+(-1-\sqrt{-3})}{2}$$

$$1+\omega+\omega^2 = \frac{2+(-1+\cancel{\sqrt{-3}}-1-\cancel{\sqrt{-3}})}{2}$$

$$1+\omega+\omega^2 = \frac{2-2}{2}$$

$$1+\omega+\omega^2 = \frac{0}{2}$$

$$1+\omega+\omega^2 = 0$$

It is proved that sum of all the cube roots of unity is zero.

(iv) Find the product of complex cube roots of unity.

Solution: Following are the complex cube roots of unity

$$\omega = \frac{-1+\sqrt{-3}}{2} \quad \omega^2 = \frac{-1-\sqrt{-3}}{2}$$

Taking product, we get

$$\omega.\omega^2 = \left(\frac{-1+\sqrt{-3}}{2}\right)\left(\frac{-1-\sqrt{-3}}{2}\right)$$

$$\omega^3 = \frac{(-1)^2 - (\sqrt{-3})^2}{4} \qquad \because (a+b)(a-b) = a^2 - b^2$$

$$\omega^3 = \frac{1-(-3)}{4}$$

$\omega^3 = \frac{1+3}{4}$

$\omega^3 = \frac{4}{4}$

$\omega^3 = 1$

Product of complex cube roots of unity is 1.

(v) Show that $x^3+y^3 = (x + y)(x + \omega y)(x + \omega^2 y)$

Solution: Let

R.H.S. $= (x + y)(x + \omega y)(x + \omega^2 y)$

$= (x+y)[x^2+\omega^2 xy+\omega xy+\omega^3 y^2]$

$= (x+y)[x^2 + xy(\omega^2+\omega)(1)y^2]$ $\boxed{\because \omega^3 = 1}$

$= (x + y)[x + xy(-1) + y^2]$ $\boxed{\because \omega+\omega^2 = -1}$

$= (x + y)(x^2 - xy + y^2)$

$= x^3 + y^3 =$ L.H.S.

(vi) Evaluate $\omega^{37} + \omega^{38} + 1$

Solution:

$\omega^{37} + \omega^{38} + 1 = \omega.\omega^{36} + \omega^2.\omega^{36} + 1$

$= \omega(\omega^3)^{12} + \omega^2(\omega^3)^{12} + 1$

$= \omega(1)^{12} + \omega^2(1)^{12} + 1$

$= \omega + \omega^2 + 1$ $(\because \omega^3 = 1)$

$= 0$ $(\because 1 + \omega + \omega^2 = 0)$

(vii) Evaluate $(1 - \omega + \omega^2)^6$

Solution:

$(1 + \omega^2 - \omega)^6 = (-\omega - \omega)^6$ $(\because \omega^2 = -\omega)$

$= (-2\omega)^6$

$= (-2)^6(\omega)^6$

$= 64\omega^6$

$= 64(\omega^3)^2$ $(\because \omega^3 = 1)$

$= 64(1)^2$

$= 64(1)$

$= 64$

(viii) If ω is cube root of unity, form an equation whose roots are 3ω and $3\omega^2$.

Solution:

Since 3ω and $3\omega^2$ are the roots of the required equation.

Sum of roots

$S = 3\omega + 3\omega^2$

$S = 3(\omega + \omega^2)$

$S = 3(-1)$

$S = -3$

Product of roots

$P = (3\omega)(3\omega^2)$

$P = 9\omega^3$

$P = 9(1)$ $(\because \omega^3 = 1)$

$P = 9$

Hence the required equation is

$x^2 - Sx + P = 0$

$x^2 - (-3)x + 9 = 0$

$x^2 + 3x + 9 = 0$

(ix) Using synthetic division, find the remainder and quotient when $(x^3 + 3x^2 + 2) \div (x - 2)$

Solution: As $x - 2 = 0 \Rightarrow x = 2$

Now write the coefficients of dividend in a row and x = 2 on the left side.

	1	3	0	2
2	↓	2	10	20
	1	5	10	22

Quotient $Q(x) = (x^2 + 5x + 10)$

Remainder $= 22$

(x) Using synthetic division, show that $x-2$ is the factor of x^3+x^2-7x+2.

Solution: Write the coefficients of dividend in a row and x = 2 on the left side.

$x^2 + x^2 - 7x + 2$

2	1	1	−7	2
	↓	2	6	−2
	1	3	−1	0

As remainder is zero, so by synthetic division $x - 2$ is factor of $x^3 + x^2 - 7x + 2$

(xi) Find the sum and product of the roots of the equation $2px^2+3qx-4r = 0$

Solution: $2px^2 + 3qx - 4r = 0$

$ax^2 + bx + c = 0$

$a = 2p$, $b = 3q$, $c = -4r$

Sum of roots $= S = \frac{-b}{a} = \frac{-3q}{2p}$

Product of roots $= P = \frac{c}{a} = \frac{-4r}{2p}$

$P = \frac{-2r}{p}$

(xii) Find $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ of the roots of the equation $x^2 - 4x + 3 = 0$

Solution: $x^2 - 4x + 3 = 0$

$ax^2 + bx + c = 0$

$\Rightarrow \quad a = 1,\ b = -4,\ c = 3$

If α and β are roots of the given equation then

Sum of roots $= \alpha + \beta = \frac{-b}{a} = \frac{-(-4)}{1} = 4$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$

Given that $\frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{\beta^2+\alpha^2}{\beta^2\alpha^2}$

$= \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$= \frac{(4)^2 - 2(3)}{(3)^2}$

$= \frac{16-6}{9}$

$= \frac{10}{9}$

(xiii) If α, β are the roots of $4x^2 - 3x + 6 = 0$ find

(a) $\alpha^2+\beta^2$ **(b)** $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ **(c)** $\alpha-\beta$

Solution:

$4x^2 - 3x + 6 = 0$

$ax^2 + bx + c = 0$

$\Rightarrow \quad a = 4,\ b = -3,\ c = 6$

As α and β are roots of the equation

Sum of roots $= \alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{4} = \frac{3}{4}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{6}{4} = \frac{3}{2}$

Now

(a) $\alpha^2 + \beta^2$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2(\alpha\beta)$

$= \left(\frac{3}{4}\right)^2 - 2\left(\frac{3}{2}\right)$

$= \frac{9}{16} - 3$

$= \frac{9-48}{16}$

$= \frac{-39}{16}$

(b) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \left[(a+b)^2 - 2ab\right] \div \alpha\beta$

$= \left[\left(\frac{3}{4}\right)^2 - 2\left(\frac{3}{2}\right)\right] \div \frac{3}{2}$

$= \left(\frac{9}{16} - 3\right) \times \frac{2}{3}$

$= \left(\frac{9-48}{16}\right) \times \frac{2}{3}$

$= \frac{-39}{16} \times \frac{2}{3}$

$= \frac{-13}{8}$

(c) $\alpha - \beta$

$(\alpha-\beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

$\sqrt{(\alpha-\beta)^2} = \sqrt{\alpha^2+\beta^2-2\alpha\beta}$

$\alpha - \beta = \sqrt{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta-2\alpha\beta}$

$\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \sqrt{\left(\frac{3}{4}\right)^2 - 4\left(\frac{3}{2}\right)}$

$= \sqrt{\frac{9}{16} - 6}$

$= \sqrt{\frac{9-96}{16}}$

$= \sqrt{\frac{-87}{16}}$

$= \frac{\sqrt{-87}}{4}$

(xiv) If α, β are the roots of $x^2 - 5x + 7 = 0$, find an equation whose roots are
(a) $-\alpha, -\beta$ **(b)** $2\alpha, 2\beta$

Solution: $x^2 - 5x + 7 = 0$

$ax^2 + bx + c = 0$

$\Rightarrow \quad a = 1, \ b = -5, \ c = 7$

Sum of roots $= S = \frac{-b}{a} = \frac{-(-5)}{1} = 5$

Product of roots $= P = \alpha\beta = \frac{c}{a} = \frac{7}{1} = 7$

Now if roots are $-\alpha, -\beta$

(a) Equation with the roots $-\alpha, -\beta$

Sum of roots $= S = (-\alpha) + (-\beta)$

$S = -(\alpha + \beta)$

$S = -(+5)$

$S = -5$

Product of roots $= P = (-\alpha)(-\beta)$

$P = \alpha\beta$

$P = 7$

So required equation

$x^2 - Sx + P = 0$

$x^2 - (-5)x + 7 = 0$

$x^2 + 5x + 7 = 0$

(b) The equation with the roots $2\alpha, 2\beta$

Sum of roots $= S = 2\alpha + 2\beta$

$= 2(\alpha + \beta)$

$= 2(5)$

$= 10$

Product of roots $= P = 2\alpha \times 2\beta$

$= 4\alpha\beta$

$= 4(7)$

$= 28$

So required equation is:

$x^2 - Sx + P = 0$

$x^2 - 10x + 28 = 0$

Q.3:Fill in the blanks

i. The discriminant of $ax^2+bx+c= 0$ is ______.

ii. If $b^2-4ac=0$ then roots of $ax^2+bx+c= 0$ are_________.

iii. If $b^2-4ac > 0$ then roots of $ax^2+bx+c= 0$ are_________.

iv. If $b^2-4ac<0$ then the root of $ax^2+bx+c= 0$ are________.

v. If $b^2-4ac>0$ and perfect square, then roots of $ax^2+bx+c= 0$ are________.

vi. If $b^2-4ac>0$ and not perfect square, then roots of $ax^2+bx+c= 0$ are________.

vii. If α, β are the roots of $ax^2+bx+c=0$, then sum of the roots is_________.

viii. If α, β are the roots of $ax^2+bx+c=0$, then product of the roots is_________.

ix. If α, β are the roots of $7x^2-5x+3=0$, then sum of the roots is_________.

x. If α, β are the roots of $5x^2+3x-9=0$, then product of the roots is_________.

xi. For a quadratic equation $ax^2+bx+c = 0$ $\frac{1}{\alpha\beta}$ is equal to_________.

xii. Cube roots of unity are _________.

xiii. Under usual notation sum of the cube roots of unity is__________.

xiv. If $1, \omega, \omega^2$ are the cube roots of unity, then ω^{-7} is equal to ________.

xv. If α, β are the roots of the quadratic equation, then the quadratic equation is written as ___________.

xvi. If 2ω and $2\omega^2$ are the roots of an equation, then equation is________.

ANSWER Key

i.	b^2-4ac	ix.	$\frac{5}{7}$
ii.	Equal	x.	$-\frac{9}{5}$
iii.	Real	xi.	$\frac{a}{c}$
iv.	Imaginary	xii.	$1,\omega,\omega^2$
v.	Rational	xiii.	Zero
vi.	Irrational	xiv.	ω^2
vii.	$-\frac{b}{a}$	xv.	$x^2-(\alpha+\beta)x+\alpha\beta=0$
viii.	$\frac{c}{a}$	xvi.	$x^2+2x+4=0$

EXERCISE 3.1

Q.1 Express the following as a ratio a : b and as a fraction in its simplest (lowest) form.

(i) Rs. 750 : Rs. 1250

$$=\frac{\text{Rs.}750}{\text{Rs.}1250}=\frac{75\div25}{125\div25}=\frac{3}{5}=3:5$$

(ii) 450 cm : 3 m ($\because$ 1m = 100 cm)

450 cm : 300 cm

$$=\frac{450\cancel{cm}}{300\cancel{cm}}=\frac{45}{30}=\frac{45\div15}{30\div15}=\frac{3}{2}=3:2$$

(iii) 4kg : 2kg 750g ($\because$ 1kg = 1000 g)

$$=4000g:(2000g+750g)$$

$$=4000g:2750g$$

$$=4000g:2750g$$

$$=\frac{400\cancel{0g}}{275\cancel{0g}}=\frac{400\div25}{275\div25}=\frac{16}{11}=16:11$$

(iv) 27min. 30 Second , 1 h

27min. 30 Second : 1 h

$$\boxed{\because\ 1h=60\min=60\times60s=3600s}$$

= (27 × 60 s + 30 s) : 3600 s

= (1620 s + 30 s) : 3600 s

= 1650 s : 3600 s

$$=\frac{1650}{3600}=\frac{165}{360}\text{ (dividing by 15)}$$

$$=\frac{11}{24}=11:24$$

(v) 75° : 225°

$$=\frac{75\times1^\circ}{225\times1^\circ}=\frac{75\div25}{225\div25}=\frac{3}{9}$$

$$=\frac{1}{3}=1:3$$

Q.2 In a class of 60 students, 25 students are girls and remaining students are boys. Compute the ratio of

(i). Boys to total students

(ii). Boys to girls

Solution:

Number of students = 60

Number of girls = 25

Number of boys = 60 – 25 = 35

(i) Ratio of boys to the total students

Number of boys : Total students

35 : 60

$$=\frac{35}{60}=\frac{35\div5}{60\div5}=\frac{7}{12}=7:12$$

(ii) Ratio of boys to the girls

Number of boys : Number of girls.

35 : 25

$$=\frac{35}{25}=\frac{35\div5}{25\div5}=\frac{7}{5}=7:5$$

Q.3 If 3(4x–5y) = 2x –7y, find the ratio x : y.

Solution: $3(4x-5y)=2x-7y$

$$12x-15y=2x-7y$$

$$12x-2x=15y-7y$$

$$10x=8y$$

$$\frac{x}{y}=\frac{8}{10}$$

$$\frac{x}{y}=\frac{8\div2}{10\div2}$$

$$\frac{x}{y}=\frac{4}{5}$$

x : y = 4 : 5

Q.4 Find the value of p, if the ratios 2p + 5 : 3p + 4 and 3 : 4 are equal.

Solution: 1st ratio = 2p + 5 : 3p + 4

2nd ratio = 3 : 4

According to given condition

$$2p+5:3p+4=3:4$$

$$\frac{2p+5}{3p+4}=\frac{3}{4}$$

$$4(2p+5)=3(3p+4)$$

$$8p+20=9p+12$$

$$20-12=9p-8p$$

$$8=p$$

$$\Rightarrow\quad \boxed{p=8}$$

Q.5 If the ratios 3x + 1 : 6 + 4x and 2 : 5 are equal. Find the value of x.

Solution: 1st Ratio = 3x + 1 : 6 + 4x

2nd Ratio = 2 : 5

According to given condition

$3x + 1 : 6 + 4x = 2 : 5$

$$\frac{3x+1}{6+4x} = \frac{2}{5}$$

$$5(3x+1) = 2(6+4x)$$

$$15x+5 = 12+8x$$

$$15x-8x = 12-5$$

$$7x = 7$$

$$x = \frac{7}{7} = 1$$

$$\boxed{x=1}$$

Q.6 Two numbers are in the ratio 5 : 8. If 9 is added to each number, we get a new ratio 8 : 11. Find the numbers.

Solution: Ratio between two numbers = 5 : 8

Number to be added = 9

New ratio = 8 : 11

Let 1st Number = 5x

2nd Number = 8x

By condition:

$5x + 9 : 8x + 9 = 8 : 11$

$$\frac{5x+9}{8x+9} = \frac{8}{11}$$

$$11(5x+9) = 8(8x+9)$$

$$55x+99 = 64x+72$$

$$99-72 = 64x-55x$$

$$27 = 9x$$

$$\frac{27}{9} = x$$

$$3 = x \Rightarrow x = 3$$

Thus 1st number = 5x = 5(3) = 15

2nd number = 8x = 8(3) = 24

Required numbers are 15 and 24.

Q.7 If 10 is added in each number of the ratio 4 : 13, we get a new ratio 1 : 2. What are the numbers?

Solution: Ratio of two numbers = 4 : 13

Number to be added = 10

New ratio = 1 : 2

Let 1st number = 4x

2nd number = 13 x

By condition:

$$4x+10 : 13x+10 = 1:2$$

$$\frac{4x+10}{13x+10} = \frac{1}{2}$$

$$2(4x+10) = 1(13x+10)$$

$$8x+20 = 13x+10$$

$$20-10 = 13x-8x$$

$$10 = 5x$$

$$\frac{10}{5} = x$$

$$2 = x \Rightarrow \boxed{x=2}$$

Thus 1st number = 4x = 4(2) = 8

2nd number = 13x = 13(2) = 26

Required numbers are 8 and 26.

Q.8 Find the cost of 8 kg of mangoes, if 5 kg of mangoes cost Rs. 250.

Solution: Mangoes = 5kg

Price = Rs. 250

Mangoes = 8kg

Let Price = x = ?

We know that

5kg : 8 kg = Rs. 250 : Rs. x

$$\frac{5}{8} = \frac{250}{x}$$

$$x \times 5 = 8 \times 250$$

$$x = \frac{8 \times 250}{5}$$

$$x = 8 \times 50$$

x = Rs.400

Q.9 If a : b = 7 : 6, find the value of 3a + 5b : 7b – 5a.

Solution: a : b = 7 : 6

$$\frac{a}{b}=\frac{7}{6}$$

$$3a+5b:7b-5a=\frac{3a+5b}{7b-5a}$$

Dividing numerator and denominator by "b".

$$=\frac{\frac{3a+5b}{b}}{\frac{7b-5a}{b}}$$

$$=\frac{3\frac{a}{b}+5\frac{b}{b}}{7\frac{b}{b}-5\frac{a}{b}}$$

Putting $\frac{a}{b}=\frac{7}{6}$, we get

$$=\frac{3\left(\frac{7}{6}\right)+5}{7-5\left(\frac{7}{6}\right)}$$

$$=\frac{\frac{21}{6}+5}{7-\frac{35}{6}}=\frac{\frac{21+30}{6}}{\frac{42-35}{6}}=\frac{51}{7}$$

3a+b: 7b–5a $=51:7$

Q.10 Complete the following:

(i) If $\frac{24}{7}=\frac{6}{x}$, then $4x=$ ....

(ii) If $\frac{5a}{3x}=\frac{15b}{y}$, then $ay=$ ....

(iii) If $\frac{9pq}{2\ell m}=\frac{18p}{5m}$, then $5q=$ ....

Solution:

(i) $\frac{24}{7}=\frac{6}{x}$

$24x=7\times6$

$6\times4x=7\times6$

$4x=\frac{7\times6}{6}$

$\boxed{4x=7}$

(ii) $\frac{5a}{3x}=\frac{15b}{y}$

$5ay=15b(3x)$

$5ay=45bx$

$ay=\frac{45bx}{5}$

$\boxed{ay=9bx}$

(iii) $\frac{9pq}{2\ell m}=\frac{18p}{5m}$

$5m(9pq)=18p(2\ell m)$

$5q=\frac{36\,p\ell m}{9pm}$

$5q=\frac{36^{4}\,p\ell m}{9\,p\,m}$

$\boxed{5q=4\ell}$

Q.11 Find x in the following proportions.

(i) $3x-2:4::2x+3:7$

(ii) $\frac{3x-1}{7}:\frac{3}{5}::\frac{2x}{3}:\frac{7}{5}$

(iii) $\frac{x-3}{2}:\frac{5}{x-1}::\frac{x-1}{3}:\frac{4}{x+4}$

(iv) $p^2+pq+q^2:x::\frac{p^3-q^3}{p+q}:(p-q)^2$

(v) $8-x:11-x::16-x:25-x$

Solution:

(i) $3x-2:4::2x+3:7$

Product of Extremes = Product of Means

$7(3x-2)=4(2x+3)$

$21x-14=8x+12$

$21x-8x=12+14$

$13x=26$

$x=\frac{26}{13}$

$\boxed{x=2}$

(ii) $\frac{3x-1}{7}:\frac{3}{5}::\frac{2x}{3}:\frac{7}{5}$

Product of Extremes = Product of Means

$$\left(\frac{3x-1}{7}\right)\left(\frac{7}{5}\right)=\left(\frac{3}{5}\right)\left(\frac{2x}{3}\right)$$

$$\frac{3x-1}{5}=\frac{2x}{5}$$

$$3x-1=\frac{2x}{5}\times 5$$

$$3x-1=2x$$

$$3x-2x=1$$

$$\boxed{x=1}$$

(iii) $\frac{x-3}{2}:\frac{5}{x-1}::\frac{x-1}{3}:\frac{4}{x+4}$

Product of Extremes = Product of Means

$$\left(\frac{x-3}{2}\right)\left(\frac{4}{x+4}\right)=\left(\frac{5}{x-1}\right)\left(\frac{x-1}{3}\right)$$

$$\frac{4x-12}{2x+8}=\frac{5}{3}$$

$$3(4x-12)=5(2x+8)$$

$$12x-36=10x+40$$

$$12x-10x=40+36$$

$$2x=76$$

$$x=\frac{76}{2}$$

$$\boxed{x=38}$$

(iv) $p^2+pq+q^2:x::\frac{p^3-q^3}{p+q}:(p-q)^2$

Product of Means = Product of Extremes

$$(x).\frac{(p^3-q^3)}{(p+q)}=(p^2+pq+q^2)(p-q)^2$$

$$x=\frac{(p^2+pq+q^2)(p-q)^2(p+q)}{p^3-q^3}$$

$$x=\frac{(p^2+pq+q^2)(p-q)(p-q)(p+q)}{(p-q)(p^2+pq+q^2)}$$

$$x=(p-q)(p+q)$$

$$\boxed{x=p^2-q^2}$$

(v) $8-x:11-x::16-x:25-x$

Product of Extremes = Product of Means

$$(8-x)(25-x)=(11-x)(16-x)$$

$$200-8x-25x+x^2=176-11x-16x+x^2$$

$$200-33x+x^2=176-27x+x^2$$

$$200-176=33x-x^2-27x+x^2$$

$$24=6x$$

$$\frac{24}{6}=x$$

$$4=x$$

$$\Rightarrow \boxed{x=4}$$

EXERCISE 3.2

Q.1 If y varies directly as x, and y = 8 when x = 2, find
(i) y in terms of x
(ii) y when x = 5
(iii) x when y = 28

Solution: $y \propto x$ | $y=8$, $x=2$ | $y=?$, $x=5$ | $x=?$, $y=28$

(i) $y \propto x$

$y = kx$ ……………..(i)

To find k,

Put $y = 8$ and $x = 2$ in equation (i)

$8 = k(2)$

$\frac{8}{2} = k$

$4 = k \Rightarrow$ $\boxed{k=4}$

(ii) To find y,

Put $k = 4$ in equation (i)

$y = kx$

$y = 4x$ …………(ii)

Now Put x = 5 in equation (ii)

$y = 4x$

$y = 4(5)$

$\boxed{y=20}$

(iii) To find x,

Put $y = 28$ in equation (ii)

$y = 4x$

$28 = 4(x)$

$28 = 4(x)$

$7 = x \Rightarrow$ $\boxed{x=7}$

Q.2 If $y \propto x$, and $y = 7$ when $x = 3$ find
(i) y in terms of x
(ii) x when y = 35 and y when x = 18

Solution:

$y \propto x$ | $y=7$, $x=3$ | $x=?$, $y=35$ | $x=18$, $y=?$

(i) $y \propto x$

$y = kx$ ……………..(i)

To find k,

Put $y = 7$ and $x = 3$ in equation (i)

$7 = k(3)$

$\frac{7}{3} = k \Rightarrow$ $\boxed{k=\frac{7}{3}}$

Put it in equation (i)

$y = kx$

$y = \frac{7}{3}x$ …………(ii)

To find *x*,

Put $y = 35$ in equation (ii)

$y = \frac{7}{3}x$

$35 = \frac{7}{3}x$

$\frac{3\times35}{7} = x$

$3\times5 = x$

$15 = x \Rightarrow$ $\boxed{x=15}$

To find *y*,

Put $k = \frac{7}{3}$ and $x = 18$ in equation (i)

$y = kx$

$y = \frac{7}{\cancel{3}}(\cancel{18}^{6})$

$y = 7(6)$

$\boxed{y=42}$

Q.3 If $R \propto T$ and $R = 5$ when $T = 8$, find the equation connecting R and T. Also find R. when T = 64 and T when R = 20.

Solution: $R \propto T$ | $R=5$, $T=8$ | $R=?$, $T=64$ | $R=20$, $T=?$

$R \propto T$

$R = KT$ ……………..(i)

To find k,

Put $R = 5$ and $T = 8$ in equation (i)

$5 = K(8)$

$\frac{5}{8} = k \Rightarrow$ $\boxed{k=\frac{5}{8}}$

Put $k=\frac{5}{8}$ in equation (i)

$R=kT$

$R=\frac{5}{8}T$ ....(ii)

To find *R*,

Put $T=64$ in equation (ii)

$R=\frac{5}{8}T$

$R=\frac{5}{8}(64)$ (8 cancelled into 64 → 8)

$R=5(8)$

$\boxed{R=40}$

To find *T*,

Put $K=\frac{5}{8}$ and $R=20$ in equation (i)

$R=KT$

$20=\frac{5}{8}(T)$

$T=\frac{20\times8}{5}$ (20 ÷ 5 → 4)

$4\times8=T$

$32=T \Rightarrow \boxed{T=32}$

Q.4 If $R\propto T^2$ and R = 8 when T = 3, find R when T = 6.

Solution: $\boxed{R\propto T^2}$ $\boxed{\begin{matrix}R=8\\T=3\end{matrix}}$ $\boxed{\begin{matrix}R=?\\T=6\end{matrix}}$

$R\propto T^2$

$R=kT^2$ ....(i)

To find k,

Put R = 8 and T = 3 in equation (i)

$8=k(3)^2$

$8=k(9)$

$\frac{8}{9}=k \Rightarrow \boxed{k=\frac{8}{9}}$

To find R,

Put $k=\frac{8}{9}$ and T = 6 in equation (i)

$R=kT^2$

$R=\frac{8}{9}(6)^2$

$R=\frac{8(36)}{9}$ (36 ÷ 9 → 4)

$R=8(4) \Rightarrow \boxed{R=32}$

Q.5 V ∝ R³ and V = 5 when R = 3, find R, when V = 625.

Solution: $\boxed{V\propto R^3}$ $\boxed{\begin{matrix}V=5\\R=3\end{matrix}}$ $\boxed{\begin{matrix}R=?\\V=625\end{matrix}}$

$V\propto R^3$

$V=kR^3$ ....(i)

To find k,

Put V = 5 and R = 3 in equation (i)

$5=k(3^3)$

$5=k(27)$

$\frac{5}{27}=k \Rightarrow k=\frac{5}{27}$

Put $k=\frac{5}{27}$ in equation (i)

$V=kR^3$

$V=\frac{5}{27}R^3$..........................(ii)

To find R,

Now put $V=625$ in equation (ii)

$(625)=\frac{5}{27}R^3$

$\frac{27\times625}{5}=R^3$ (625 ÷ 5 → 125)

$27\times125=R^3$

$R^3 = 3^3\times5^3$

$R^3 = (3\times5)^3$

$R^3=(15)^3$

Taking cube root of both sides

$\sqrt[3]{R^3}=\sqrt[3]{15^3}$

$R^{3\times\frac{1}{3}}=15^{3\times\frac{1}{3}}$

$\boxed{R=15}$

Q.6 If w varies directly as u^3 and w = 81 when u = 3. Find w, when u = 5.

Solution: $\boxed{w \propto u^3}$ $\boxed{\begin{matrix} w = 81 \\ u = 3 \end{matrix}}$ $\boxed{\begin{matrix} w = ? \\ u = 5 \end{matrix}}$

$$w \propto u^3$$
$$w = ku^3 \quad \ldots\ldots\ldots\ldots\ldots(i)$$

To find k,

Put $w = 81$ and u = 3 in equation (i)

$$81 = k(3)^3$$
$$81 = k(27)$$
$$\frac{81}{27} = k$$
$$3 = k \quad \Rightarrow \quad \boxed{k = 3}$$

Put $k = 3$ in equation (i)

$$w = ku^3$$
$$w = 3u^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

To find w,

Now put u = 5 in equation (ii)

$$w = 3(5)^3$$
$$w = 3(125)$$
$$\boxed{w = 375}$$

Q.7 If y varies inversely as x and y = 7 when x = 2, find y, when x = 126.

Solution: $\boxed{y \propto \frac{1}{x}}$ $\boxed{\begin{matrix} y = 7 \\ x = 2 \end{matrix}}$ $\boxed{\begin{matrix} y = ? \\ x = 126 \end{matrix}}$

$$y \propto \frac{1}{x}$$
$$y = \frac{k}{x} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

To find k,

Put y = 7 and x = 2

$$7 = \frac{k}{2}$$
$$7 \times 2 = k$$
$$14 = k \quad \Rightarrow \quad \boxed{k = 14}$$

Put $k = 14$ in equation (i)

$$y = \frac{k}{x} = \frac{14}{x}$$
$$\Rightarrow y = \frac{14}{x} \ldots\ldots\ldots(ii)$$

To find y,

Now put x = 126 in equation (ii)

$$y = \frac{14}{x} = \frac{14}{126} = \frac{\cancel{14}^{1}}{\cancel{126}_{9}}$$
$$\boxed{y = \frac{1}{9}}$$

Q.8 If $y \propto \frac{1}{x}$ and y = 4 when x = 3, find x when y = 24.

Solution: $\boxed{y \propto \frac{1}{x}}$ $\boxed{\begin{matrix} y = 4 \\ x = 3 \end{matrix}}$ $\boxed{\begin{matrix} x = ? \\ y = 24 \end{matrix}}$

$$y \propto \frac{1}{x}$$
$$y = \frac{k}{x} \quad \ldots\ldots\ldots\ldots\ldots (i)$$

To find k,

Put y = 4 and x = 3 in equation (i)

$$4 = \frac{k}{3}$$
$$4 \times 3 = k$$
$$12 = k \quad \Rightarrow \quad \boxed{k = 12}$$

Put k = 12 in equation (i)

$$y = \frac{k}{x}$$
$$y = \frac{12}{x} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

To find x,

Now put y = 24 in equation (ii)

$$24 = \frac{12}{x}$$
$$24x = 12$$
$$x = \frac{12}{24} = \frac{\cancel{12}^{1}}{\cancel{24}^{2}}$$
$$\boxed{x = \frac{1}{2}}$$

Q.9 If $w \propto \frac{1}{z}$ and w = 5 when z = 7, find w when $z = \frac{175}{4}$.

Solution: $\boxed{w \propto \frac{1}{z}}$ $\boxed{w=5,\ z=7}$ $\boxed{w=?,\ z=\frac{175}{4}}$

$$w \propto \frac{1}{z} \quad \text{(given)}$$

$$w = \frac{k}{z} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put w = 5 and z = 7 in equation (i)

$$5 = \frac{k}{7}$$

$$5 \times 7 = k$$

$$35 = k \quad \Rightarrow \quad \boxed{k = 35}$$

Put K = 35 in equation (i)

$$w = \frac{k}{z} = \frac{35}{z} \Rightarrow w = \frac{35}{z} \ldots\ldots\ldots\text{(ii)}$$

To find w,

Now put $z = \frac{175}{4}$ in equation (ii)

$$w = \frac{35}{z} = 35 \div z$$

$$w = 35 \div \frac{175}{4}$$

$$w = 35 \times \frac{4}{175}$$

$$w = \frac{140}{175} \quad 35^{1} \times \frac{4}{175_{5}}$$

$$\boxed{w = \frac{4}{5}}$$

Q.10 $A \propto \frac{1}{r^2}$ and A = 2 when r = 3, find r when A = 72.

Solution: $\boxed{A \propto \frac{1}{r^2}}$ $\boxed{A = 2,\ r = 3}$ $\boxed{A = 72,\ r = ?}$

$$A \propto \frac{1}{r^2}$$

$$A = \frac{k}{r^2} \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put A =2 and r = 3 in equation (i)

$$2 = \frac{k}{(3)^2}$$

$$2 = \frac{k}{9}$$

$$2 \times 9 = k$$

$$18 = k \quad \Rightarrow \quad \boxed{k = 18}$$

Put k = 18 in equation (i)

$$A = \frac{k}{r^2}$$

$$A = \frac{18}{r^2} \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

To find r,

Now put A = 72 in equation (ii)

$$72 = \frac{18}{r^2}$$

$$72r^2 = 18$$

$$r^2 = \frac{18}{72}$$

$$r^2 = \frac{1}{4}$$

Taking square root of both sides

$$\sqrt{r^2} = \pm\sqrt{\frac{1}{4}}$$

$$\boxed{r = \pm\frac{1}{2}}$$

Q.11 $a \propto \frac{1}{b^2}$ and a = 3 when b = 4, find a, when b = 8.

Solution: $\boxed{a \propto \frac{1}{b^2}}$ $\boxed{a = 3,\ b = 4}$ $\boxed{a = ?,\ b = 8}$

$$a \propto \frac{1}{b^2}$$

$$a = \frac{k}{b^2} \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

To find k,

Put a = 3 and b = 4 in equation (i)

$3 = \frac{k}{(4)^2}$

$3 = \frac{k}{16}$

$3 \times 16\ k$

$48 = k \Rightarrow \boxed{k = 48}$

Put K = 48 in equation ....(i)

$a = \frac{k}{b^2} \Rightarrow a = \frac{48}{b^2}$ ....(ii)

To find a,

Now, put b = 8 in equation .....(ii)

$a = \frac{48}{b^2} = \frac{48}{(8)^2} = \frac{48}{64}$

$\boxed{a = \frac{3}{4}}$

Q.12 $V \propto \frac{1}{r^3}$ **and V = 5 when r = 3, find V when r = 6 and r when V = 320.**

Solution:

$\boxed{V \propto \frac{1}{r^3}}$ $\boxed{\begin{matrix} V = 5 \\ r = 3 \end{matrix}}$ $\boxed{\begin{matrix} V = ? \\ r = 6 \end{matrix}}$ $\boxed{\begin{matrix} v = 320 \\ r = ? \end{matrix}}$

$V \propto \frac{1}{r^3}$

$V = \frac{k}{r^3}$ ....(i)

To find k,

Put V = 5 and r = 3 in equation (i)

$5 = \frac{k}{(3)^3}$

$5 = \frac{k}{27}$

$5 \times 27 = k \Rightarrow \boxed{k = 135}$

Put k = 135 in equation .....(i)

$V = \frac{135}{r^3}$ ....(ii)

To find V, Put r = 6 in equation (ii)

$V = \frac{135}{(6)^3} = \frac{135}{216}$

$\boxed{V = \frac{5}{8}}$ *(Dividing by 27)*

To find r, put v = 320 in equation (ii)

$V = \frac{135}{r^3}$

$320 = \frac{135}{r^3}$

$r^3 = \frac{135}{320} = \frac{27}{64}$

$r^3 = \frac{3^3}{4^3}$

$r^3 = \left(\frac{3}{4}\right)^3$ taking cube root

$\sqrt[3]{r^3} = \sqrt[3]{\left(\frac{3}{4}\right)^3}$

$\boxed{r = \frac{3}{4}}$

Q.13 $m \propto \frac{1}{n^3}$ **and m = 2 when n = 4 find m when n = 6 and n when m = 432.**

Solution:

$\boxed{m \propto \frac{1}{n^3}}$ $\boxed{\begin{matrix} m = 2 \\ n = 4 \end{matrix}}$ $\boxed{\begin{matrix} m = ? \\ n = 6 \end{matrix}}$ $\boxed{\begin{matrix} n = ? \\ m = 432 \end{matrix}}$

$m \propto \frac{1}{n^3}$

$m = \frac{k}{n^3}$ ....(i)

To find k,

Put m = 2 and n = 4 in equation (i)

$2 = \frac{k}{(4)^3}$

$2 = \frac{K}{64}$

$2 \times 64 = k$

$128 = k \Rightarrow \boxed{k = 128}$

Put k = 128 in equation ....(i)

$m = \frac{128}{n^3}$ ....(ii)

To find m,

put n = 6 in equation (ii)

$m = \frac{128}{(6)^3} = \frac{128}{216} = \frac{32}{54} = \frac{16}{27}$

$\boxed{m = \frac{16}{27}}$

To find n,

Put m = 432 in equation (ii)

$$m = \frac{128}{n^3}$$

$$432 = \frac{128}{n^3}$$

$$n^3 = \frac{128}{432} \quad (\textit{dividing by } 16)$$

$$n^3 = \frac{8}{27}$$

$$n^3 = \frac{2^2}{3^3}$$

$$n^3 = \left(\frac{2}{3}\right)^3$$

Taking cube of root

$$\sqrt[3]{n^3} = \sqrt[3]{\left(\frac{2}{3}\right)^3}$$

$$\boxed{n = \frac{2}{3}}$$

EXERCISE 3.3

Q1: Find a third proportional

(i) 6, 12

Solution:

Let third proportional is x then 6,12, x

By proportion.

$6 : 12 :: 12 : x$

Product of Extremes = Product of Means

$6(x) = 12 \times 12$

$6(x) = 144$

$x = \frac{144}{6} = 24$

$\boxed{x = 24}$

(ii) $a^3, 3a^2$

Solution:

Let 3rd proportional is x then $a^3, 3a^2, x$

By proportion

$a^3 : 3a^2 :: 3a^2 : x$

Product of Extremes = Product of Means

$xa^3 = (3a^2)(3a^2)$

$xa^3 = 9a^4$

$x = \frac{9a^4}{a^3}$

$x = 9a^{4-3}$

$\boxed{x = 9a}$

(iii) $a^2 - b^2, a - b$

Solution:

Let 3rd proportional is x then $a^2 - b^2, a - b, x$

By proportion

$a^2-b^2 : a-b :: a - b : x$

Product of Extremes = Product of Means

$x(a^2 - b^2) = (a-b)(a-b)$

$x = \frac{(a-b)(a-b)}{a^2-b^2}$

$x = \frac{(a-b)\cancel{(a-b)}}{(a+b)\cancel{(a-b)}}$

$\boxed{x = \frac{a-b}{a+b}}$

(iv) $(x-y)^2, x^3-y^3$

Solution:

Let 3rd proportional is "a" then $(x-y)^2, x^3 - y^3, a$

By proportion:

$(x-y)^2 : x^3 - y^3 :: x^3 - y^3 : a$

Product of Extremes = Product of Means

$a(x-y)^2 = (x^3 - y^3)(x^3 - y^3)$

$a = \frac{(x^3-y^3)^2}{(x-y)^2}$

$a = \frac{\left[(x-y)(x^2+xy+y^2)\right]^2}{(x-y)^2}$

$a = \frac{\cancel{(x-y)^2}(x^2+xy+y^2)^2}{\cancel{(x-y)^2}}$

$\boxed{a = (x^2+xy+y^2)^2}$

(v) $(x+y)^2, x^2 - xy - 2y^2$

Solution:

Let 3rd proportional is "a"

Then $(x + y)^2, x^2 - xy - 2y^2, a$

By Proportion:

$(x+y)^2 : x^2 - xy - 2y^2 :: x^2 - xy - 2y^2 : a$

Product of Extremes = Product of Means:

$a(x+y)^2 = (x^2 - xy - 2y^2)(x^2 - xy - 2y^2)$

$a = \frac{(x^2-xy-2y^2)^2}{(x+y)^2}$

$a = \frac{(x^2-xy-y^2-y^2)^2}{(x+y)^2}$

$a = \frac{(x^2-y^2-xy-y^2)^2}{(x+y)^2}$

$a = \frac{\left[(x+y)(x-y)-y(x+y)\right]^2}{(x+y)^2}$

$a = \frac{\left[(x+y)(x-y-y)\right]^2}{(x+y)^2}$

$= \frac{\cancel{(x+y)^2}(x-2y)^2}{\cancel{(x+y)^2}}$

$\boxed{a = (x-2y)^2}$

(vi) $\dfrac{p^2-q^2}{p^3+q^3}, \dfrac{p-q}{p^2-pq+q^2}$

Solution:

Let 3rd proportional is x

Then $\dfrac{p^2-q^2}{p^3+q^3}, \dfrac{p-q}{p^2-pq+q^2}, x$

By proportion.

$$\frac{p^2-q^2}{p^3+q^3} : \frac{p-q}{p^2-pq+q^2} :: \frac{p-q}{p^2-pq+q^2} : x$$

Product of Extremes = Product of Means.

$$x.\frac{p^2-q^2}{p^3+q^3} = \frac{p-q}{p^2-pq+q^2} \times \frac{p-q}{p^2-pq+q^2}$$

$$x.\frac{p^2-q^2}{p^3+q^3} = \left[\frac{(p-q)}{(p^2-pq+q^2)}\right]^2$$

$$x = \frac{(p-q)^2}{(p^2-pq+q^2)^2} \times \frac{p^3+q^3}{(p^2-q^2)}$$

$$x = \frac{\cancel{(p-q)}(p-q)}{(p^2-pq+q^2)^{\cancel{2}}} \times \frac{\cancel{(p+q)}\,\cancel{(p^2-pq+q^2)}}{\cancel{(p+q)}\,\cancel{(p-q)}}$$

$$\boxed{x = \frac{p-q}{p^2-pq+q^2}}$$

Q.2 Find a fourth proportional

(i) 5, 8, 15

Solution:

Let 4th proportional is x then 5, 8, 15, x

By proportion

5 : 8 : : 15 : x

Product of Extremes = Product of Means

$5(x) = 8(15)$

$$x = \frac{8(\cancel{15}^{\,3})}{\cancel{5}1}$$

$x = 8(3)$

$\boxed{x = 24}$

(ii) $4x^4, 2x^3, 18x^5$

Solution:

Let 4th proportional is "a" then $4x^4, 2x^3, 18x^5$, a

By proportion

$4x^4 : 2x^3 : : 18x^5 : a$

Product of Extreme = Product of Means.

$a(4x^4) = 2x^3(18x^5)$

$$a = \frac{36x^8}{4x^4}$$

$a = 9x^{8-4}$

$\boxed{a = 9x^4}$

(iii) $15a^5b^6, 10a^2b^5, 21a^3b^3$

Solution:

Let 4th proportional is x

then $15a^5b^6, 10a^2b^5, 21a^3b^3$, x

By proportion

$15a^5b^6 : 10a^2b^5 : : 21a^3b^3 : x$

Product of Extremes = Product of Means

$x(15a^5b^6) = (10a^2b^5)(21a^3b^3)$

$$x = \frac{14\,\cancel{210}\,\cancel{a^5}\,b^8}{\cancel{15}\,\cancel{a^5}\,b^6}$$

$x = 14b^{8-6}$

$\boxed{x = 14b^2}$

(iv) $x^2 - 11x + 24; (x-3), (5x^4 - 40x^3)$

Solution:

Let 4th proportional is "a"

$x^2 - 11x + 24, (x-3), (5x^4 - 40x^3)$, a

By proportion

$x^2 - 11x + 24 : (x-3) : : 5x^4 - 40x^3 : a$

Product of Extremes = Product of Means

$a(x^2 - 11x + 24) = (x-3)(5x^4 - 40x^3)$

$$a = \frac{(x-3)(5x^4-40x^3)}{x^2-11x+24}$$

$$a = \frac{(x-3).5x^3(x-8)}{x^2-3x-8x+24}$$

$$a = \frac{5x^3(x-3)(x-8)}{x(x-3)-8(x-3)}$$

$$a = \frac{5x^3\cancel{(x-3)}\,\cancel{(x-8)}}{\cancel{(x-3)}\,\cancel{(x-8)}}$$

$\boxed{a = 5x^3}$

(v) $p^3+q^3, p^2-q^2, p^2-pq+q^2$

Solution:

Let 4th proportional is x

$p^3+q^3, p^2-q^2, p^2-pq+q^2, x$

By proportion

$p^3+q^3, p^2-q^2, p^2-pq+q^2 : x$

Product of Extremes = Product of Means.

$x(p^3+q^3) = (p^2-q^2)(p^2-pq+q^2)$

$$x = \frac{(p^2-q^2)(p^2-pq+q^2)}{p^3+q^3}$$

$$x = \frac{(p+q)(p-q)(p^2-pq+q^2)}{(p+q)(p^2-pq+q^2)}$$

$$\boxed{x = (p-q)}$$

(vi) $(p^2-q^2)(p^2+pq+q^2), p^3+q^3, p^3-q^3$

Solution:

Let 4th proportional is x.

Then $(p^2-q^2)(p^2+pq+q^2), p^3+q^3, p^3-q^3, x$

By proportion:

$(p^2-q^2)(p^2+pq+q^2) : p^3+q^3 :: p^3-q^3 : x$

Product of Extremes = Product of Means:

$x\ (p^2-q^2)(p^2+pq+q^2) = (p^3+q^3)(p^3-q^3)$

$$x = \frac{(p^3+q^3)(p^3-q^3)}{(p^2-q^2)(p^2+pq+q^2)}$$

$$x = \frac{(p+q)(p^2-pq+q^2)(p-q)(p^2+pq+q^2)}{(p+q)(p-q)(p^2+pq+q^2)}$$

$$\boxed{x = (p^2-pq+q^2)}$$

Q.3: Find mean proportional:

(i) 20, 45

Solution:

Let mean proportional is m

then 20, m, 45

By proportion

$20 : m :: m : 45$

Product of Means = Product of Extremes

$m.m = 20 \times 45$

$m^2 = 900$

Taking square root

$\sqrt{m^2} = \pm\sqrt{900}$

$$\boxed{m = \pm 30}$$

(ii) $20x^3y^5$, $5x^7y$

Solution:

Let mean proportional is m

then $20x^3y^5$, m , $5x^7y$

By proportion,

$20x^3y^5 : m :: m : 5x^7y$

Product of Means = Product of Extremes

$m.m = (20x^3y^5)(5x^7y)$

$m^2 = 100x^{10}y^6$

Taking square root of both sides

$\sqrt{m^2} = \pm\sqrt{100x^{10}y^6}$

$m = \pm\sqrt{100}.\sqrt{x^{10}}.\sqrt{y^6}$

$m = \pm\sqrt{100}.\sqrt{x^{10}}.\sqrt{y^6}$

$m = \pm 10x^{10\times\frac{1}{2}}.y^{6\times\frac{1}{2}}$

$$\boxed{m = \pm 10x^5y^3}$$

(iii) $15p^4qr^3, 135q^5r^7$

Solution:

Let mean proportional is m

then $15p^4qr^3$, m , $135q^5r^7$

By proportion

$15p^4qr^3 : m :: m : 135q^5r^7$

Product of Means = Product of Extremes

$m.m = (15p^4qr^3)(135q^5r^7)$

$m^2 = 2025p^4q^6r^{10}$

Taking square root

$\sqrt{m^2} = \pm\sqrt{2025p^4q^6r^{10}}$

$m = \pm\sqrt{2025}\sqrt{p^4}.\sqrt{q^6}.\sqrt{r^{10}}$

$m = \pm 45p^{4\times\frac{1}{2}}.q^{6\times\frac{1}{2}}.r^{10\times\frac{1}{2}}$

$$\boxed{m = \pm 45p^2.q^3.r^5}$$

(iv) $x^2 - y^2, \frac{x-y}{x+y}$

Solution:

Let mean proportional is m.

then $x^2 - y^2, m, \frac{x-y}{x+y}$

By proportion

$$x^2 - y^2 : m :: m : \frac{x-y}{x+y}$$

Product of Means = Product of Extremes

$$m.m = (x^2 - y^2)\frac{(x-y)}{x+y}$$

$$m^2 = \frac{(x+y)(x-y)(x-y)}{(x+y)}$$

$$m^2 = (x-y)^2$$

Taking square root

$$\sqrt{m^2} = \pm\sqrt{(x-y)^2}$$

$$\boxed{m = \pm(x-y)}$$

Q.4 Find the values of the letter involved in the following continued proportions

(i) 5, p, 45

Solution:

By continued proportion

$$5 : p :: p : 45$$

Product of Means = Product of Extremes

$$p.p = 5 \times 45$$

$$p^2 = 225$$

Taking square root of both sides

$$\sqrt{p^2} = \pm\sqrt{225}$$

$$\boxed{p = \pm 15}$$

(ii) 8, x, 18

Solution:

By continued proportion

$$8 : x :: x : 18$$

Product of Means = Product of Extremes

$$x.x = 8 \times 18$$

$$x^2 = 144$$

Taking square root

$$\sqrt{x^2} = \pm\sqrt{144}$$

$$\boxed{x = \pm 12}$$

(iii) 12, 3p – 6, 27

Solution:

By continued proportion

$$12 : 3p - 6 :: 3p - 6 : 27$$

Product of Means = Product of Extremes.

$$(3p-6)(3p-6) = 12 \times 27$$

$$(3p-6)^2 = 324$$

Taking square root of both sides

$$\sqrt{(3p-6)^2} = \pm\sqrt{324}$$

$$3p - 6 = \pm 18$$

$3p - 6 = 18$ or $3p - 6 = -18$

$3p = 18 + 6$ or $3p = -18 + 6$

$3p = 24$ or $3p = -12$

$p = \frac{24}{3}$ or $p = \frac{-12}{3}$

$p = 8$ or $p = -4$

(iv) 7, m – 3, 28

Solution:

By continued proportion

$$7 : m - 3 :: m - 3 : 28$$

Product of Means = Product of Extremes

$$(m-3)(m-3) = 7 \times 28$$

$$(m-3)^2 = 196$$

Taking square root of both sides

$$\sqrt{(m-3)^2} = \pm\sqrt{196}$$

$$m - 3 = \pm 14$$

$m - 3 = 14$ or $m - 3 = -14$

$m = 14 + 3$ or $m = -14 + 3$

$m = 17$ or $m = -11$

EXERCISE 3.4

Q.1 Prove that a : b = c : d , if

(i) $\frac{4a+5b}{4a-5b}=\frac{4c+5d}{4c-5d}$

(ii) $\frac{2a+9b}{2a-9b}=\frac{2c+9d}{2c-9d}$

(iii) $\frac{ac^2+bd^2}{ac^2-bd^2}=\frac{c^3+d^3}{c^3-d^3}$

(iv) $\frac{a^2c+b^2d}{a^2c-b^2d}=\frac{ac^2+bd^2}{ac^2-bd^2}$

(v) $pa+qb : pa-qb = pc+qd : pc-qd$

(vi) $\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$

(vii) $\frac{2a+3b+2c+3d}{2a+3b-2c-3d}=\frac{2a-3b+2c-3d}{2a-3b-2c+3d}$

(viii) $\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}$

Solutions:

(i) $\mathbf{\frac{4a+5b}{4a-5b}=\frac{4c+5d}{4c-5d}}$

Solution:

$$\frac{4a+5b}{4a-5b}=\frac{4c+5d}{4c-5d}$$

By componendo – dividendo theorem

$$\frac{(4a+5b)+(4a-5b)}{(4a+5b)-(4a-5b)}=\frac{(4c+5d)+(4c-5d)}{(4c+5d)-(4c-5d)}$$

$$\frac{4a+\cancel{5b}+4a-\cancel{5b}}{\cancel{4a}+5b-\cancel{4a}+5b}=\frac{4c+\cancel{5d}+4c-\cancel{5d}}{\cancel{4c}+5d-\cancel{4c}+5d}$$

$$\frac{8a}{10b}=\frac{8c}{10d}$$

By multiplying both sides by $\frac{10}{8}$

$$\frac{\cancel{10}}{\cancel{8}}\frac{\cancel{8}}{\cancel{10}}\frac{a}{b}=\frac{\cancel{10}}{\cancel{8}}\frac{\cancel{8}c}{\cancel{10}d}$$

$\frac{a}{b}=\frac{c}{d} \Rightarrow a:b=c:d$ Hence proved

(ii) $\dfrac{2a+9b}{2a-9b}=\dfrac{2c+9d}{2c-9d}$

Solution:

$$\frac{2a+9b}{2a-9b}=\frac{2c+9d}{2c-9d}$$

By componendo – dividendo theorem

$$\frac{(2a+9b)+(2a-9b)}{(2a+9b)-(2a-9b)}=\frac{(2c+9d)+(2c-9d)}{(2c+9d)-(2c-9d)}$$

$$\frac{2a+\cancel{9b}+2a-\cancel{9b}}{\cancel{2a}+9b-\cancel{2a}+9b}=\frac{2c+\cancel{9d}+2c-\cancel{9d}}{\cancel{2c}+9d-\cancel{2c}+9d}$$

$$\frac{4a}{18b}=\frac{4c}{18d}$$

Multiplying both sides by $\dfrac{18}{4}$

$$\frac{\cancel{18}}{\cancel{4}}\times\frac{\cancel{4}a}{\cancel{18}b}=\frac{\cancel{4}c}{\cancel{18}d}.\frac{\cancel{18}}{\cancel{4}}$$

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow$ $a:b=c:d$

Hence proved

(iii) $\dfrac{ac^2+bd^2}{ac^2-bd^2}=\dfrac{c^3+d^3}{c^3-d^3}$

Solution:

$$\frac{ac^2+bd^2}{ac^2-bd^2}=\frac{c^3+d^3}{c^3-d^3}$$

By componendo – dividendo theorem

$$\frac{(ac^2+bd^2)+(ac^2-bd^2)}{(ac^2+bd^2)-(ac^2-bd^2)}=\frac{(c^3+d^3)+(c^3-d^3)}{(c^3+d^3)-(c^3-d^3)}$$

$$\frac{ac^2+\cancel{bd^2}+ac^2-\cancel{bd^2}}{\cancel{ac^2}+bd^2-\cancel{ac^2}+bd^2}=\frac{c^3+\cancel{d^3}+c^3-\cancel{d^3}}{\cancel{c^3}+d^3-\cancel{c^3}+d^3}$$

$$\frac{\cancel{2}ac^2}{\cancel{2}bd^2}=\frac{\cancel{2}c^3}{\cancel{2}d^3}$$

$$\frac{ac^2}{bd^2}=\frac{c.c^2}{d.d^2}$$

Multiplying both sides by $\dfrac{d^2}{c^2}$

$$\frac{a\cancel{c^2}}{b\cancel{d^2}}\times\frac{\cancel{d^2}}{\cancel{c^2}}=\frac{c\cancel{c^2}}{d\cancel{d^2}}\times\frac{\cancel{d^2}}{\cancel{c^2}}$$

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow$ $a:b=c:d$ Hence proved

(iv) $\dfrac{a^2c+b^2d}{a^2c-b^2d}=\dfrac{ac^2+bd^2}{ac^2-bd^2}$

Solution:

$$\frac{a^2c+b^2d}{a^2c-b^2d}=\frac{ac^2+bd^2}{ac^2-bd^2}$$

By componendo – dividendo theorem

$$\frac{(a^2c+b^2d)+(a^2c-b^2d)}{(a^2c+b^2d)-(a^2c-b^2d)}=\frac{(ac^2+bd^2)+(ac^2-bd^2)}{(ac^2+bd^2)-(ac^2-bd^2)}$$

$$\frac{a^2c+\cancel{b^2d}+a^2c-\cancel{b^2d}}{\cancel{a^2c}+b^2d-\cancel{a^2c}+b^2d}=\frac{ac^2+\cancel{bd^2}+ac^2-\cancel{bd^2}}{\cancel{ac^2}+bd^2-\cancel{ac^2}+bd^2}$$

$$\frac{\cancel{2}a^2c}{\cancel{2}b^2d}=\frac{\cancel{2}ac^2}{\cancel{2}bd^2}$$

$$\frac{a.ac}{b.bd}=\frac{ac.c}{bd.d}$$

Multiplying both sides by $\dfrac{bd}{ac}$

$$\frac{a.\cancel{ac}}{b.\cancel{bd}}\times\frac{\cancel{bd}}{\cancel{ac}}=\frac{\cancel{bd}}{\cancel{ac}}.\frac{\cancel{ac}.c}{\cancel{bd}.d}$$

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow$ $a:b=c:d$ Hence proved

(v) $pa+qb:pa-qb=pc+qd:pc-qd$

Solution:

$$pa+qb:pa-qb=pc+qd:pc-qd$$

OR

$$\frac{pa+qb}{pa-qb}=\frac{pc+qd}{pc-qd}$$

By componendo– dividendo theorem

$$\frac{(pa+qb)+(pa-qb)}{(pa+qb)-(pa-qb)}=\frac{(pc+qd)+(pc-qd)}{(pc+qd)-(pc-qd)}$$

$$\frac{pa+\cancel{qb}+pa-\cancel{qb}}{\cancel{pa}+qb-\cancel{pa}+qb}=\frac{pc+\cancel{qd}+pc-\cancel{qd}}{\cancel{pc}+qd-\cancel{pc}+qd}$$

$$\frac{\cancel{2}pa}{\cancel{2}qb}=\frac{\cancel{2}pc}{\cancel{2}qd}$$

$$\frac{pa}{qb}=\frac{pc}{qd}$$

Multiplying both sides by $\frac{q}{p}$

$$\frac{\cancel{q}}{\cancel{p}}.\frac{\cancel{p}a}{\cancel{q}b}=\frac{\cancel{q}}{\cancel{p}}.\frac{\cancel{p}c}{\cancel{q}d}$$

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow$ $a:b=c:d$ Hence proved

(vi) $$\mathbf{\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}}$$

Solution:

$$\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$$

By componendo dividendo theorem

$$\frac{(a+b+c+d)+(a+b-c-d)}{(a+b+c+d)-(a+b-c-d)}=\frac{(a-b+c-d)+(a-b-c+d)}{(a-b+c-d)-(a-b-c+d)}$$

$$\frac{a+b+\cancel{c}+\cancel{d}+a+b-\cancel{c}-\cancel{d}}{\cancel{a}+\cancel{b}+c+d-\cancel{a}-\cancel{b}+c+d}=\frac{a-b+\cancel{c}-\cancel{d}+a-b-\cancel{c}+\cancel{d}}{\cancel{a}-\cancel{b}+c-d-\cancel{a}+\cancel{b}+c-d}$$

$$\frac{2a+2b}{2c+2d}=\frac{2a-2b}{2c-2d}$$

$$\frac{\cancel{2}(a+b)}{\cancel{2}(c+d)}=\frac{\cancel{2}(a-b)}{\cancel{2}(c-d)}$$

By alternendo theorem

$$\frac{a+b}{a-b}=\frac{c+d}{c-d}$$

Again by componendo– dividendo theorem

$$\frac{(a+b)+(a-b)}{(a+b)-(a-b)}=\frac{(c+d)+(c-d)}{(c-d)-(c-d)}$$

$$\frac{a+\cancel{b}+a-\cancel{b}}{\cancel{a}+b-\cancel{a}+b}=\frac{c+\cancel{d}+c-\cancel{d}}{\cancel{c}+d-\cancel{c}+d}$$

$$\frac{\cancel{2}a}{\cancel{2}b}=\frac{\cancel{2}c}{\cancel{2}d}\quad\Rightarrow\quad\frac{a}{b}=\frac{c}{d}\ \Rightarrow a:b=c:d$$

(vii) $$\mathbf{\frac{2a+3b+2c+3d}{2a+3b-2c-3d}=\frac{2a-3b+2c-3d}{2a-3b-2c+3d}}$$

Solution:

$$\frac{2a+3b+2c+3d}{2a+3b-2c-3d}=\frac{2a-3b+2c-3d}{2a-3b-2c+3d}$$

By componendo–dividendo theorem

$$\frac{(2a+3b+2c+3d)+(2a+3b-2c-3d)}{(2a+3b+2c+3d)-(2a+3b-2c-3d)}$$

$$=\frac{(2a-3b+2c-3d)+(2a-3b-2c+3d)}{(2a-3b+2c-3d)-(2a-3b-2c+3d)}$$

$$\frac{2a+3b+\cancel{2c}+\cancel{3d}+2a+3b-\cancel{2c}-\cancel{3d}}{\cancel{2a}+\cancel{3b}+2c+3d-\cancel{2a}-\cancel{3b}+2c+3d}$$

$$=\frac{2a-3b+\cancel{2c}-\cancel{3d}+2a-3b-\cancel{2c}+\cancel{3d}}{\cancel{2a}-\cancel{3b}+2c-3d-\cancel{2a}+\cancel{3b}+2c-3d}$$

$$\frac{4a+6b}{4c+6d}=\frac{4a-6b}{4c-6d}$$

By alternando theorem

$$\frac{4a+6b}{4a-6b}=\frac{4c+6d}{4c-6d}$$

Again by componendo–dividendo theorem

$$\frac{(4a+6b)+(4a-6b)}{(4a+6b)-(4a-6b)}=\frac{(4c+6d)+(4c-6d)}{(4c+6d)-(4c-6d)}$$

$$\frac{4a+\cancel{6b}+4a-\cancel{6b}}{\cancel{4a}+6b-\cancel{4a}+6b}=\frac{4c+\cancel{6d}+4c-\cancel{6d}}{\cancel{4c}+6d-\cancel{4c}+6d}$$

$$\frac{8a}{12b}=\frac{8c}{12d}$$

Multiplying both sides by $\frac{12}{8}$

$$\frac{\cancel{8}a}{\cancel{12}b}.\frac{\cancel{12}}{\cancel{8}}=\frac{\cancel{12}}{\cancel{8}}.\frac{\cancel{8}c}{\cancel{12}d}$$

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow$ $a:d=c:d$ Hence proved

(viii) $$\mathbf{\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}}$$

Solution:

$$\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}$$

By componendo – dividendo theorem

$$\frac{(a^2+b^2)+(a^2-b^2)}{(a^2+b^2)-(a^2-b^2)}=\frac{(ac+bd)+(ac-bd)}{(ac+bd)-(ac-bd)}$$

$$\frac{a^2+\cancel{b^2}+a^2-\cancel{b^2}}{\cancel{a^2}+b^2-\cancel{a^2}+b^2}=\frac{ac+\cancel{bd}+ac-\cancel{bd}}{\cancel{ac}+bd-\cancel{ac}+bd}$$

$$\frac{\cancel{2}a^2}{\cancel{2}b^2}=\frac{\cancel{2}ac}{\cancel{2}bd}$$

$$\frac{a.a}{b.a}=\frac{ac}{bd}$$

Multiplying both sides by "$\frac{b}{a}$"

$$\frac{\cancel{b}}{\cancel{a}}.\frac{\cancel{a}.a}{\cancel{b}.a}=\frac{\cancel{b}}{\cancel{a}}\frac{\cancel{a}c}{\cancel{b}d}$$

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow$ $a:b=c:d$ Hence proved

Q.2 Use componendo–dividendo theorem to find the values of the following.

(i) Find the value of

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}, \text{ if } x=\frac{4yz}{y+z}$$

Solution: $x=\dfrac{4yz}{y+z}$ .... (i)

Dividing the equation (i) by "2y"

$$\frac{x}{2y}=\frac{4yz}{(y+z).2y}$$

$$\frac{x}{2y}=\frac{2z}{y+z}$$

By componendo – dividendo theorem

$$\frac{x+2y}{x-2y}=\frac{2z+(y+z)}{2z-(y+z)}$$

$$\frac{x+2y}{x-2y}=\frac{2z+y+z}{2z-y-z}$$

$$\frac{x+2y}{x-2y}=\frac{3z+y}{z-y} \quad \text{................ (ii)}$$

Now dividing the eq (i) by "2z"

$$\frac{x}{2z}=\frac{4yz}{(y+z).2z}$$

$$\frac{x}{2z}=\frac{2y}{y+z}$$

By componendo – dividendo theorem

$$\frac{x+2z}{x-2z}=\frac{2y+(y+z)}{2y-(y+z)}$$

$$\frac{x+2z}{x-2z}=\frac{2y+y+z}{2y-y-z}$$

$$\frac{x+2z}{x-2z}=\frac{3y+z}{y-z} \quad \text{................ (iii)}$$

Adding eq. (ii) and (iii)

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=\frac{3z+y}{z-y}+\frac{3y+z}{y-z}$$

$$=\frac{3z+y}{-1(y-z)}+\frac{3y+z}{y-z}$$

$$=\frac{-1(3z+y)}{y-z}+\frac{3y+z}{y-z}$$

$$=\frac{-3z-y+3y+z}{y-z}$$

$$=\frac{2y-2z}{y-z}$$

$$=\frac{2\cancel{(y-z)}}{\cancel{(y-z)}}$$

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=2$$

(ii) Find the value of

$$\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}, \text{ if } m=\frac{10np}{n+p}$$

Solution: $m=\dfrac{10np}{n+p}$ .... (i)

Dividing equation (i) by "5n"

$$\frac{m}{5n}=\frac{10np}{(n+p)5n}$$

$$\frac{m}{5n}=\frac{2p}{n+p}$$

By componendo – dividendo theorem

$$\frac{m+5n}{m-5n}=\frac{2p+(n+p)}{2p-(n+p)}$$

$$\frac{m+5n}{m-5n}=\frac{2p+n+p}{2p-n-p}$$

$$\frac{m+5n}{m-5n}=\frac{3p+n}{p-n} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

Now, dividing equation (i) by "5p"

$$\frac{m}{5p}=\frac{10np}{(n+p)5p}$$

$$\frac{m}{5p}=\frac{2n}{n+p}$$

By componendo– dividendo theorem

$$\frac{m+5p}{m-5p}=\frac{2n+(n+p)}{2n-(n+p)}$$

$$\frac{m+5p}{m-5p}=\frac{2n+n+p}{2n-n-p}$$

$$\frac{m+5p}{m-5p}=\frac{3n+p}{n-p} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

Adding equation (ii) and (iii)

$$\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}=\frac{3p+n}{p-n}+\frac{3n+p}{n-p}$$

$$=\frac{3p+n}{-1(n-p)}+\frac{3n+p}{n-p}$$

$$=\frac{-(3p+n)}{n-p}+\frac{3n+p}{n-p}$$

$$=\frac{-3p-n+3n+p}{n-p}$$

$$=\frac{2n-2p}{n-p}$$

$$=\frac{2\cancel{(n-p)}}{\cancel{(n-p)}}$$

$$\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}=2$$

(iii) Find the value of $\frac{x-6a}{x+6a}-\frac{x+6b}{x-6b}$ **if** $x=\frac{12ab}{a-b}$

Solution: $x=\frac{12ab}{a-b} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$

Dividing equation (i) by 6a

$$\frac{x}{6a}=\frac{12ab}{(a-b).6a}$$

$$\frac{x}{6a}=\frac{2b}{a-b}$$

By componendo– dividendo theorem

$$\frac{x+6a}{x-6a}=\frac{2b+(a-b)}{2b-(a-b)}$$

$$\frac{x+6a}{x-6a}=\frac{2b+a-b}{2b-a+b}$$

$$\frac{x+6a}{x-6a}=\frac{a+b}{3b-a}$$

By invertendo theorem

$$\frac{x-6a}{x+6a}=\frac{3b-a}{a+b} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Now, dividing the equation (i) by 6b.

$$\frac{x}{6b}=\frac{12ab}{(a-b).6b}$$

$$\frac{x}{6b}=\frac{2a}{a-b}$$

By componendo – dividendo theorem

$$\frac{x+6b}{x-6b}=\frac{2a+(a-b)}{2a-(a-b)}$$

$$\frac{x+6b}{x-6b}=\frac{2a+a-b}{2a-a+b}$$

$$\frac{x+6b}{x-6b}=\frac{3a-b}{a+b} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

Subtracting equation (iii) from (ii)

$$\frac{x-6a}{x+6a}-\frac{x+6b}{x-6b}=\frac{3b-a}{a+b}-\frac{3a-b}{a+b}$$

$$=\frac{(3b-a)-(3a-b)}{a+b}$$

$$=\frac{3b-a-3a+b}{a+b}$$

$$=\frac{4b-4a}{a+b}$$

$$=\frac{4(b-a)}{(a+b)}$$

(iv) Find the value of

$$\frac{x-3y}{x+3y}-\frac{x+3z}{x-3z}, \text{ if } x=\frac{3yz}{y-z}$$

Solution: $x=\frac{3yz}{y-z}$ .... (i)

Dividing equation (i) by "3y"

$$\frac{x}{3y}=\frac{3yz}{(y-z).3y}$$

$$\frac{x}{3y}=\frac{z}{y-z}$$

By componendo – dividendo theorem

$$\frac{x+3y}{x-3y}=\frac{z+(y-z)}{z-(y-z)}$$

$$\frac{x+3y}{x-3y}=\frac{\cancel{z}+y-\cancel{z}}{z-y+z}$$

$$\frac{x+3y}{x-3y}=\frac{y}{2z-y}$$

By invertendo theorem

$$\frac{x-3y}{x+3y}=\frac{2z-y}{y} \text{ .... (ii)}$$

Now dividing equation (i) by "3z"

$$\frac{x}{3z}=\frac{3yz}{(y-z).3z}$$

$$\frac{x}{3z}=\frac{y}{y-z}$$

By componendo–dividendo theorem

$$\frac{x+3z}{x-3z}=\frac{y+(y-z)}{y-(y-z)}$$

$$\frac{x+3z}{x-3z}=\frac{y+y-z}{\cancel{y}-\cancel{y}+z}$$

$$\frac{x+3z}{x-3z}=\frac{2y-z}{z} \text{ .... (iii)}$$

Subtracting equation (iii) from (ii)

$$\frac{x-3y}{x+3y}-\frac{x+3z}{x-3z}=\frac{2z-y}{y}-\frac{2y-z}{z}$$

$$=\frac{z(2z-y)-y(2y-z)}{yz}$$

$$=\frac{2z^2-\cancel{yz}-2y^2+\cancel{yz}}{yz}$$

$$=\frac{2z^2-2y^2}{yz}=\frac{2(z^2-y^2)}{yz}$$

(v) Find the value of

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}, \text{ if } s=\frac{6pq}{p-q}$$

Solution:

$s=\frac{6pq}{p-q}$ ....(i)

Dividing equation (i) by "3P"

$$\frac{s}{3p}=\frac{6pq}{(p-q).3p}$$

$$\frac{s}{3p}=\frac{2q}{p-q}$$

By componendo–dividendo theorem

$$\frac{s+3p}{s-3p}=\frac{2q+(p-q)}{2q-(p-q)}$$

$$\frac{s+3p}{s-3p}=\frac{2q+p-q}{2q-p+q}$$

$$\frac{s+3p}{s-3p}=\frac{p+q}{3q-p}$$

By invertendo theorem

$$\frac{s-3p}{s+3p}=\frac{3q-p}{p+q} \text{ .... (ii)}$$

Now dividing equation (i) by "3q"

$$\frac{s}{3q}=\frac{2\cancel{6}p\cancel{q}}{(p-q).\cancel{3}\cancel{q}}$$

$$\frac{s}{3q}=\frac{2p}{p-q}$$

By componendo – dividendo theorem

$$\frac{s+3q}{s-3q}=\frac{2p+(p-q)}{2p-(p-q)}$$

$$\frac{s+3q}{s-3q}=\frac{2p+p-q}{2p-p+q}$$

$$\frac{s+3q}{s-3q}=\frac{3p-q}{p+q}\text{.......................}(iii)$$

Adding equation (ii) and (iii)

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=\frac{3q-p}{p+q}+\frac{3p-q}{p+q}$$

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=\frac{3q-p+3p-q}{(p+q)}$$

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=\frac{2p+2q}{(p+q)}$$

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=\frac{2\cancel{(p+q)}}{\cancel{(p+q)}}$$

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=2$$

(vi) Solve $\frac{(\mathbf{x}-\mathbf{2})^2-(\mathbf{x}-\mathbf{4})^2}{(\mathbf{x}-\mathbf{2})^2+(\mathbf{x}-\mathbf{4})^2}=\frac{\mathbf{12}}{\mathbf{13}}$

Solution:

$$\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2}=\frac{12}{13}$$

By componendo–dividendo theorem

$$\frac{[(x-2)^2-(x-4)^2]+[(x-2)^2+(x-4)^2]}{[(x-2)^2-(x-4)^2]-[(x-2)^2+(x-4)^2]}=\frac{12+13}{12-13}$$

$$\frac{(x-2)^2-\cancel{(x-4)^2}+(x-2)^2+\cancel{(x-4)^2}}{\cancel{(x-2)^2}-(x-4)^2-\cancel{(x-2)^2}-(x-4)^2}=\frac{25}{-1}$$

$$\frac{\cancel{2}(x-2)^2}{\cancel{-}\cancel{2}(x-4)^2}=\cancel{-}25$$

$$\left(\frac{x-2}{x-4}\right)^2=25$$

Taking square root of both side

$$\sqrt{\left(\frac{x-2}{x-4}\right)^2}=\pm\sqrt{25}$$

$$\frac{x-2}{x-4}=\pm5$$

$\frac{x-2}{x-4}=5$ or $\frac{x-2}{x-4}=-5$

x – 2 = 5 (x – 4) or x – 2 = – 5 (x – 4)

x – 2 = 5x – 20 or x – 2 = – 5x + 20

20 – 2 = 5x – x or x + 5x = 20 + 2

18 = 4x or 6x = 22

$\frac{\overset{9}{\cancel{18}}}{\underset{2}{\cancel{4}}}=x$ or $x=\frac{22}{6}$

$x=\frac{9}{2}$ or $x=\frac{11}{3}$

$$S.S=\left\{\frac{9}{2},\frac{11}{3}\right\}$$

(vii) Solve $\frac{\sqrt{\mathbf{x^2+2}}+\sqrt{\mathbf{x^2-2}}}{\sqrt{\mathbf{x^2+2}}-\sqrt{\mathbf{x^2-2}}}=\frac{\mathbf{2}}{\mathbf{1}}$

Solution:

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}}=\frac{2}{1}$$

By componendo– dividendo theorem

$$\frac{\left(\sqrt{x^2+2}+\sqrt{x^2-2}\right)+\left(\sqrt{x^2+2}-\sqrt{x^2-2}\right)}{\left(\sqrt{x^2+2}+\sqrt{x^2-2}\right)-\left(\sqrt{x^2+2}-\sqrt{x^2-2}\right)}=\frac{2+1}{2-1}$$

$$\frac{\sqrt{x^2+2}+\cancel{\sqrt{x^2-2}}+\sqrt{x^2+2}-\cancel{\sqrt{x^2-2}}}{\cancel{\sqrt{x^2+2}}+\sqrt{x^2-2}-\cancel{\sqrt{x^2+2}}+\sqrt{x^2-2}}=\frac{3}{1}$$

$$\frac{\cancel{2}\sqrt{x^2+2}}{\cancel{2}\sqrt{x^2-2}}=3$$

$$\sqrt{\frac{x^2+2}{x^2-2}}=3$$

Taking square of both sides

$$\left(\sqrt{\frac{x^2+2}{x^2-2}}\right)^{\cancel{2}}=(3)^2$$

$$\frac{x^2+2}{x^2-2}=9$$

$x^2 + 2 = 9 (x^2 – 2)$

$x^2+2 = 9x^2 – 18$

$2 + 18 = 9x^2 – x^2$

$20 = 8x^2$

$x^2=\frac{\overset{5}{\cancel{20}}}{\underset{2}{\cancel{8}}} \Rightarrow x^2=\frac{5}{2}$

Taking square root

$\sqrt{x^2}=\pm\sqrt{\frac{5}{2}} \Rightarrow x=\pm\sqrt{\frac{5}{2}}$

$$S.S=\left\{\pm\sqrt{\frac{5}{2}}\right\}$$

(viii) Solve $\dfrac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}}=\dfrac{1}{3}$

Solution:

$$\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}}=\frac{1}{3}$$

By componendo–dividendo theorem

$$\frac{\left(\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}\right)+\left(\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}\right)}{\left(\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}\right)-\left(\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}\right)}=\frac{1+3}{1-3}$$

$$\frac{\sqrt{x^2+8p^2}-\cancel{\sqrt{x^2-p^2}}+\sqrt{x^2+8p^2}+\cancel{\sqrt{x^2-p^2}}}{\cancel{\sqrt{x^2+8p^2}}-\sqrt{x^2-p^2}-\cancel{\sqrt{x^2+8p^2}}-\sqrt{x^2-p^2}}=\frac{4}{-2}$$

$$\frac{\cancel{2}\sqrt{x^2+8p^2}}{\cancel{-}\cancel{2}\sqrt{x^2-p^2}}=\cancel{-}2$$

$$\sqrt{\frac{x^2+8p^2}{x^2-p^2}}=2$$

Taking square of both sides

$$\left(\sqrt{\frac{x^2+8p^2}{x^2-p^2}}\right)^2=(2)^2$$

$$\frac{x^2+8p^2}{x^2-p^2}=4$$

$x^2+8p^2 = 4(x^2-p^2)$

$x^2+8p^2 = 4x^2-4p^2$

$8p^2+4p^2 = 4x^2-x^2$

$12p^2 = 3x^2$

$\Rightarrow$ $3x^2 = 12p^2$

$$x^2=\frac{12p^2}{3}$$

$x^2 = 4p^2$

Taking square root

$$\sqrt{x^2}=\pm\sqrt{4p^2}$$

$x = \pm 2p$

S.S = $\{\pm 2p\}$

(ix) Solve $\dfrac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3}=\dfrac{13}{14}$

Solution:

$$\frac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3}=\frac{13}{14}$$

By componendo–dividendo theorem

$$\frac{\left[(x+5)^3-(x-3)^3\right]+\left[(x+5)^3+(x-3)^3\right]}{\left[(x+5)^3-(x-3)^3\right]-\left[(x+5)^3+(x-3)^3\right]}=\frac{13+14}{13-14}$$

$$\frac{(x+5)^3-\cancel{(x-3)^3}+(x+5)^3+\cancel{(x-3)^3}}{\cancel{(x+5)^3}-(x-3)^3-\cancel{(x+5)^3}-(x-3)^3}=\frac{27}{-1}$$

$$\frac{\cancel{2}(x+5)^3}{\cancel{-}\cancel{2}(x-3)^3}=\cancel{-}27$$

$$\left(\frac{x+5}{x-3}\right)^3=27$$

Taking cube root

$$\sqrt[3]{\left(\frac{x+5}{x-3}\right)^3}=\sqrt[3]{27}$$

$$\left(\frac{x+5}{x-3}\right)^{\cancel{3}\times\frac{1}{\cancel{3}}}=(3)^{\cancel{3}\times\frac{1}{\cancel{3}}}$$

$$\frac{x+5}{x-3}=3$$

$(x+5) = 3(x-3)$

$x+5 = 3x-9$

$5+9 = 3x-x$

$14 = 2x$

$$\frac{14}{2}=x$$

$7 = x$

$\Rightarrow$ $x = 7$

S.S = $\{7\}$

EXERCISE 3.5

Q.1 If s varies directly as u^2 and inversely as v and s = 7, when u = 3, v = 2. Find the value of s when u = 6 and v = 10

Solution: $s \propto \frac{u^2}{v}$ | $s = 7, u = 3, v = 2$ | $s = ?, u = 6, v = 10$

$$s \propto \frac{u^2}{v}$$

$$s = \frac{ku^2}{v} \quad \text{.............................} \quad (i)$$

To find k,

Put s = 7 =, u = 3, v = 2 in equation (i)

$$7 = \frac{k(3)^2}{2}$$

$$7\times 2 = k(9)$$

$$\frac{14}{9} = k \quad \Rightarrow \quad k = \frac{14}{9}$$

Put $k = \frac{14}{9}$ in equation (i)

$$s = \frac{14u^2}{9v} \text{.........................}(ii)$$

To find s,

Now put, u=6 and v=10 in equation (i)

$$s = \frac{14u^2}{9v}$$

$$s = \frac{14(6)^2}{9(10)}$$

$$s = \frac{14\times \cancel{36}^{2}}{\cancel{90}_{5}} \quad \text{(dividing by 18)}$$

$$s = \frac{14\times 2}{5}$$

$$\boxed{s = \frac{28}{5}}$$

Q.2 If w varies jointly as x, y^2 and z and w = 5 when x = 2, y = 3, z = 10. Find w when x = 4, y = 7 and z = 3.

Solution:

$w \propto xy^2z$ | $w = 5, x = 2, y = 3, z = 10$ | $w = ?, x = 4, y = 7, z = 3$

$$w \propto xy^2z$$

$$w = kxy^2z \quad \text{......................} \quad (i)$$

To find k,

Put w =5, x = 2, y = 3 and z =10 in equation (i)

$$5 = k(2)(3)^2(10)$$

$$5 = k(20)(9)$$

$$5 = k(180)$$

$$\frac{5}{180} = k$$

$$\Rightarrow \quad k = \frac{\cancel{5}^{1}}{\cancel{180}_{36}}$$

$$k = \frac{1}{36}$$

Put $k = \frac{1}{36}$ in equation (i)

$$w = \frac{1}{36}xy^2z \text{............}(ii)$$

To find w,

Now put x = 4, y = 7, z = 3 in equation (ii)

$$w = \frac{1}{36}xy^2z$$

$$w = \frac{1}{36}(4)(7)^2(3)$$

$$w = \frac{1}{36}(4)(49)(3)$$

$$w = \frac{1}{\cancel{36}_{3}}(49)\times \cancel{12}^{1}$$

$$w = \frac{1}{3}(49)$$

$$\boxed{w = \frac{49}{3}}$$

Q.3 If y varies directly as x^3 and inversely as z^2 and t, and y = 16 when x = 4, z = 2, t = 3. Find the value of y when x = 2, z = 3 and t = 4.

Solution:

$y \propto \frac{x^3}{z^2t}$	$y=16, x=4$ $z=2, t=3$	$y=?, x=2$ $z=3, t=4$

$$y \propto \frac{x^3}{z^2t}$$

$$y = \frac{kx^3}{z^2t} \quad \text{.....................(i)}$$

To find k,

Put y = 16, x = 4, z = 2 and t = 3 in equation (i)

$$16 = \frac{k(4)^3}{(2)^2(3)}$$

$$16 = \frac{k(64)}{4\times 3}$$

$$\frac{16\times 4\times 3}{64} = k$$

$$\frac{\cancel{64}}{\cancel{64}}\times 3 = k \quad \Rightarrow \quad k = 3$$

Put k = 3, in equation (i)

$$y = \frac{3x^3}{z^2t} \text{.................(ii)}$$

To find y,

Now, put x = 2, z = 3 and t = 4 in equation (ii)

$$y = \frac{3\times(2)^3}{(3)^2(4)}$$

$$y = \frac{3\times 8}{9\times 4}$$

$$y = \frac{24}{36}$$

$$y = \frac{\overset{2}{\cancel{24}}}{\underset{3}{\cancel{36}}} \; (\textit{dividing by } 12)$$

$$\boxed{y = \frac{2}{3}}$$

Q.4 If u varies directly as x^2 and inversely as product yz^3, and u=2, when x = 8, y = 7, z = 2. Find the value of u when x =6, y=3, z = 2.

Solution:

$u \propto \frac{x^2}{yz^3}$	$u=2, x=8$ $y=7, z=2$	$u=?, x=6$ $y=3, z=2$

$$u \propto \frac{x^2}{yz^3}$$

$$u = \frac{kx^2}{yz^3} \quad \text{...............(i)}$$

To find k,

Put u = 2, x = 8, y = 7 and z = 2 in equation(i)

$$2 = \frac{k(8)^2}{7(2)^3}$$

$$2 = \frac{k64}{7(8)}$$

$$\frac{2(8)(7)}{64} = k$$

$$\frac{\overset{1}{\cancel{16}}(7)}{\underset{4}{\cancel{64}}} = k$$

$$\frac{7}{4} = k \quad \Rightarrow \quad k = \frac{7}{4}$$

Put $k = \frac{7}{4}$ in equation (i)

$$u = \frac{7x^2}{4yz^3} \text{..................(ii)}$$

To find u,

Now put x = 6, y = 3, z = 2 in equation (ii)

$$u = \frac{7x^2}{4yz^3} = \frac{7(6)^2}{4(3)(2)^3}$$

$$u = \frac{7(\overset{3}{\cancel{36}})}{\underset{1}{\cancel{12}}\times 8}$$

$$u = \frac{7(3)}{8}$$

$$\boxed{u = \frac{21}{8}}$$

Q.5 If v varies directly as the product xy^3 and inversely as z^2 and v = 27 when x = 7, y = 6, z = 7. Find the value of V when x = 6; y = 2, z = 3.

Solution: $v \propto \frac{xy^3}{z^2}$ | $v = 27, x = 7, y = 6, z = 7$ | $v = ?, x = 6, y = 2, z = 3$

$$v \propto \frac{xy^3}{z^2}$$

$$v = \frac{kxy^3}{z^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put v = 27, x = 7, y = 6, z = 7 in equation(i)

$$27 = \frac{k(7)(6)^3}{(7)^2}$$

$$27 = \frac{k(216)}{7}$$

$$\frac{\overset{1}{\cancel{27}} \times 7}{\underset{8}{\cancel{216}}} = k$$

$$\Rightarrow \quad k = \frac{7}{8}$$

Put $k = \frac{7}{8}$ in equation (i)

$$v = \frac{7xy^3}{8z^2} \ldots\ldots\ldots\ldots \text{(ii)}$$

To find v,

Now, put x = 6, y = 2, z = 3 in equation (ii).

$$v = \frac{7}{8}\frac{(6)(2)^3}{(3)^2}$$

$$v = \frac{7}{\cancel{8}}\frac{(6)(\cancel{8})}{9}$$

$$v = \frac{\cancel{42}^{14}}{\cancel{9}_3}$$

$$\Rightarrow \quad \boxed{v = \frac{14}{3}}$$

Q.6 If w varies inversely as the cube of U, and w = 5 when U = 3. Find w, when U = 6.

Solution: $w \propto \frac{1}{u^3}$ | $w = 5, u = 3$ | $w = ?, u = 6$

$$w \propto \frac{1}{u^3}$$

$$w = \frac{k}{u^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put w = 5 and u = 3 in equation (i)

$$5 = \frac{k}{(3)^3}$$

$$5 = \frac{k}{27}$$

$$\Rightarrow \quad k = 27 \times 5$$

$$k = 135$$

Put k = 135 in equation (i)

$$w = \frac{k}{u^3}$$

$$w = \frac{135}{u^3} \ldots\ldots\ldots\ldots \text{(ii)}$$

To find w,

Now, Put u = 6 in equation (ii)

$$w = \frac{135}{(6)^3}$$

$$w = \frac{135}{216}$$

$$w = \frac{5 \times \cancel{27}}{8 \times \cancel{27}}$$

$$\boxed{w = \frac{5}{8}}$$

EXERCISE 3.6

Q.1 If a:b=c:d, (a,b,c,d ≠0), then show that

(i) $\frac{4a-9b}{4a+9b}=\frac{4c-9d}{4c+9d}$

(ii) $\frac{6a-5b}{6a+5b}=\frac{6c-5d}{6c+5d}$

(iii) $\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$

(iv) $a^6+c^6 : b^6+d^6 = a^3c^3 : b^3d^3$

(v) $p(a+b)+qb : p(c+d)+qd = a : c$

(vi) $a^2+b^2 : \frac{a^3}{a+b} = c^2+d^2 : \frac{c^3}{c+d}$

(vii) $\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$

Solution:

(i) $\mathbf{\frac{4a-9b}{4a+9b}=\frac{4c-9d}{4c+9d}}$

Let $a : b = c : d = k$

$\frac{a}{b}=\frac{c}{d}=k$

$\frac{a}{b}=k,\ and\ \frac{c}{d}=k$

$\boxed{a = bk}$ and $\boxed{c = dk}$

$L.H.S=\frac{4a-9b}{4a+9b}$

$=\frac{4bk-9b}{4bk+9b}$

$=\frac{b(4k-9)}{b(4k+9)}$

$L.H.S=\frac{4k-9}{4k+9}$ .... (i)

Now , taking

$R.H.S=\frac{4c-9d}{4c+9d}$

$=\frac{4dk-9d}{4dk+9d}$

$=\frac{d(4k-9)}{d(4k+d)}$

$R.H.S=\frac{4k-9}{4k+9}$ .... (ii)

From equation (i) and (ii)

L.H.S = R.H.S

So $\dfrac{4a-9b}{4a+9b}=\dfrac{4c-9d}{4c+9d}$

Hence proved

(ii) $\mathbf{\dfrac{6a-5b}{6a+5b}=\dfrac{6c-5d}{6c+5d}}$

Let a : b = c : d = k

$\dfrac{a}{b}=\dfrac{c}{d}=k$

$\Rightarrow \dfrac{a}{b}=k \quad : \quad \dfrac{c}{d}=k$

$\boxed{a=bk}$; $\boxed{c=dk}$

Let $L.H.S=\dfrac{6a-5b}{6a+5b}$

$=\dfrac{6bk-5b}{6bk+5b}$

$=\dfrac{b(6k-5)}{b(6k+5)}$

$L.H.S=\dfrac{6k-5}{6k+5}$ .... (i)

Now $R.H.S=\dfrac{6c-5d}{6c+5d}$

$=\dfrac{6dk-5d}{6dk+5d}$

$=\dfrac{d(6k-5)}{d(6k+5)}$

$R.H.S=\dfrac{6k-5}{6k+5}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

So, $\dfrac{6a-5b}{6a+5b}=\dfrac{6c-5d}{6c+5d}$

Hence proved

(iii) $\mathbf{\dfrac{a}{b}=\sqrt{\dfrac{a^2+c^2}{b^2+d^2}}}$

Let a : b = c : d = k

$\dfrac{a}{b}=\dfrac{c}{d}=k$

$\dfrac{a}{b}=k \quad : \quad \dfrac{c}{d}=k$

$\boxed{a=bk}$; $\boxed{c=dk}$

Let $L.H.S=\dfrac{a}{b}=\dfrac{\not{b}k}{\not{b}}$

L.H.S = k ———— (i)

Now $R.H.S=\sqrt{\dfrac{a^2+c^2}{b^2+d^2}}$

$=\sqrt{\dfrac{b^2k^2+d^2k^2}{b^2+d^2}}$

$=\sqrt{\dfrac{k^2\cancel{(b^2+d^2)}}{\cancel{(b^2+d^2)}}}$

$=\sqrt{k^2}$

$R.H.S=k$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

Hence proved

So $\dfrac{a}{b}=\sqrt{\dfrac{a^2+c^2}{b^2+d^2}}$

(iv) $\mathbf{a^6+c^6:b^6+d^6=a^3c^3:b^3d^3}$

Let a : b = c : d = k

$\dfrac{a}{b}=\dfrac{c}{d}=k$

$\dfrac{a}{b}=k;\dfrac{c}{d}=k$

Let L.H.S = $a^6+c^6:b^6+d^6$

$=\dfrac{a^6+c^6}{b^6+d^6}=\dfrac{(bk)^6+(dk)^6}{b^6+d^6}$

$=\dfrac{b^6k^6+d^6k^6}{b^6+d^6}=\dfrac{k^6\cancel{(b^6+d^6)}}{\cancel{(b^6+d^6)}}$

L.H.S = k^6 .... (i)

Now, R.H.S = $a^3c^3:b^3d^3$

$=\dfrac{a^3c^3}{b^3d^3}=\dfrac{(bk)^3.(dk)^3}{b^3d^3}$

$=\dfrac{b^3k^3.d^3k^3}{b^3d^3}=\dfrac{\cancel{b^3d^3}.k^{3+3}}{\cancel{b^3d^3}}$

R.H.S = k^6 .... (ii)

From (i) and (ii)

L.H.S = R.H.S

So $a^6+c^6:b^6+d^6=a^3c^3:b^3d^3$

Hence proved

(v) $\mathbf{p(a+b)+qb : p(c+d)+qd = a : c}$

Let $a : b = c : d = k$

$$\frac{a}{b}=\frac{c}{d}=k$$

$$\frac{a}{b}=k \;;\; \frac{c}{d}=k$$

$$\boxed{a=bk} \;;\; \boxed{c=dk}$$

Let $L.H.S = p(a+b)+qb : p(c+d)+qd$

$$=\frac{p(a+b)+qb}{p(c+d)+qd}$$

$$=\frac{p(bk+b)+qb}{p(dk+d)+qd}$$

$$=\frac{pb(k+1)+qb}{pd(k+1)+qd}$$

$$=\frac{b[p(k+1)+q]}{d[p(k+1)+q]}$$

$$L.H.S=\frac{b}{d} \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

$$R.H.S = a : c = \frac{a}{c}$$

$$R.H.S=\frac{bk}{dk}$$

$$R.H.S=\frac{b}{d} \quad \ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From (i) and (ii)

L.H.S = R.H.S

$$p(a+b)+qb : p(c+d)+qd = a : b$$

Hence proved

(vi) $\mathbf{a^2+b^2 : \frac{a^3}{a+b} = c^2+d^2 : \frac{c^3}{c+d}}$

Let $a : b = c : d = k$

$$\frac{a}{b}=\frac{c}{d}=k$$

$$\frac{a}{b}=k \;;\; \frac{c}{d}=k$$

$$\boxed{a=bk} \;;\; \boxed{c=dk}$$

Let $L.H.S=a^2+b^2:\frac{a^3}{a+b}$

$$=\left[(bk)^2+b^2\right]\div\frac{(bk)^3}{bk+b}$$

$$=(b^2k^2+b^2)\times\frac{(bk+b)}{b^3k^3}$$

$$=b^2(k^2+1)\times\frac{b(k+1)}{b^3k^3}$$

$$=\frac{b^3(k^2+1)(k+1)}{b^3k^3}$$

$$L.H.S=\frac{(k^2+1)(k+1)}{k^3} \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Now $R.H.S=c^2+d^2:\frac{c^3}{c+d}$

$$=\left[(dk)^2+d^2\right]\div\frac{(dk)^3}{(dk+d)}$$

$$=(d^2k^2+d^2)\times\frac{(dk+d)}{d^3k^3}$$

$$=d^2(k^2+1)\times\frac{d(k+1)}{d^3k^3}$$

$$=\frac{d^3(k^2+1)(k+1)}{d^3k^3}$$

$$R.H.S=\frac{(k^2+1)(k+1)}{k^3} \quad \ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From (i) and (ii)

L.H.S = R.H.S

So $a^2+b^2:\frac{a^3}{a+b}=c^2+d^2:\frac{c^3}{c+d}$

Hence proved

(vii) $\mathbf{\frac{a}{a-b}:\frac{a+b}{b}=\frac{c}{c-d}:\frac{c+d}{d}}$

Let $a : b = c : d = k$

$$\frac{a}{b}=\frac{c}{d}=k$$

$$\frac{a}{b}=k, \quad \frac{c}{d}=k$$

$$\boxed{a=bk}, \quad \boxed{c=dk}$$

Let $L.H.S = \frac{a}{a-b} : \frac{a+b}{b}$

$= \frac{bk}{bk-b} \div \frac{bk+b}{b}$

$= \frac{\cancel{b}k}{\cancel{b}(k-1)} \times \frac{\cancel{b}}{\cancel{b}(k+1)}$

$= \frac{k}{(k)^2-(1)^2}$

L.H.S $= \frac{k}{k^2-1}$ .... (i)

Now $R.H.S = \frac{c}{c-d} : \frac{c+d}{d}$

$= \frac{dk}{dk-d} \div \frac{dk+d}{d}$

$= \frac{\cancel{d}k}{\cancel{d}(k-1)} \times \frac{\cancel{d}}{\cancel{d}(k+1)}$

$= \frac{k}{(k)^2-(1)^2}$

$R.H.S = \frac{k}{k^2-1}$ .... (ii)

From (i) and (ii)
L.H.S = R.H.S

So $\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$

Hence proved

Q.2 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ $(a, b, c, d, e, f \neq 0)$, then show that

(i) $\frac{a}{b} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

(ii) $\frac{ac+ce+ea}{bd+df+fb} = \left[\frac{ace}{bdf}\right]^{2/3}$

(iii) $\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

Solution:

(i) $\frac{a}{b} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$\frac{a}{b} = k$, $\frac{c}{d} = k$, $\frac{e}{f} = k$

$\boxed{a = bk\ ,\ c = dk\ ,\ e = fk}$

$L.H.S = \frac{a}{b} = \frac{\cancel{b}k}{\cancel{b}} = k$

L.H.S = k .... (i)

Now $R.H.S = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

$= \sqrt{\frac{b^2k^2+d^2k^2+f^2k^2}{b^2+d^2+f^2}}$

$= \sqrt{\frac{k^2\cancel{(b^2+d^2+f^2)}}{\cancel{(b^2+d^2+f^2)}}}$

$= \sqrt{k^2}$

R.H.S = k .... (ii)

From (i) and (ii)
R.H.S = R.H.S

i.e $\frac{a}{b} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

(ii) $\frac{\mathbf{ac+ce+ea}}{\mathbf{bd+df+fb}} = \left(\frac{\mathbf{ace}}{\mathbf{bdf}}\right)^{\frac{2}{3}}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$\frac{a}{b} = k,$ $\frac{c}{d} = k,$ $\frac{e}{f} = k$

$\boxed{a = bk;\quad c = dk,\quad e = fk}$

Let $L.H.S = \frac{ac+ce+ea}{bd+df+fb}$

$= \frac{bk(dk)+dk(fk)+fk(bk)}{bd+df+fb}$

$= \frac{k^2bd+k^2df+k^2fb}{bd+df+fb}$

$= \frac{k^2\cancel{(bd+df+fb)}}{\cancel{(bd+df+fb)}}$

$L.H.S = k^2$ .... (i)

Now, $R.H.S = \left(\frac{ace}{bdf}\right)^{\frac{2}{3}}$

$$= \left(\frac{bk.dk.fk}{bdf}\right)^{\frac{2}{3}} = \left(\frac{k^3.bdf}{bdf}\right)^{\frac{2}{3}}$$

$$= \left(k^3\right)^{\frac{2}{3}} = k^{3\times\frac{2}{3}}$$

R.H.S = k^2 …………… (ii)

From (i) and (ii)

L.H.S = R.H.S

So, $\frac{ac+ce+ea}{bd+df+fb} = \left(\frac{ace}{bdf}\right)^{\frac{2}{3}}$

Hence proved.

(iii) $\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$\frac{a}{b} = k, \quad \frac{c}{d} = k, \quad \frac{e}{f} = k$

$\boxed{a = bk, \quad c = dk, \quad e = fk}$

Let $L.H.S = \frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb}$

$$= \frac{bk.dk}{bd} + \frac{dk.fk}{df} + \frac{fk.bk}{fb}$$

$$= \frac{k^2 \cancel{bd}}{\cancel{bd}} + \frac{k^2 \cancel{df}}{\cancel{df}} + \frac{k^2 \cancel{fb}}{\cancel{fb}}$$

$= k^2 + k^2 + k^2$

L.H.S = $3k^2$ …………… (i) Now

$R.H.S = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

$$= \frac{\cancel{b^2}k^2}{\cancel{b^2}} + \frac{\cancel{d^2}k^2}{\cancel{d^2}} + \frac{\cancel{f^2}k^2}{\cancel{f^2}}$$

$= k^2 + k^2 + k^2$

R.H.S $= 3k^2$ …………… (ii)

From (i) and (ii)

L.H.S = R.H.S

$\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{df} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

EXERCISE 3.7

Q. 1 The surface area A of a cube varies directly as the square of the length ℓ of an edge and A = 27 square units when ℓ = 3 units. Find (i) A when ℓ = 4 units (ii) ℓ when A = 12 sq. units.

Solution: $A \propto \ell^2$ | $A = 27$ units , $\ell = 3$ units

(i) A = ? , ℓ = 4 units

(ii) ℓ = ? , A = 12 units

$A \propto \ell^2$

$A = k\ell^2$ …………….. (i)

To find k,

Put A = 27 and $\ell = 3$ in equation (i)

$27 = k(3)^2$

$27 = k(9)$

$\frac{27}{9} = k$

$3 = k$

$\Rightarrow$ $k = 3$

(i) To find A,

Put k = 3 and $\ell = 4$ in equation (i)

$A = 3(4)^2$

$A = 3(16)$

$A = 48$ square units

(ii) To find ℓ,

Put k = 3 and A = 12 in equation (i)

$A = K\ell^2$

$12 = 3\ell^2$

$\frac{12}{3} = \ell^2$

$\ell^2 = 4$

$\sqrt{\ell^2} = \pm\sqrt{4}$

$\ell = \pm 2$ units

Since length is always taken positive so

$\ell = 2$ units

Q. 2 The surface area S of the sphere varies directly as the square of radius r, and S = 16 π when r = 2. Find r when S = 36π.

Solution:

$\boxed{S \propto r^2}$ $\boxed{S = 16\pi,\ r = 2}$ $\boxed{r = ?\ ,\ S = 36\pi}$

$$S \propto r^2$$

$$S = kr^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put s = 16 π and r = 2 in equation (i)

$$16\pi = k(2)^2$$

$$16\pi = k(4)$$

$$\frac{4\cancel{16}\pi}{1\cancel{4}} = k$$

$$4\pi = k$$

$\Rightarrow$ $\boxed{k = 4\pi}$

To find s,

Put $k = 4\pi$ in equation (i)

$$S = kr^2$$

$$S = 4\pi r^2 \ldots\ldots\ldots \text{(ii)}$$

To find r,

Now put $S = 36\pi$ in equation (ii)

$$36\pi = 4\pi r^2$$

$$\frac{36\pi}{4\pi} = r^2$$

$$\frac{9\cancel{36}\cancel{\pi}}{1\cancel{4}\cancel{\pi}} = r^2$$

$$r^2 = 9$$

$$r^2 = 9$$

Taking square root

$$\sqrt{r^2} = \pm\sqrt{9}$$

$$r = \pm 3$$

Since r is radius of sphere which is positive quantity so $\boxed{r = 3 \text{ units}}$

Q.3 In Hook's law the force F applied to stretch a spring varies directly as the amount of elongation S, and F = 32 ℓb when S = 1.6 in. Find (i) S when F = 50 ℓb (ii) F when S=0.8 in.

Solution: $\boxed{F \propto S}$ $\boxed{F = 32\ \ell b\ ,\ S = 1.6 \text{ inch}}$

$\boxed{\text{(i)}\ S = ?\ ,\ F = 50\ \ell b}$

$\boxed{\text{(ii)}\ F = ?,\ S = 0.8 \text{inch}}$

$$F \propto S$$

$$F = kS \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

put F = 32 and s = 1.6 in equation (i)

$$32 = k(1.6)$$

$$\frac{32}{1.6} = k$$

$$20 = k$$

$$k = 20$$

(i) To find s,

Put $k = 20$ and $F = 50$ in equation. (i)

$$F = kS$$

$$50 = 20(S)$$

$$\frac{50}{20} = S$$

$$S = \frac{5}{2}$$

$$S = 2.5 \text{ inch}$$

(ii) To find F,

Put $K = 20$ and $S = 0.8$ inch in equation (i)

$$F = kS$$

$$F = 20(0.8)$$

$$F = {}^{2}\cancel{20}\,\frac{8}{\cancel{10}_1}$$

$$F = 2 \times 8$$

$$F = 16\ \ell b$$

Q.4 **The intensity I of the light from a given source varies inversely as the square of the distance d from it. If the intensity is 20 candlepower at a distance of 12 ft. from the source, Find the intensity at a point 8ft. from the source.**

Solution: $I \propto \frac{1}{d^2}$ | I = 20 cp, d = 12 ft | I = ?, d = 8 ft

$$I \propto \frac{1}{d^2}$$

$$I = \frac{k}{d^2} \quad \ldots\ldots\ldots\ldots (i)$$

To find k,

Put I = 20 and d = 12 ft in equation (i)

$$20 = \frac{k}{(12)^2}$$

$$20 = \frac{k}{144}$$

$$20 \times 144 = k$$

$$2880 = k$$

$$k = 2880$$

To find I,

Now Put k = 2880 and d = 8 ft in equation (i)

$$I = \frac{k}{d^2}$$

$$I = \frac{2880}{(8)^2}$$

$$I = \frac{2880}{64}$$

$I = 45$ Candle power

Q.5 The pressure P in a body of fluid, varies directly as the depth d. If the pressure exerted on the bottom of a tank by a column of fluid 5ft. high is 2.25 lb/sq. in, how deep must he fluid be to exert a pressure of 9 lb/sq.in?

Solution:

$p \propto d$

d = 5ft, p = 2.25 ℓb/sq.inch

d = ?, p = 9 ℓb/sq.inch

$$p \propto d$$

$$p = kd \quad \ldots\ldots\ldots\ldots (i)$$

To find k,

Put p = 2.25 and d = 5 in equation (i)

$$2.25 = k(5)$$

$$\frac{2.25}{5} = k$$

$\Rightarrow$ $k = 0.45$

To find d,

Put k = 0.45 and p = 9 in equation (i)

$$p = kd$$

$$9 = 0.45\,(d)$$

$$\frac{9}{0.45} = d$$

$$d = \frac{900}{45} \Rightarrow \boxed{d = 20ft}$$

Q.6 **Labour costs c varies jointly as the number of workers n and the average number of days d, if the cost of 800 workers for 13 days is Rs. 286000, then find the labour cost of 600 workers for 18 days.**

Solution: $c \propto nd$

c = Rs.286000, n = 800, d = 13 | c = ?, n = 600, d = 18

$$c = knd \quad \ldots\ldots (i)$$

To find k,

Put the values of c, n and d in equation (i)

$$286000 = k(800)(13)$$

$$\frac{286000}{800 \times 13} = k$$

$$27.5 = k \Rightarrow \boxed{k = 27.5}$$

To find C,

Put k = 27.5, n = 600 and d = 18 in equation (i)

$$c = knd$$

$$c = 27.5 \times 600 \times 18$$

$$\boxed{c = \text{Rs. } 297000}$$

Q.7 The supporting load c of a pillar varies as the fourth power of its diameter d and inversely as the square of its length ℓ. A pillar of diameter 6 inch and of height 30 feet will support a load of 63 tons. How high a 4 inch pillar must be to support a load of 28 tons?

Solution:

$c \propto \frac{d^4}{\ell^2}$	d = 6 inch	ℓ = ?
	ℓ = 30ft	d = 4 inch
	c = 63 tons	c = 28 tons

$$c \propto \frac{d^4}{\ell^2}$$

$$c = \frac{kd^4}{\ell^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put the values of d, ℓ and c in equation (i)

$$63 = \frac{k(6)^4}{(30)^2}$$

$$63 = \frac{k(1296)}{900}$$

$$\frac{63 \times 900}{1296} = k$$

$$\frac{56700}{1296} = k$$

$$43.75 = k$$

$$k = 43.75$$

To find ℓ,

Put K = 43.75 , d = 4 and c = 28 in equation (i)

$$c = \frac{kd^4}{\ell^2}$$

$$28 = \frac{43.75 \times (4)^4}{\ell^2}$$

$$\ell^2 \times 28 = 43.75 \times 256$$

$$\ell^2 = \frac{11200}{28}$$

$$\ell^2 = 400$$

Taking square root of both side

$$\sqrt{\ell^2} = \pm\sqrt{400}$$

$$\ell = \pm 20$$

Since length is always positive so

$$\boxed{\ell = 20\,ft}$$

Q.8 The time T required for an elevator to lift a weight varies jointly as the weight w and the lifting depth d varies inversely as the power p of the motor. If 25 sec. are required for a 4-hp motor to lift 500 ℓb through 40 ft, what power is required to lift 800 ℓb, through 120 ft in 40 sec?

Solution: $\boxed{T \propto \frac{wd}{P}}$

T = 25s, p = 4hp	p = ?, w = 800ℓb
w = 500ℓb, d = 40ft	d = 120ft, T = 40sec

$$T \propto \frac{wd}{p}$$

$$T = \frac{kwd}{p} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put the values of T, P, W and d in equation (i)

$$25 = \frac{k(500)(40)}{4}$$

$$\frac{25 \times 4}{500 \times 40} = k$$

$$\frac{100}{20000} = k$$

$$\frac{1}{200} = k \quad \Rightarrow \quad \boxed{k = \frac{1}{200}}$$

To find P,

Put, $k = \frac{1}{200}$, w = 800 ℓb, d = 120 ft,

T = 40s in equation (i)

$$T = \frac{kwd}{p}$$

$$40 = \frac{1}{200} \times \frac{800 \times 120}{p}$$

$$40 = \frac{480}{p}$$

$$p = \frac{480}{40}$$

$$p = \frac{480}{40} = 12$$

$$\Rightarrow \quad \boxed{p = 12hp}$$

Q.9 The kinetic energy (K.E) of a body varies jointly as the mass "m" of the body and the square of its velocity "v". If the kinetic energy is 4320 ft/ℓb when the mass is 45 ℓb and the velocity is 24 ft/sec. Determine the kinetic energy of a 3000 ℓb automobile travelling 44 ft/ sec.

Solution: $\boxed{K.E \propto mv^2}$

$K.E = 4320\ ft/\ell b$	$K.E = ?$
$m = 45\ \ell b$	$m = 3000\ \ell b$
$v = 24\ ft/s$	$v = 44\ ft/\text{sec}$

$K.E \propto mv^2$

$K.E = \text{kmv}^2$ .... (i)

To find K,

Put K.E = 4320, m = 45 and v = 24 in equation (i)

$4320 = k(45)(24)^2$

$4320 = k(45 \times 576)$

$4320 = k\ 25920$

$\frac{4320}{25920} = k$

$\boxed{k = \frac{432}{2592}}$

To find K. E,

Put $k = \frac{432}{2592}$, m = 3000, v = 44 in equation (i)

$K.E = Kmv^2$

$K.E = \frac{432}{2592} \times 300 \times (44)^2$

$K.E = \frac{432}{2592} \times 300 \times 1936$

$K.E = \frac{250905600}{2592}$

$\boxed{K.E = 96,8000\ ft/\ell b}$

MISCELLANEOUS EXERCISE – 3

Q. 1 Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

1. If $u \propto v^2$, then
(a) $u \propto v^2$ (b) $u = kv^2$
(c) $uv^2 = k$ (d) $uv^2 = 1$

2. The third proportional of x^2 and y^2 is
(a) $\frac{y^2}{x^2}$ (b) x^2y^2
(c) $\frac{y^4}{x^2}$ (d) $\frac{y^2}{x^4}$

3. The fourth proportional w of x:y::v:w is........
(a) $\frac{xy}{v}$ (b) $\frac{vy}{x}$
(c) xyv (d) $\frac{x}{vy}$

4. If a: b=x: y, then alternando property is
(a) $\frac{a}{x}=\frac{b}{y}$ (b) $\frac{a}{b}=\frac{x}{y}$
(c) $\frac{a+b}{b}=\frac{x+y}{y}$ (d) $\frac{a-b}{x}=\frac{x-y}{y}$

5. In continued proportion a: b = b: c, ac= b^2, b is said to be ___ proportional.
(a) third (b) fourth
(c) means (d) none of these

6. In continued proportion a: b = b: c, c is said to be __ proportional to a and b.
(a) third (b) fourth
(c) means (d) None of these

7. If a : b = x : y, then invertendo property is
(a) $\frac{a}{x}=\frac{b}{y}$ (b) $\frac{a}{a-b}=\frac{x}{x-y}$
(c) $\frac{a+b}{b}=\frac{x+y}{y}$ (d) $\frac{b}{a}=\frac{y}{x}$

8. In a ratio a: b, a is called
(a) relation (b) antecedent
(c) consequent (d) None of these

9. If $\frac{a}{b}=\frac{c}{d}$, then componendo property is
(a) $\frac{a}{a+b}=\frac{c}{c+d}$ (b) $\frac{a}{a-b}=\frac{c}{c-d}$
(c) $\frac{ad}{bc}$ (d) $\frac{a-b}{b}=\frac{c-d}{d}$

10. If $\frac{24}{7}=\frac{6}{x}$, then 4x = ...
(a) 7 (b) $\frac{7}{4}$
(c) 4 (d) $\frac{42}{24}$

11. In a ratio x: y, y is called
(a) relation (b) antecedent
(c) consequent (d) None of these

12. In a proportion a: b:: c: d, a and d are called
(a) means
(b) extremes
(c) forth proportional
(d) None of these

13. In a proportion a:b::c: d, b and c are called
(a) means
(b) extremes
(c) fourth proportional
(d) None of these

14. Find x in proportion 4:x::5:15
(a) $\frac{75}{4}$ (b) $\frac{4}{3}$
(c) $\frac{3}{4}$ (d) 12

15. If $y^2 \propto \frac{1}{x^3}$, then
(a) $y^2=\frac{k}{x^3}$ (b) $y^2=\frac{1}{x^3}$
(c) $y^2=x^2$ (d) $y^2=kx^3$

16. If w varies inversely as p^2, then k = ...
(a) $\frac{w}{P^2}$ (b) wp^2
(c) $\frac{P^2}{w}$ (d) wp

17. If 6: m:: 9: 12, then m = ...
(a) 6 (b) 9
(c) 1 (d) 8

18. If x and y varies directly, then x = ...
(a) y (b) ky
(c) $\frac{k}{y}$ (d) k

19. If $\frac{u}{v}=\frac{v}{w}=k$, then
(a) $u=wk^2$ (b) $u=vk^2$
(c) $u=w^2k$ (d) $u=v^2k$

20. If $\frac{5a}{3x}=\frac{15b}{y}$, then ay = ... 03(114)
(a) $\frac{9bx}{y}$ (b) $\frac{9y}{9b}$
(c) $5ay=45bx$ (d) 9bx

21. Fourth proportional of 5, 8, 15 is ....
(a) 120 (b) 40
(c) 24 (d) 20

22. Newton's law of Gravitation is an example of
(a) variation
(b) direct variation
(c) inverse variation
(d) joint variation

23. The relation between radius and circumference of a circle is an example of ....
(a) variation
(b) direct variation
(c) inverse variation
(d) joint variation

24. If v varies directly as u^3, then u^3 = ...
(a) vk (b) $\frac{k}{v}$
(c) $\frac{v}{k}$ (d) vk^3

25. The simplest form of the ratio $\frac{(x+y)(x^2+xy+y^2)}{x^3-y^3}$ is.........
(a) $\frac{x-y}{x+y}$ (b) $\frac{x+y}{x-y}$
(c) 1 (d) 2

26. A third proportional of 12 and 4, is...........
    (a) $\frac{3}{4}$ (b) $\frac{4}{3}$
    (c) 12 (d) 16
27. What is the value of p in the continued proportion of 5, p, 45?
    (a) 225 (b) ±50
    (c) ±15 (d) ±9
28. Fourth proportional of $4x^4$, $2x^3$, $18x^5$ is ....
    (a) $36x^8$ (b) $9x^2$
    (c) $9x^{12}$ (d) $9x^4$
29. Mean Proportional of 20 and 45 is ...
    (a) ±30 (b) ±25
    (c) ±20 (d) ±15
30. The mean proportional of $9p^6q^4$ and r^8 is.........
    (a) $\pm3p^3q^2r^4$ (b) $\pm9p^6q^2r^8$
    (c) $\pm9p^3q^2r^4$ (d) $\pm3p^6q^4q^8$
31. There are .... types of variation.
    (a) one (b) two
    (c) three (d) four
32. In proportion 7:4::p:8, p =.........
    (a) 1 (b) 28
    (c) 14 (d) 56
33. The mean proportional of $4m^2n^4$ and p^6 is...........
    (a) $\pm2mnp$ (b) $\pm mnp$
    (c) $\pm\frac{2m^2n}{P^3}$ (d) $\pm2mn^2p^3$
34. The continued proportion of 4, m, 9 is............
    (a) 4 : m : : m : 9
    (b) 4 : 9 : : 9 : m
    (c) 9 : 4 : : 4 : m
    (d) 9 : 4 : : m : m
35. What is the value of P in continued proportion of 12, p, 3?
    (a) ±4 (b) ±6
    (c) ±30 (d) ±2
36. The fourth proportional of 15, 6, 5 is ...
    (a) 30 (b) 15
    (c) 2 (d) 1
37. What is the value of x in the continued proportion of 8, x, 18?
    (a) ±144 (b) ±8
    (c) ±18 (d) ±12
38. If $\frac{9pq}{2\ell m} = \frac{18p}{5m}$, then 5q = ...
    (a) 4m (b) 4p
    (c) 4ℓ (d) 4q
39. Third proportional of 6, 12 is ....
    (a) 24 (b) 2
    (c) 18 (d) 84
40. Mean proportional of $20x^3y^5$, $5x^7y$
    (a) $\pm10x^5y^6$ (b) $\pm10x^5y^3$
    (c) $\pm10x^{10}y^6$ (d) $100x^{10}y^6$
41. Third proportional of a^3, $3a^2$ is ....
    (a) $3a^5$ (b) 9a
    (c) $9a^4$ (d) $9a^7$

ANSWER KEY

1.	b	2.	c	3.	b	4.	a	5.	c
6.	a	7.	d	8.	b	9.	a	10.	a
11.	c	12.	b	13.	a	14.	d	15.	a
16.	b	17.	d	18.	b	19.	a	20.	d
21.	c	22.	d	23.	b	24.	c	25.	b
26.	b	27.	c	28.	d	29.	a	30.	a
31.	b	32.	c	33.	d	34.	a	35.	b
36.	c	37.	d	38.	c	39.	a	40.	b
41.	b								

Q.2: Write short answer of the following questions:

(i) Define ratio and give one example.

Answer: A relation between two quantities of the same kind is called ratio and ratio has no unit. If 'a' and 'b' are two quantities of the same kind and b is not zero, then their ratio is defined as a : b or $\frac{a}{b}$ or $a \div b$.

Example: If a hockey team wins 4 matches and losses 5, then ratio of the games won to the games lost is 4 : 5 or in fraction $\frac{4}{5}$.

(ii) Define proportion.

Proportion:

A proportion is a statement, which is expressed as an equivalence of two ratios.

If two ratios a : b and c : d are equal, then we can write a : b = c : d

Symbolically the proportion of a, b, c and d is written as

a : b : : c : d

(iii) Define direct variation.

Direct variation:

If two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in the other quantity in the same ratio, then this variation is called direct variation.

If a quantity y varies directly with regard to a quantity x, we say that y is directly proportional to x and is written as

$y \propto x$ or $y = kx$ where $k \neq 0$.

(iv) Define inverse variation.

Inverse Variation:

If two quantities are related in such a way that when one quantity increases, the other decreases in the same ratio is called inverse variation.

If a quantity y varies inversely with regard to quantity x, we say that y is inversely proportional to x or y varies inversely as x and is written as

$$y \propto \frac{1}{x} \quad \text{or} \quad y = \frac{k}{x}$$

i.e $xy = k$, where $k \neq 0$ is the constant of variation.

(v) State theorem of componedndo–dividendo.

Theorem of componendo–dividendo

If a : b = c : d, then componendo–dividendo theorem is written as:

(i) a + b : a − b = c + d : c − d

(ii) a − b : a + b = c − d : c + d

(vi) Find x, if 6:x:: 3: 5.

Solution:

6 : x : : 3 : 5

Product of Means = Product of Extremes:

$$3(x) = 6 \times 5$$
$$3x = 30$$
$$x = \frac{30}{3}$$
$$\boxed{x = 10}$$

(vii) If x and y^2 varies directly, and x = 27 when y = 4. Find the value of y when x=3.

Solution:

$\boxed{x \propto y^2}$ $\boxed{\begin{matrix} x = 27 \\ y = 4 \end{matrix}}$ $\boxed{\begin{matrix} y = ? \\ x = 3 \end{matrix}}$

$$y^2 \propto x$$
$$y^2 = kx \text{ ....(i)}$$

To find K,

Put x = 27 and y = 4 in equation (i)

$$(4)^2 = k(27)$$
$$\frac{16}{27} = k \Rightarrow \boxed{k = \frac{16}{27}}$$

To find y,

Put $k = \frac{16}{27}$ and x = 3 in equation (i)

$$y^2 = kx$$
$$y^2 = \frac{16}{27} \times 3$$
$$y^2 = \frac{16}{9}$$
$$\sqrt{y^2} = \pm\sqrt{\frac{16}{9}} \Rightarrow \boxed{y = \pm\frac{4}{3}}$$

(viii) If u and v varies invarsely, and u = 8, when v = 3, find v when u = 12.

Solution:

$\boxed{v \propto \frac{1}{u}}$ $\boxed{\begin{matrix} u=8 \\ v=3 \end{matrix}}$ $\boxed{\begin{matrix} v=? \\ u=12 \end{matrix}}$

$$v \propto \frac{1}{u}$$

$$v = \frac{k}{u} \text{.............(i)}$$

To find k,

Put u = 8 and v = 3 in equation (i)

$$3 = \frac{k}{8}$$

$$3 \times 8 = k$$

$$24 = k$$

$$\Rightarrow \boxed{k = 24}$$

To find y,

Put k = 24 and u = 12 in equation (i)

$$v = \frac{k}{u}$$

$$v = \frac{24}{12}$$

$$\boxed{v = 2}$$

(ix) Find a fourth proportional to 8, 7, 6.

Solution:

Let 4th proportional is x.

then 8, 7, 6, x

By Proportion

$$8 : 7 : : 6 : x$$

Product of Extremes = Product of Means:

$$8\,(x) = 7 \times 6$$

$$x = \frac{42}{8}$$

$$\boxed{x = \frac{21}{4}}$$

(x) Find a mean proportional to 16 and 49.

Solution:

Let mean proportional is m,

Then 16, m, 49

By proportion

$$16 : m : : m : 49$$

Product of Means = Product of Extremes:

$$m.\,m = 16 \times 49$$

$$m^2 = 4^2 \times 7^2$$

$$m^2 = (4 \times 7)^2$$

$$m^2 = (28)^2$$

Taking square root

$$\sqrt{m^2} = \pm\sqrt{(28)^2}$$

$$\boxed{m = \pm 28}$$

(xi) Find third proportional to 28 and 4.

Solution:

Let 3rd proportional is x

Then 28, 4, x

By Proportion

$$28 : 4 : : 4 : x$$

Product of Extremes = Product of Means.

$$28\,(x) = 4 \times 4$$

$$x = \frac{16}{28}$$

$$\boxed{x = \frac{4}{7}}$$

(xii) If $y \propto \frac{x^2}{z}$ and y = 28 when x = 7, z = 2, then find y.

Solution:

$\boxed{x \propto \frac{x^2}{z}}$ $\boxed{\begin{matrix} y=28 \\ x=7 \\ z=2 \end{matrix}}$ $\boxed{y=?}$

$$y \propto \frac{x^2}{z}$$

$$y = \frac{kx^2}{z} \text{..................(i)}$$

To find k,

Put y = 28, x = 7 and z = 2 in equation (i)

$$28 = \frac{k(7)^2}{2}$$

$$28 \times 2 = k\,(49)$$

$$\frac{56}{49} = k$$

$$k = \frac{\cancel{56}\,8}{\cancel{49}\,7}$$

$$\boxed{k = \frac{8}{7}}$$

To find y,

Put $k = \frac{8}{7}$ in equation (i)

$$y = k\frac{x^2}{z}$$

$$y = \frac{8}{7}\frac{x^2}{z}$$

(xiii) If $z \propto xy$ and z = 36, when x = 2, y = 3, then find z.

Solution:

$z \propto xy$ | $z = 36, x = 2, y = 3$ | $z = ?$

$$z \propto xy$$

$$z = kxy \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

put z = 36; x = 2 and y = 3 in equation (i)

$$36 = k(2)(3)$$

$$36 = k(6)$$

$$\frac{36}{6} = k$$

$$6 = k$$

$$\Rightarrow \boxed{k = 6}$$

To find z,

Put k = 6 in equation (i)

$$z = kxy$$

$$\boxed{z = 6xy}$$

(xiv) If $w \propto \frac{1}{v^2}$ and w = 2 when v = 3 then find w.

Solution:

$w \propto \frac{1}{v^2}$ | $w = 2, v = 3$ | $w = ?$

$$w \propto \frac{1}{v^2}$$

$$w = \frac{k}{v^2} \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find K,

Put w = 2 and v = 3 in equation (i)

$$2 = \frac{k}{(3)^2}$$

$$2 = \frac{k}{9}$$

$$2 \times 9 = k$$

$$18 = k$$

$$\Rightarrow \boxed{k = 18}$$

To find w,

Put k = 18 in equation (i)

$$w = \frac{k}{v^2}$$

$$w = \frac{18}{v^2}$$

Fill in the blanks

1. The simplest form of the ratio $\frac{(x+y)(x^2+xy+y^2)}{x^3-y^3}$ is_______.
2. In a ratio x: y; x is called _____.
3. In a ratio a: b , b is called_____.
4. In a proportion a : b:: x:y, a and y are called______.
5. In a proportion p : q :: m : n ; q and m are called________.
6. In proportion 7:4::p:8, p =____.
7. If 6: m:: 9: 12, then m = _______.
8. If x and y varies directly, the x = _____.
9. If v varies directly as u^3, then u^3 = _______.
10. If w varies inversely as p^2, then k = ______.
11. A third proportional of 12 and 4, is______.
12. The fourth proportional of 15, 6, 5 is _____.
13. The mean proportional of $4m^2n^4$ and p^6 is________.
14. The continued proportion of 4, m and 9 is_____.

ANSWER KEY

1	$\frac{x+y}{x-y}$	2	Antecedent	3	Consequent
4	Extremes	5	Means	6	P = 14
7	m = 8	8	ky	9	$\frac{v}{k}$
10	p^2w	11	$\frac{4}{3}$	12	2
13	$\pm 2mn^2p^3$	14	m = ± 6		

EXERCISE 4.1

Resolve into partial fractions.

Q.1 $\dfrac{7x-9}{(x+1)(x-3)}$

Solution: $\dfrac{7x-9}{(x+1)(x-3)}$

Let $\dfrac{7x-9}{(x+1)(x-3)} = \dfrac{A}{x+1} + \dfrac{B}{x-3}$ ...... (i)

Multiplying equation (i) by (x + 1) (x – 3)

$7x - 9 = A(x-3) + B(x+1)$ ...... (ii)

As equation (ii) is an identity which is true for all values of x.

Put $x - 3 = 0$ i.e $x = 3$ and

Put $x + 1 = 0$ i.e $x = -1$

Putting $x = 3$ and $x = -1$ in (ii) we get

For $x = 3$	For $x = -1$
$7(3)-9 = +B(3+1)$	$7(-1)-9 = A(-1-3)$
$21 - 9 = 4B$	$-7-9 = -4A$
$12 = 4B$	$-16 = -4A$
$\Rightarrow \boxed{B=3}$	$\Rightarrow \boxed{A=4}$

Putting the value of A and B in equation (i)

We get the required partial fractions as.

$$\frac{4}{x+1} + \frac{3}{x-3}$$

Thus $\dfrac{7x-9}{(x+1)(x-3)} = \dfrac{4}{x+1} + \dfrac{3}{x-3}$

Q.2 $\dfrac{x-11}{(x-4)(x+3)}$

Solution: $\dfrac{x-11}{(x-4)(x+3)}$

Let $\dfrac{x-11}{(x-4)(x+3)} = \dfrac{A}{x-4} + \dfrac{B}{x+3}$ .....(i)

Multiplying by (x–4)(x+3) on both sides, we get

$x - 11 = A(x+3) + B(x-4)$....... (ii)

As equation (ii) is an identity which is true for all value of x.

Putting $x + 3 = 0$ i.e $x = -3$

and $x - 4 = 0$ i.e $x = 4$ in (ii) we get

For $x = -3$	For $x = 4$
$-3-11 = B(-3-4)$	$4-11 = A(4+3)$
$-14 = -7B$	$-7 = 7A$
$\Rightarrow \boxed{B=2}$	$\Rightarrow \boxed{A=-1}$

Hence the required partial fractions are

$$\frac{x-11}{(x-4)(x+3)} = \frac{-1}{x-4} + \frac{2}{x+3}$$

Q.3 $\dfrac{3x-1}{x^2-1}$

Solution: $\dfrac{3x-1}{x^2-1}$

$$\frac{3x-1}{x^2-1} = \frac{3x-1}{(x-1)(x+1)}$$

Let $\dfrac{3x-1}{(x-1)(x+1)} = \dfrac{A}{x-1} + \dfrac{B}{x+1}$ ......(i)

Multiplying both sides by (x–1)(x+1), we get

$3x - 1 = A(x+1) + B(x-1)$ .... (ii)

As equation (ii) is an identity which is true for all values of x.

Let $x + 1 = 0$ i.e $x = -1$ and

$x - 1 = 0$ i.e $x = 1$

Putting $x = -1$ and $x = 1$ in (ii) We get

For $x = 1$	For $x = -1$
$3(1) - 1 = A(1+1)$	$3(-1) - 1 = B(-1-1)$
$3 - 1 = 2A$	$-3 - 1 = -2B$
$2 = 2A$	$-4 = -2B$
$\Rightarrow \boxed{A=1}$	$\Rightarrow \boxed{B=2}$

Hence the required partial fractions are

$$\frac{3x-1}{(x-1)(x+1)} = \frac{1}{x-1} + \frac{2}{x+1}$$

Q.4 $\dfrac{x-5}{x^2+2x-3}$

Solution: $\dfrac{x-5}{x^2+2x-3}$

$$\frac{x-5}{x^2+2x-3} = \frac{x-5}{x^2+3x-x-3}$$
$$= \frac{x-5}{x(x+3)-1(x+3)}$$
$$= \frac{x-5}{(x-1)(x+3)}$$
$$\frac{x-5}{(x-1)(x+3)} = \frac{A}{x-1}+\frac{B}{x+3} \;\ldots\ldots\text{(i)}$$

Multiplying both sides by (x–1) (x+3), we get

$$x-5 = A(x+3)+B(x-1) \ldots\ldots\text{(ii)}$$

As equation (ii) is an identity which is true for all values of x.

Let $x+3=0 \Rightarrow x=-3$

and $x-1=0 \Rightarrow x=1$

Putting x = – 3 and x=1 in equation (ii) we get

For x = –3	For x = 1
$-3-5 = +B(-3-1)$	$1-5 = A(1+3)$
$-8 = -4B$	$-4 = 4A$
$B = \frac{-8}{-4}$	$A = \frac{-4}{4}$
$\Rightarrow \boxed{B=2}$	$\Rightarrow \boxed{A=-1}$

Hence the required partial fractions are

$$\frac{x-5}{x^2+2x-3} = \frac{-1}{x-1}+\frac{2}{x+3}$$

Q.5 $\dfrac{3x+3}{(x-1)(x+2)}$

Solution: $\dfrac{3x+3}{(x-1)(x+2)}$

Let $\dfrac{3x+3}{(x-1)(x+2)} = \dfrac{A}{x-1}+\dfrac{B}{x+2} \;\ldots\ldots\text{(i)}$

Multiplying both sides by (x–1)(x+2), we get

$$3x+3 = A(x+2)+B(x-1)\ldots\ldots\text{(ii)}$$

As equation (ii) is an identity which is true for all values of x.

Let $x-1=0$ i.e $x = 1$

and $x+2=0$ i.e $x = -2$

Putting x =1and x = –2 in equation (ii) we get

For x = 1	For x = –2
$3(1)+3 = A(1+2)$	$3(-2)+3 = B(-2-1)$
$3+3 = 3A$	$-6+3 = -3B$
$6 = 3A$	$-3 = -3B$
$A = \frac{6}{3}$	$B = \frac{-3}{-3}$
$\Rightarrow \boxed{A=2}$	$\Rightarrow \boxed{B=1}$

Hence the required partial fractions are

$$\frac{3x+3}{(x-1)(x+2)} = \frac{2}{x-1}+\frac{1}{x+2}$$

Q.6 $\dfrac{7x-25}{(x-4)(x-3)}$

Solution: $\dfrac{7x-25}{(x-4)(x-3)}$

Let $\dfrac{7x-25}{(x-4)(x-3)} = \dfrac{A}{x-4}+\dfrac{B}{x-3}$

Multiplying both sides by(x–4) (x – 3), we get

$$7x-25 = A(x-3)+B(x-4)\ldots\ldots\text{(ii)}$$

As equation (ii) is an identity which is true for all values of x.

Let $x-3=0$ i.e $x = 3$

and $x-4=0$ i.e $x = 4$

Putting x = 3 and x = 4 in equation (ii) we get

For x = 3	For x = 4
$7(3)-25 = B(3-4)$	$7(4)-25 = A(4-3)$
$21-25 = -B$	$28-25 = 1A$
$-4 = -B$	$3 = A$
$\Rightarrow \boxed{B=4}$	$\Rightarrow \boxed{A=3}$

Hence the required partial fractions are

$$\frac{7x-25}{(x-4)(x-3)} = \frac{3}{x-4}+\frac{4}{x-3}$$

Q.7 $\dfrac{x^2+2x+1}{(x-2)(x+3)}$

Solution: $\dfrac{x^2+2x+1}{(x-2)(x+3)}$ is an improper fraction. First we resolve it into proper fraction.

By long division we get

$$x^2+x-6\overline{)x^2+2x+1}\quad \text{quotient } 1$$
$$\pm x^2 \pm x \mp 6$$
$$x+7$$

We have $\dfrac{x^2+2x+1}{x^2+x-6} = 1+\dfrac{x+7}{x^2+x-6}$

Let $\dfrac{x+7}{(x-2)(x+3)} = \dfrac{A}{x-2}+\dfrac{B}{x+3}$ ....(i)

Multiplying both sides by (x–2) (x +3), we get

$x + 7 = A\,(x+3) + B(x-2)$......... (ii)

As equation (ii) is an identity which is true for all values of x.

Let $x+3=0$ i.e $x = -3$

and $x-2=0$ i.e $x = 2$

Putting x = –3 and x = 2 in equation (ii) we get

For $x = -3$	For $x = 2$
$-3+7 = B(-3-2)$	$2+7 = A\,(2+3)$
$4 = -5B$	$9 = 5A$
$\Rightarrow \boxed{B = -\dfrac{4}{5}}$	$\Rightarrow \boxed{A = \dfrac{9}{5}}$

Hence the required partial fractions are

$$\frac{x^2+2x+1}{(x-2)(x+3)} = 1+\frac{9}{5(x-2)}-\frac{4}{5(x+3)}$$

Q.8 $\dfrac{6x^3+5x^2-7}{3x^2-2x-1}$

Solution: $\dfrac{6x^3+5x^2-7}{3x^2-2x-1}$ is an improper fraction.

First we resolve it into proper fraction.

$$3x^2-2x-1\overline{)6x^3+5x^2-7}\quad \text{quotient } 2x+3$$
$$\pm 6x^3 \mp 4x^2 \mp 2x$$
$$9x^2+2x-7$$
$$\pm 9x^2 \mp 6x \mp 3$$
$$8x-4$$

$$\frac{6x^3+5x^2-7}{3x^2-2x-1} = (2x+3)+\frac{8x-4}{(3x+1)(x-1)}$$

Now, Let $\dfrac{8x-4}{(3x+1)(x-1)} = \dfrac{A}{3x+1}+\dfrac{B}{x-1}$ ....(i)

Multiplying both sides by(3x+1) (x–1), we get

$8x-4 = A\,(x-1) + B(3x+1)$...... (ii)

As equation (ii) is an identity which is true for all values of x.

Let $x-1=0$ i.e $x = 1$

and $3x+1=0$ i.e $x = -\dfrac{1}{3}$

Putting $x = 1$ and $x = \dfrac{-1}{3}$ in equation (ii) we get

For $x = 1$	For $x = \dfrac{-1}{3}$
$8(1)-4 = B\,[3(1)+1]$	$8\left(\dfrac{-1}{3}\right)-4 = A\left(\dfrac{-1}{3}-1\right)$
$-4 = 4B$	$\dfrac{-8}{3}-4 = \dfrac{A(-1-3)}{3}$
$4 = 4B$	$\dfrac{-8-12}{3} = \dfrac{A(-4)}{3}$
$\Rightarrow 4B = 4$	$\dfrac{-20}{3} = \dfrac{-4}{3}A$
$B = \dfrac{4}{4}$	
$\Rightarrow \boxed{B = 1}$	$\Rightarrow \boxed{A = 5}$

Hence the required partial functions are

$$\frac{6x^3+5x^2-7}{3x^2-2x-1} = 2x+3+\frac{5}{3x+1}+\frac{1}{x-1}$$

EXERCISE 4.2

Resolve into partial fractions:

Q.1 $\dfrac{x^2-3x+1}{(x-1)^2(x-2)}$

Solution:

Let $\dfrac{x^2-3x+1}{(x-1)^2(x-2)}=\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x-2}$ ...(i)

Multiplying both sides by $(x-1)^2(x-2)$,we get

$x^2-3x+1= A(x-1)(x-2)+B(x-2)+C(x-1)^2$ ..(ii)

$x^2-3x+1= A(x^2-3x+2)+B(x-2)+C(x^2-2x+1)$

Putting $x - 1 = 0$ i.e $x = 1$ in (ii) we get

$(1)^2 - 3(1) + 1 = \ (1 - 2)$

$1 - 3 + 1 = B(-1)$

$-1 = -B$

$\Rightarrow B = 1$

Putting $x - 2 = 0$ i.e $x = 2$ in (ii) we get

$(2)^2 - 3(2) + 1 = C(2 - 1)^2$

$4 - 6 + 1 = C$

$-1 = C$

Equating the coefficient of x^2 in (ii) we get

$1=A+C$

$1 = A - 1$

$\Rightarrow$ $A = 1 + 1$

$\boxed{A = 2}$

Hence the required partial fractions are

$$\frac{x^2-3x+1}{(x-1)^2(x-2)} = \frac{2}{x-1}+\frac{1}{(x-1)^2}-\frac{1}{x-2}$$

Q.2 $\dfrac{x^2+7x+11}{(x+2)^2(x+3)}$

Solution:

Let $\dfrac{x^2+7x+11}{(x+2)^2(x+3)}=\dfrac{A}{x+2}+\dfrac{B}{(x+2)^2}+\dfrac{C}{x+3}$ ...(i)

Multiplying both sides by $(x+2)^2(x+3)$

$\Rightarrow x^2+7x+11=A(x+2)(x+3)+B(x+3)+C(x+2)^2$

$x^2+7x+11=A(x^2+5x+6)+B(x+3)+C(x^2+4x+4)$..(ii)

Putting $x + 2 = 0$ i.e $x = -2$ in (ii) we get

$(-2)^2+7(-2)+11=B(-2+3)$

$4 - 14 + 11 = B$

$\Rightarrow$ $\boxed{B = 1}$

Putting $x+3 = 0$ i.e $x= -3$ in (ii) we get

$(-3)^2+7(-3)+11=C(-3+2)^2$

$9-21+11=C(-1)^2$

$20-21=C(1)$

$-1=C \quad \Rightarrow \quad \boxed{C=-1}$

Equating coefficient of x^2 in (ii) we get

$A+C=1$

$A-1=1$

$A=1+1$

$\boxed{A=2}$

Hence the required partial fractions are:

$$\frac{x^2+7x+11}{(x+2)^2(x+3)}=\frac{2}{x+2}+\frac{1}{(x+2)^2}-\frac{1}{x+3}$$

Q.3 $\dfrac{9}{(x-1)(x+2)^2}$

Solution:

Let $\dfrac{9}{(x-1)(x+2)^2}=\dfrac{A}{x-1}+\dfrac{B}{x+2}+\dfrac{C}{(x+2)^2}$.... (i)

Multiplying both sides by $(x-1)(x+2)^2$, we get

$9=A(x+2)^2+B(x-1)(x+2)+C(x-1)$..(ii)

Putting $x-1=0$ i.e $x=1$ in (ii) we get

$9=A(1+2)^2$

$9=A(3)^2$

$9=9A$

$\Rightarrow \quad \boxed{A=1}$

Putting $x+2=0$ i.e $x=-2$ in (ii) we get

$9=C(-2-1)$

$9=-3C$

$\Rightarrow \quad \boxed{C=-3}$

Equating the coefficient of x^2 in (ii) we get

$A+B=0$

$B=-A$

$\boxed{B=-1}$

Hence the partial fractions are

$$\frac{9}{(x-1)(x+2)^2}=\frac{1}{x-1}-\frac{1}{x+2}-\frac{3}{(x+2)^2}$$

Q.4 $\dfrac{x^4+1}{x^2(x-1)}$

Solution: $\dfrac{x^4+1}{x^2(x-1)}=\dfrac{x^4+1}{x^3-x^2}$ is an improper fraction. First we resolve it into proper fraction.

$$\begin{array}{r|l} & x+1 \\ x^3-x^2 & x^4+1 \\ & \pm x^4 \mp x^3 \\ \hline & x^3+1 \\ & \pm x^3 \mp x^2 \\ \hline & x^2+1 \end{array}$$

$$\frac{x^4+1}{x^2(x-1)}=(x+1)+\frac{x^2+1}{x^2(x-1)} \text{ .... (i)}$$

Let $\dfrac{x^2+1}{x^2(x-1)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x-1}$ .... (ii)

Multiplying both sides by $x^2(x-1)$ we get

$x^2+1=A(x)(x-1)+B(x-1)+Cx^2$......(iii)

Putting $x=0$ in (iii) we get

$0+1=B(0-1)$

$1=-B$

$\Rightarrow \quad \boxed{B=-1}$

Putting $x-1=0$ i.e $x=1$ in (iii) we get

$(1)^2+1=C(1)^2$

$1+1=C(1)$

$2=C$

$\Rightarrow \quad \boxed{C=2}$

Equating the coefficient of x^2 in (iii) we get

$A+C=1$

$A+2=1$

$A=1-2$

$\Rightarrow \quad \boxed{A=-1}$

Putting the value of A, B and C in equation(ii)

Thus required partial fractions are

$$\frac{x^4+1}{x^2(x-1)}=(x+1)-\frac{1}{x}-\frac{1}{x^2}+\frac{2}{x-1}$$

Q.5 $\dfrac{7x+4}{(3x+2)(x+1)^2}$

Solution: $\dfrac{7x+4}{(3x+2)(x+1)^2}$

Let $\dfrac{7x+4}{(3x+2)(x+1)^2}=\dfrac{A}{3x+2}+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}$.....(i)

Multiplying both sides by $(3x+2)(x+1)^2$ we get

$7x+4=A(x+1)^2+B(3x+2)(x+1)+C(3x+2)$... (ii)

Putting $3x+2=0$ i.e $x=\dfrac{-2}{3}$ in (ii) we get

$$7\left(\frac{-2}{3}\right)+4 = A\left(\frac{-2}{3}+1\right)^2$$
$$\frac{-14}{3}+4 = A\left(\frac{-2+3}{3}\right)^2$$
$$\frac{-14+12}{3} = A\left(\frac{1}{3}\right)^2$$
$$\frac{-2}{3} = \frac{1}{9}A$$
$$-18 = 3A$$
$$A = \frac{-18}{3}$$
$$\Rightarrow \boxed{A=-6}$$

Putting $x+1=0$ i.e $x=-1$ in (ii) we get

$$7(-1)+4 = C(3(-1)+2)$$
$$-7+4 = -C$$
$$\Rightarrow -3 = -C$$
$$\Rightarrow \boxed{C=3}$$

Equating the coefficient of x^2 we get

$$A+3B=0$$
$$-6+3B=0$$
$$3B=6$$
$$B=\frac{6}{3} \Rightarrow \boxed{B=2}$$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{7x+4}{(3x+2)(x+1)^2} = \frac{-6}{3x+2}+\frac{2}{x+1}+\frac{3}{(x+1)^2}$$

Q.6 $\dfrac{1}{(x-1)^2(x+1)}$

Solution: $\dfrac{1}{(x-1)^2(x+1)}$

Let $\dfrac{1}{(x-1)^2(x+1)} = \dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x+1}$ .....(i)

Multiplying both sides by $(x-1)^2(x+1)$ we get

$1 = A(x-1)(x+1)+B(x+1)+C(x-1)^2$...(ii)

Putting $x-1=0$ i.e $x=1$ in (ii) we get

$$1 = B(1+1)$$
$$1=2B \Rightarrow \boxed{B=\frac{1}{2}}$$

Putting $x+1=0$ i.e $x=-1$ in (ii) we get

$$1 = C(-1-1)^2$$
$$1 = C(-2)^2$$
$$1 = 4C \Rightarrow \boxed{C=\frac{1}{4}}$$

Equating the coefficient of x^2 in (ii) we get

$$A+C=0$$
$$A=-C$$
$$A=-\left(\frac{1}{4}\right) \Rightarrow \boxed{A=\frac{-1}{4}}$$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{1}{(x-1)^2(x+1)} = \frac{-1}{4(x-1)}+\frac{1}{2(x-1)^2}+\frac{1}{4(x+1)}$$

Q.7 $\dfrac{3x^2+15x+16}{(x+2)^2}$

Solution: $\dfrac{3x^2+15x+16}{(x+2)^2} = \dfrac{3x^2+15x+16}{x^2+4x+4}$

The given fraction is improper fraction. First we resolve it into proper fraction.

By long division,

$$\begin{array}{r} 3 \\ x^2+4x+4\overline{)\,3x^2+15x+16} \\ \pm 3x^2 \pm 12x \pm 12 \\ \hline 3x+4 \end{array}$$

$$\frac{3x^2+15x+16}{(x+2)^2} = 3+\frac{3x+4}{x^2+4x+4} \quad ....(i)$$

Let $\dfrac{3x+4}{(x+2)^2} = \dfrac{A}{x+2}+\dfrac{B}{(x+2)^2}$ ....(ii)

Multiplying both sides by $(x+2)^2$ we get

$3x+4 = A(x+2)+B$....... (iii)

Putting $x+2=0$ i.e $x=-2$ in (iii) we get

$$3(-2)+4 = B$$
$$-6+4 = B$$
$$\Rightarrow \boxed{B=-2}$$

Equating the coefficient of 'x' we get

$$3 = A$$
$$\Rightarrow \boxed{A=3}$$

Putting the value of A and B in equation (ii) and using equation (i) we get required partial fractions.

$$\frac{3x^2+15x+16}{(x+2)^2} = 3+\frac{3}{x+2}-\frac{2}{(x+2)^2}$$

Q.8 $\frac{1}{(x^2-1)(x+1)}$

Solution: $\frac{1}{(x^2-1)(x+1)}=\frac{1}{(x-1)(x+1)(x+1)}$

$= \frac{1}{(x-1)(x+1)^2}$

Let $\frac{1}{(x-1)(x+1)^2}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$ ....(i)

Multiplying both sides by (x–1) $(x+1)^2$ we get

$1= A(x+1)^2 + B(x+1)(x-1)+C(x-1)$ .....(ii)

Putting $x-1=0$ i.e $x=1$ in (ii) we get

$1 = A(1+1)^2$

$1 = A(2)^2$

$1 = 4A$

$\Rightarrow$ $\boxed{A = \frac{1}{4}}$

Putting $x+1=0$ i.e $x=-1$ in (ii) we get

$1 = C(-1-1)$

$1 = -2C$

$\Rightarrow$ $\boxed{C = \frac{-1}{2}}$

Equating the coefficient of x^2 in equation (ii) we get $A+B=0$

$B = -A$

$B = -\left(\frac{1}{4}\right)$

$\boxed{B = -\frac{1}{4}}$

Putting the value of A and B in equation (ii) we get required partial fractions.

$$\frac{1}{(x-1)(x+1)^2}=\frac{1}{4(x-1)}-\frac{1}{4(x+1)}-\frac{1}{2(x+1)^2}$$

EXERCISE 4.3

Resolve into partial fractions.

Q.1 $\dfrac{3x-11}{(x+3)(x^2+1)}$

Solution: $\dfrac{3x-11}{(x+3)(x^2+1)}$

Let $\dfrac{3x-11}{(x+3)(x^2+1)} = \dfrac{A}{x+3} + \dfrac{Bx+C}{x^2+1}$ ... (i)

Multiplying both sides $(x+3)(x^2+1)$, we get

$3x-11 = A(x^2+1) + (Bx+C)(x+3)$... (ii)

$3x-11 = A(x^2+1) + Bx(x+3) + C(x+3)$... (iii)

Putting $x+3=0$ i.e $x=-3$ in (ii), we get

$$3(-3)-11 = A[(-3)^2+1]$$
$$-9-11 = A(9+1)$$
$$-20 = 10A$$
$$A = \frac{-20}{10}$$
$$\Rightarrow \boxed{A=-2}$$

Now equating the coefficients of x^2 and x we get from equation (iii)

$A+B = 0$
$-2+B = 0$
$B = 2$
$\Rightarrow \boxed{B=2}$

$3B+C = 3$
$3(2)+C = 3$
$6+C = 3$
$C = 3-6$
$\Rightarrow \boxed{C=-3}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{3x-11}{(x+3)(x^2+1)} = \frac{-2}{x+3} + \frac{2x-3}{x^2+1}$$

Q.2 $\dfrac{3x+7}{(x^2+1)(x+3)}$

Solution: $\dfrac{3x+7}{(x^2+1)(x+3)}$

Let $\dfrac{3x+7}{(x^2+1)(x+3)} = \dfrac{Ax+B}{x^2+1} + \dfrac{C}{x+3}$ ......(i)

Multiplying both sides by $(x^2+1)(x+3)$

$3x+7 = (Ax+B)(x+3) + C(x^2+1)$

$3x+7 = Ax(x+3) + B(x+3) + C(x^2+1)$ .....(ii)

Putting $x+3 = 0$ i.e $x = -3$ in (ii), we get

$$3(-3)+7 = C[(-3)^2+1]$$
$$-9+7 = C(9+1)$$
$$-2 = 10C$$
$$\Rightarrow C = \frac{-2}{10}$$
$$\boxed{C = \frac{-1}{5}}$$

Now equating the coefficients of x^2 and x in equation (iii) we get

$A+C = 0$
$A + \left(\dfrac{-1}{5}\right) = 0$
$A - \dfrac{1}{5} = 0$
$\Rightarrow \boxed{A = \dfrac{1}{5}}$

$3A+B = 3$
$3\left(\dfrac{1}{5}\right)+B = 3$
$B = 3 - \dfrac{3}{5}$
$B = \dfrac{15-3}{5}$
$\Rightarrow \boxed{B = \dfrac{12}{5}}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{3x+7}{(x^2+1)(x+3)} = \frac{x+12}{5(x^2+1)} - \frac{1}{5(x+3)}$$

Q.3 $\dfrac{1}{(x+1)(x^2+1)}$

Solution: $\dfrac{1}{(x+1)(x^2+1)}$

Let $\dfrac{1}{(x+1)(x^2+1)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+1}$ ....(i)

Multiplying both sides by (x+1) (x^2+1),we get

$1 = A(x^2+1) + (Bx+C)(x+1)$

$1 = A(x^2+1) + Bx(x+1) + C(x+1)$... (ii)

Putting $x+1=0$ i.e $x=-1$ in (ii), we get

$1 = A[(-1)^2+1]$

$1 = A(1+1)$

$1 = 2A$

$\boxed{A = \dfrac{1}{2}}$

Equating the coefficients of x^2 and x in equation (ii) we get

$A + B = 0$

$\dfrac{1}{2} + B = 0$

$\Rightarrow \boxed{B = -\dfrac{1}{2}}$

$B + C = 0$

$-\dfrac{1}{2} + C = 0$

$\Rightarrow \boxed{C = \dfrac{1}{2}}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{1}{(x+1)(x^2+1)} = \frac{1}{2(x+1)} - \frac{x-1}{2(1+x^2)}$$

Q.4 $\dfrac{9x-7}{(x+3)(x^2+1)}$

Solution: $\dfrac{9x-7}{(x+3)(x^2+1)}$

Let $\dfrac{9x-7}{(x+3)(x^2+1)} = \dfrac{A}{x+3} + \dfrac{Bx+C}{x^2+1}$ .....(i)

Multiplying both sides by (x+3)(x^2+1) we get

$9x-7 = A(x^2+1) + (Bx+C)(x+3)$

$9x-7 = A(x^2+1) + Bx(x+3) + C(x+3)$.... (ii)

Putting $x+3=0$ i.e $x=-3$ in (ii), we get

$9(-3) - 7 = A[(-3)^2+1]$

$-27 - 7 = A(9+1)$

$-34 = 10A$

$\Rightarrow A = \dfrac{-34}{10} \Rightarrow \boxed{A = \dfrac{-17}{5}}$

Equating coefficients of x^2 and x in equation (ii) we get

$A + B = 0$

$\dfrac{-17}{5} + B = 0$

$\Rightarrow \boxed{B = \dfrac{17}{5}}$

$3B + C = 9$

$3\left(\dfrac{17}{5}\right) + C = 9$

$\dfrac{51}{5} + C = 9$

$C = 9 - \dfrac{51}{5}$

$C = \dfrac{45-51}{5}$

$\Rightarrow \boxed{C = \dfrac{-6}{5}}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{9x-7}{(x+3)(x^2+1)} = \frac{-17}{5(x+3)} + \frac{17x-6}{5(x^2+1)}$$

Q.5 $\dfrac{3x+7}{(x+3)(x^2+4)}$

Solution: $\dfrac{3x+7}{(x+3)(x^2+4)}$

Let $\dfrac{3x+7}{(x+3)(x^2+4)} = \dfrac{A}{x+3} + \dfrac{Bx+C}{x^2+4}$ … (i)

Multiplying both sides by $(x+3)(x^2+4)$ we get

$3x + 7 = A(x^2+4) + (Bx+C)(x+3)$

$3x + 7 = A(x^2+4) + Bx(x+3) + C(x+3)$ …(ii)

Putting $x + 3 = 0$ i.e $x = -3$ in (ii) we get

$3(-3) + 7 = A((-3)^2 + 4)$

$-9 + 7 = A(9+4)$

$-2 = 13A$

$\Rightarrow \boxed{A = \dfrac{-2}{13}}$

Equating the coefficients of x^2 and x in equation (ii) we get

$A + B = 0$

$\dfrac{-2}{13} + B = 0$

$B = \dfrac{2}{13}$

$\Rightarrow \boxed{B = \dfrac{2}{13}}$

$3B + C = 3$

$3\left(\dfrac{2}{13}\right) + C = 3$

$\dfrac{6}{13} + C = 3$

$C = 3 - \dfrac{6}{13}$

$C = \dfrac{39-6}{13}$

$\Rightarrow \boxed{C = \dfrac{33}{13}}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{3x+7}{(x+3)(x^2+4)} = \frac{-2}{13(x+3)} + \frac{2x+33}{13(x^2+4)}$$

Q.6 $\dfrac{x^2}{(x+2)(x^2+4)}$

Solution: $\dfrac{x^2}{(x+2)(x^2+4)}$

Let $\dfrac{x^2}{(x+2)(x^2+4)} = \dfrac{A}{x+2} + \dfrac{Bx+C}{x^2+4}$ ….(i)

Multiplying both sides by $(x+2)(x^2+4)$ we get

$x^2 = A(x^2+4) + (Bx+C)(x+2)$

$x^2 = A(x^2+4) + Bx(x+2) + C(x+2)$…. (ii)

Putting $x + 2 = 0$ i.e $x = -2$ in (ii) we get

$(-2)^2 = A[(-2)^2 + 4]$

$4 = A(4+4)$

$4 = 8A$

$\Rightarrow A = \dfrac{4}{8}$

$\boxed{A = \dfrac{1}{2}}$

Equating the coefficients of x^2 and x in equation (ii) we get

$A + B = 1$

$\dfrac{1}{2} + B = 1$

$B = 1 - \dfrac{1}{2}$

$\Rightarrow \boxed{B = \dfrac{1}{2}}$

$2B + C = 0$

$\cancel{2}\left(\dfrac{1}{\cancel{2}}\right) + C = 0$

$1 + C = 0$

$\Rightarrow \boxed{C = -1}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{1}{2(x+2)} + \frac{x-2}{2(x^2+4)}$$

Q.7 $\frac{1}{x^3+1}$

Solution: $\frac{1}{x^3+1}$

$$\frac{1}{x^3+1} = \frac{1}{(x+1)(x^2-x+1)}$$

Let $\frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$ .....(i)

Multiplying both sides by $(x+1)(x^2-x+1)$, we get

$1 = A(x^2-x+1) + (Bx+C)(x+1)$

$1 = A(x^2-x+1) + Bx(x+1) + C(x+1)$...(ii)

Putting $x+1=0$ i.e $x=-1$ in (ii) we get

$1 = A\,[(-1)^2-(-1)+1]$

$1 = A(1+1+1)$

$1 = 3A$

$\Rightarrow$ $\boxed{A = \frac{1}{3}}$

Comparing the coefficients of x^2 and x in equation (ii) we get

$A+B=0$	$-A+B+C=0$
$\frac{1}{3}+B=0$	$-\frac{1}{3}-\frac{1}{3}+C=0$
$\Rightarrow B=\frac{-1}{3}$	$\frac{-2}{3}+C=0$
$\boxed{B=\frac{-1}{3}}$	$\Rightarrow \boxed{C=\frac{2}{3}}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{1}{3(x+1)} - \frac{x-2}{3(x^2-x+1)}$$

Q.8 $\frac{x^2+1}{x^3+1}$

Solution: $\frac{x^2+1}{x^3+1}$

$$\frac{x^2+1}{x^3+1} = \frac{x^2+1}{(x+1)(x^2-x+1)}$$

Let $\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$ .....(i)

Multiplying both sides by $(x+1)(x^2-x+1)$, we get

$x^2+1 = A(x^2-x+1) + (Bx+C)(x+1)$

$x^2+1 = A(x^2-x+1) + Bx(x+1) + C(x+1)$...(ii)

Putting $x+1=0$ i.e $x=-1$ in (ii) we get

$(-1)^2+1 = A\,[(-1)^2-(-1)+1]$

$1+1 = A(1+1+1)$

$2 = 3A$

$\Rightarrow$ $\boxed{A = \frac{2}{3}}$

Equating the coefficients of x^2 and x in equation (ii) we get

$A+B=1$	$-A+B+C=0$
$\frac{2}{3}+B=1$	$\frac{-2}{3}+\frac{1}{3}+C=0$
$B=1-\frac{2}{3}$	$\frac{-1}{3}+C=0$
$\Rightarrow \boxed{B=\frac{1}{3}}$	$\Rightarrow \boxed{C=\frac{1}{3}}$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{x^2+1}{x^3+1} = \frac{2}{3(x+1)} + \frac{x+1}{3(x^2-x+1)}$$

EXERCISE 4.4

Q.1 $\dfrac{x^3}{(x^2+4)^2}$

Solution: $\dfrac{x^3}{(x^2+4)^2}$

Let $\dfrac{x^3}{(x^2+4)^2} = \dfrac{Ax+B}{x^2+4} + \dfrac{Cx+D}{(x^2+4)^2}$ ……. (i)

Multiplying both sides by $(x^2 + 4)^2$, we get

$x^3 = (Ax+B)(x^2+4) + (Cx+D)$

$x^3 = Ax(x^2+4) + B(x^2+4) + (Cx+D)$ ……..(ii)

Equating the coefficients of x^3, x^2, x and constants, we get

Coefficients of x^3: $A = 1$

Coefficients of x^2: $B = 0$

Coefficients of x: $4A + C = 0$

$4(1) + C = 0$

$\Rightarrow C = -4$

Constants: $4B + D = 0$

$4(0) + D = 0$

$\Rightarrow D = 0$

Putting the value of A,B,C and D in equation(i) we get required partial fractions.

$$\frac{x^3}{(x^2+4)^2} = \frac{x}{x^2+4} - \frac{4x}{(x^2+4)^2}$$

Q.2 $\dfrac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}$

Solution: $\dfrac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}$

Let $\dfrac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+1} + \dfrac{Dx+E}{(x^2+1)^2}$ …(i)

Multiplying both sides by $(x+1)(x^2+1)^2$ we get

$x^4+3x^2+x+1 = A(x^2+1)^2 + (Bx+C)(x+1)(x^2+1)$

$+(Dx+E)(x+1).....(ii)$

$x^4 + 3x^2 + x + 1 = A(x^4+2x^2+1)+Bx(x^3+x^2+x+1)$
$+C(x^3+x^2+x+1)+Dx(x+1) + E(x+1)$

$x^4 + 3x^2 + x + 1 = A(x^4+2x^2+1)+B(x^4+x^3+x^2+x)$
$+C(x^3+x^2+x+1)+D(x^2+x)+E(x+1)..$ (iii)

Putting $x+1=0$ i.e $x=-1$ in eq.(ii), we get

$(-1)^4 + 3(-1)^2 + (-1) + 1 = A[(-1)^2+1]^2$

$1 + 3(1) - 1 + 1 = A(1+1)^2$

$4 = 4A$

$\Rightarrow \boxed{A = 1}$

Now equating the coefficients of x^4, x^3, x^2, x and constants, we get from equation (iii)

Coefficients of x^4: $A + B = 1$

$1 + B = 1$

$B = 1 - 1$

$\Rightarrow \boxed{B = 0}$

Coefficients of x^3: $B + C = 0$

$0 + C = 0$

$\Rightarrow \boxed{C = 0}$

Coefficients of x^2: $2A + B + C + D = 3$

$2(1) + 0 + 0 + D = 3$

$D = 3 - 2$

$\boxed{D = 1}$

Coefficients of x: $B + C + D + E = 1$

$0 + 0 + 1 + E = 1$

$E = 1 - 1$

$\Rightarrow \boxed{E = 0}$

Putting the value of A, B, C and D in equation(i) we get required partial fractions.

$$\frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2} = \frac{1}{x+1} + \frac{x}{(x^2+1)^2}$$

Q.3 $\dfrac{x^2}{(x+1)(x^2+1)^2}$

Solution: $\dfrac{x^2}{(x+1)(x^2+1)^2}$

Let $\dfrac{x^2}{(x+1)(x^2+1)^2} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+1} + \dfrac{Dx+E}{(x^2+1)^2}$ .. (i)

Multiply both sides by $(x+1)(x^2+1)^2$ we get

$x^2 = A(x^2+1)^2 + (Bx+C)(x+1)(x^2+1)$
$+ (Dx+E)(x+1)$.......(ii)

$x^2 = A(x^4+2x^2+1)+Bx(x^3+x^2+x+1)$
$+C(x^3+x^2+x+1)+Dx(x+1)+E(x+1)$

$x^2 = A(x^4+2x^2+1)+B(x^4+x^3+x^2+x)$
$+C(x^3+x^2+x+1)+D(x^2+x)+E(x+1)$...(iii)

Putting $x+1=0$ i.e $x=-1$ in equation (ii) we get

$(-1)^2 = A[(-1)^2+1]^2$

$1 = A(1+1)^2$

$1 = 4A \Rightarrow \boxed{A = \frac{1}{4}}$

Now equating the coefficients of x^4, x^3, x^2, x and constants we get from equation (iii)

Coefficients of x^4: $A + B = 0$

$\frac{1}{4} + B = 0 \Rightarrow \boxed{B = -\frac{1}{4}}$

Coefficients of x^3 : $B + C = 0$

$-\frac{1}{4} + C = 0 \Rightarrow \boxed{C = \frac{1}{4}}$

Coefficients of x^2: $2A + B + C + D = 1$

$2\left(\frac{1}{4}\right) - \frac{1}{4} + \frac{1}{4} + D = 1$

$\frac{1}{2} + D = 1$

$D = 1 - \frac{1}{2}$

$$\boxed{D = \frac{1}{2}}$$

Coefficients of x: $B + C + D + E = 0$

$$-\frac{1}{4}+\frac{1}{4}+\frac{1}{2}+E = 0$$

$$\frac{1}{2}+E = 0$$

$$\boxed{E = -\frac{1}{2}}$$

Putting the value of A, B, C, D and E in equation(i) we get required partial fractions.

$$\frac{x^2}{(x+1)(x^2+1)^2} = \frac{1}{4(x+1)} - \frac{x-1}{4(x^2+1)} + \frac{x-1}{2(x^2+1)^2}$$

Q.4 $\dfrac{x^2}{(x-1)(x^2+1)^2}$

Solution: $\dfrac{x^2}{(x-1)(x^2+1)^2}$

Let $\dfrac{x^2}{(x-1)(x^2+1)^2} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+1} + \dfrac{Dx+E}{(x^2+1)^2}$ ..(i)

Multiplying both sides by $(x-1)(x^2+1)^2$, we get

$$x^2 = A(x^2+1)^2 + (Bx+C)(x-1)(x^2+1) + (Dx+E)(x-1) \quad \text{....(ii)}$$

$$x^2 = A(x^4+2x^2+1) + Bx(x-1)(x^2+1) + C(x-1)(x^2+1) + Dx(x-1) + E(x-1)$$

$$x^2 = A(x^4+2x^2+1) + B(x^4-x^3+x^2-x) + C(x^3-x^2+x-1) + D(x^2-x) + E(x-1) \quad \text{... (iii)}$$

Putting $x - 1 = 0$ i.e $x = 1$ in equation (ii) we get

$$(1)^2 = A[(1)^2+1]^2$$

$$1 = A(1+1)^2$$

$$1 = 4A \Rightarrow \boxed{A = \frac{1}{4}}$$

Now equating the coefficients of x^4, x^3, x^2 and x in equation (iii) we get

Coefficients of x^4: $A + B = 0$

$$\frac{1}{4}+B = 0$$

$$\Rightarrow \boxed{B = -\frac{1}{4}}$$

Coefficients of x^3: $-B + C = 0$

$$-\left(-\frac{1}{4}\right)+C = 0$$

$$\Rightarrow C = -\frac{1}{4}$$

Coefficients of x^2: $2A + B - C + D = 1$

$$2\left(\frac{1}{4}\right) - \frac{1}{4} - \left(\frac{-1}{4}\right) + D = 1$$

$$\frac{1}{2} - \frac{1}{4} + \frac{1}{4} + D = 1$$

$$D = 1 - \frac{1}{2}$$

$$D = \frac{2-1}{2}$$

$$\boxed{D = \frac{1}{2}}$$

Coefficients of x: $-B + C - D + E = 0$

$$-\left(-\frac{1}{4}\right) - \frac{1}{4} - \frac{1}{2} + E = 0$$

$$\frac{1}{4} - \frac{1}{4} - \frac{1}{2} + E = 0$$

$$\frac{-1}{2} + E = 0$$

$$\boxed{E = \frac{1}{2}}$$

Putting the value of A, B, C, D and E in equation(i) we get required partial fractions.

$$\frac{x^2}{(x-1)(x^2+1)^2} = \frac{1}{4(x-1)} - \frac{x+1}{4(x^2+1)} + \frac{x+1}{2(x^2+1)^2}$$

Q.5 $\frac{x^4}{(x^2+2)^2}$

Solution: $\frac{x^4}{(x^2+2)^2}$

$\frac{x^4}{(x^2+2)^2} = \frac{x^4}{x^4+4x^2+4}$ is an improper fraction. First we resolve it into proper fraction.

$$\begin{array}{r|l} & 1 \\ x^4+4x^2+4 & x^4 \\ & \pm x^4 \pm 4x^2 \pm 4 \\ \hline & -4x^2-4 \end{array}$$

$$\frac{x^4}{(x^2+2)^2} = 1 + \frac{-4x^2-4}{(x^2+2)^2}$$

Let $\frac{-4x^2-4}{(x^2+2)^2} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2}$ .....(i)

Multiplying both sides by $(x^2+2)^2$ we get

$-4x^2-4 = (Ax+B)(x^2+2) + (Cx+D)$

$-4x^2-4 = A(x^3+2x) + B(x^2+2) + Cx + D$......(ii)

Equating the coefficients of x^3, x^2, x and constants in equation (ii) we get

Coefficients of x^3: $A = 0$

Coefficients of x^2: $B = -4$

Coefficients of x: $2A + C = 0$

$2(0) + C = 0$

$\Rightarrow \boxed{C = 0}$

Constants : $2B + D = -4$

$2(-4) + D = -4$

$-8 + D = -4$

$D = 8 - 4$

$\boxed{D = 4}$

Putting the value of A, B, C and D in equation(i) we get required partial fractions.

$$\frac{x^4}{(x^2+2)^2} = 1 + \frac{-4}{x^2+2} + \frac{4}{(x^2+2)^2}$$

$$\frac{x^4}{(x^2+2)^2} = 1 - \frac{4}{x^2+2} + \frac{4}{(x^2+2)^2}$$

Q.6 $\frac{x^5}{(x^2+1)^2}$

Solution: $\frac{x^5}{(x^2+1)^2}$

$\frac{x^5}{(x^2+1)^2} = \frac{x^5}{x^4+2x^2+1}$ is an improper fraction.

First we resolve it into proper fraction

$$\begin{array}{r|l} & x \\ x^4+2x^2+1 & \cancel{x^5} \\ & \pm \cancel{x^5} \pm 2x^3 \pm x \\ \hline & -2x^3 - x \end{array}$$

$$\frac{x^5}{(x^2+1)^2} = x + \frac{-2x^3-x}{(x^2+1)^2}$$

Let $\frac{-2x^3-x}{(x^2+1)^2} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}$ ....(i)

Multiplying both sides by $(x^2+1)^2$ we get

$-2x^3 - x = (Ax+B)(x^2+1) + (Cx+D)$

$-2x^3-x = A(x^3+x) + B(x^2+1) + Cx + D$

Equating the coefficients of x^3, x^2, x and constants we get

Coefficients of x^3 : $A = -2$

Coefficients of x^2 : $B = 0$

Coefficients of x : $A + C = -1$

$-2 + C = -1$

$C = -1 + 2$

$\Rightarrow \boxed{C = 1}$

Constants : $B + D = 0$

$0 + D = 0$

$\Rightarrow D = 0$

Hence the required partial fractions are

$$\frac{x^5}{(x^2+1)^2} = x + \frac{-2x}{x^2+1} + \frac{x}{(x^2+1)^2}$$

$$\Rightarrow \frac{x^5}{(x^2+1)^2} = x - \frac{2x}{x^2+1} + \frac{x}{(x^2+1)^2}$$

MISCELLANEOUS EXERCISE – 4

Q. 1 Multiple Choice Questions:

Four possible answers are given for the following questions. Tick (✓) the correct answer.

1. $(x+3)^2 = x^2 + 6x + 9$ is
   (a) a linear equation
   (b) an equation
   (c) an identity
   (d) none of these

2. $\dfrac{2x+1}{(x+1)(x-1)}$ is
   (a) an improper fraction
   (b) an equation
   (c) a proper fraction
   (d) none of these

3. $\dfrac{x^3+1}{(x-1)(x+2)}$ is
   (a) a proper fraction
   (b) an improper fraction
   (c) an identity
   (d) a constant term

4. **A fraction in which the degree of numerator is less than the degree of the denominator is called**
   (a) an equation
   (b) an improper fraction
   (c) an identity
   (d) a proper fraction

5. **A function of the form $f(x) = \dfrac{N(x)}{D(x)}$, with $D(x) \neq 0$, where N(x) and D(x) are polynomials in x is called**
   (a) an identity
   (b) an equation
   (c) a fraction
   (d) none of these

6. **The identity $(5x+4)^2 = 25x^2 + 40x + 16$ is true for**
   (a) one value of x
   (b) two values of x
   (c) all values of x
   (d) none of these

7. **A fraction in which the degree of the numerator is greater or equal to the degree of denominator is called**
   (a) a proper fraction
   (b) an improper fraction
   (c) an equation
   (d) algebraic relation

8. **Partial fractions of $\dfrac{x-2}{(x-1)(x+2)}$ are of the form**
   (a) $\dfrac{A}{x-1}+\dfrac{B}{x+2}$ (b) $\dfrac{Ax}{x-1}+\dfrac{B}{x+2}$
   (c) $\dfrac{A}{x-1}+\dfrac{Bx+C}{x+2}$ (d) $\dfrac{Ax+B}{x-1}+\dfrac{C}{x+2}$

9. **Partial fractions of $\dfrac{x+2}{(x+1)(x^2+2)}$ are of the form**
   (a) $\dfrac{A}{x+1}+\dfrac{B}{x^2+2}$
   (b) $\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+2}$
   (c) $\dfrac{Ax+B}{x+1}+\dfrac{C}{x^2+2}$
   (d) $\dfrac{A}{x+1}+\dfrac{Bx}{x^2+2}$

10. **Partial fractions of $\dfrac{x^2+1}{(x+1)(x-1)}$ are of the form**
   (a) $\dfrac{A}{x+1}+\dfrac{B}{x-1}$
   (b) $1+\dfrac{A}{x+1}+\dfrac{Bx+C}{x-1}$
   (c) $1+\dfrac{A}{x+1}+\dfrac{B}{x-1}$
   (d) $\dfrac{Ax+B}{(x+1)}+\dfrac{C}{x-1}$

ANSWER KEY

1.	c	2.	c	3.	b	4.	d	5.	c
6.	c	7.	b	8.	a	9.	b	10.	c

Q. 2 Write short answers of the following questions:

(i) Define a rational fraction.

An expression of the form $\frac{N(x)}{D(x)}$ with $D(x) \neq 0$ and N(x) and D(x) are polynomials in x with real coefficients, is called a rational fraction. Every fractional expression can be expressed as a quotient of two polynomials.

(ii) What is a proper fraction?

A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called a proper fraction if degree of the polynomial N(x) in the numerator is less than the degree of the polynomial D(x) in the denominator.

(iii) What is an improper fraction?

A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called an improper fraction if degree of the polynomial N(x) is greater or equal to the degree of the polynomial D(x) e.g $\frac{x^2+1}{x-1}$

(iv) What are partial fractions?

Every proper fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ can be resolved into an algebraic sum of components fractions. These components fractions of a resultant fraction are called its partial fractions.

(v) How can we make partial fractions of $\frac{x-2}{(x+2)(x+3)}$?

Solution: $\frac{x-2}{(x+2)(x+3)}$

Let $\frac{x-2}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$ ......(i)

Multiplying both sides by (x+2)(x+3), we get

$x-2 = A(x+3) + B(x+2)$ .... (ii)

As both sides of the identity are equal for all values of x,

Put x + 2 = 0 i.e x = − 2 in equation (ii), we get

$-2-2 = A(-2+3)$

$-4 = A$

$\Rightarrow \boxed{A = -4}$

Now put x+3=0 i.e x= −3 in equation (ii) we get

$-3-2 = B(-3+2)$

$-5 = -B$

$\Rightarrow \boxed{B = 5}$

Putting the value of A and B in equation(i) we get required partial fractions.

$$\frac{x-2}{(x+2)(x+3)} = \frac{-4}{x+2} + \frac{5}{x+3}$$

(vi) Resolve $\frac{1}{x^2-1}$ into partial fractions.

Solution: $\frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)}$

Let $\frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$ .... (i)

Multiplying both sides By (x − 1) (x + 1), we get

$1 = A(x+1) + B(x-1)$ ....(ii)

As both sides of identity are equal for all values of x

Putting x− 1=0 i.e x = 1in equation (ii) we get

$1 = A(1+1)$

$1 = 2A$

$\Rightarrow \boxed{A = \frac{1}{2}}$

Putting x+1 = 0 i.e x = − 1 in equation (ii) we get

$1 = B(-1-1)$

$1 = -2B$

$\Rightarrow \boxed{B = -\frac{1}{2}}$

Putting the value of A and B in equation(i) we get required partial fractions.

$$\frac{1}{x^2-1} = \frac{1}{2(x-1)} - \frac{1}{2(x+1)}$$

(vii) . Find partial fractions of $\frac{3}{(x+1)(x-1)}$

Solution: $\frac{3}{(x+1)(x-1)}$

Let $\frac{3}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$ ....(i)

Multiplying both sides by (x+1) (x – 1), we get

$3 = A(x-1) + B(x+1)$.............(ii)

As both sides of the identity are equal for all values of x.

Put x + 1= 0 i.e x = –1put in equation (ii) we get

$3 = A(-1-1)$

$3 = -2A \Rightarrow \boxed{A = \frac{-3}{2}}$

Now put x – 1 = 0 i.e x = 1 in equation (ii) we get

$\Rightarrow 3 = B(1+1)$

$3 = 2B \Rightarrow \boxed{B = \frac{3}{2}}$

Putting the value of A and B in equation(i) we get required partial fractions.

$$\frac{3}{(x+1)(x-1)} = \frac{-3}{2(x+1)} + \frac{3}{2(x-1)} = \frac{3}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

(viii) Resolve $\frac{x}{(x-3)^2}$ into partial fractions.

Solution: $\frac{x}{(x-3)^2}$

Let $\frac{x}{(x-3)^2} = \frac{A}{x-3} + \frac{B}{(x-3)^2}$ ....(i)

Multiplying both sides by $(x-3)^2$, we get

$x = A(x-3) + B$ ....(ii)

As both sides of the identity are equal for all values of x,

Put x – 3 = 0 i.e x =3 in equation (ii) we get

$3 = B$

$\Rightarrow \boxed{B = 3}$

Now comparing the coefficients of x, we have

$\Rightarrow \boxed{A = 1}$

Putting the value of A and B in equation(i) we get required partial fractions.

$$\frac{x}{(x-3)^2} = \frac{1}{x-3} + \frac{3}{(x-3)^2}$$

(ix) How we can make the partial fractions of $\frac{x}{(x+a)(x-a)}$?

Solution: $\frac{x}{(x+a)(x-a)}$

Let $\frac{x}{(x+a)(x-a)} = \frac{A}{x+a} + \frac{B}{x-a}$ ....(i)

Multiplying both sides by (x + a) (x – a), we get

$x = A(x-a) + B(x+a)$ .... (ii)

As both sides of the identity are equal for all values of x,

Put x + a = 0 i.e x = –a put in equation (ii) we get

$-a = A(-a-a)$

$-a = -2aA$

$\Rightarrow A = \frac{-a}{-2a}$

$\Rightarrow \boxed{A = \frac{1}{2}}$

Now put x – a = 0 i.e x = a in equation (ii) we get

$a = B(a+a)$

$a = 2aB$

$\Rightarrow B = \frac{a}{2a}$

$\Rightarrow \boxed{B = \frac{1}{2}}$

Putting the value of A and B in equation(i) we get required partial fractions.

$$\frac{x}{(x+a)(x-a)} = \frac{1}{2(x+a)} + \frac{1}{2(x-a)}$$

$$= \frac{1}{2}\left(\frac{1}{x+a} + \frac{1}{x-a}\right)$$

(x) Whether $(x+3)^2 = x^2 + 6x + 9$ is an identity?

Answer:

Yes $(x+3)^2 = x^2 + 6x + 9$ is an identity because it is true for all the values of x.

EXERCISE 5.2

Q.1 If X = {1, 3, 5, 7, …., 19}
Y = {0, 2, 4, 6, 8, …, 20}
Z = {2, 3, 5, 7, 11, 13, 17, 19, 23}, then find the following:

(i) $X\cup(Y\cup Z)$ (ii) $(X\cup Y)\cup Z$
(iii) $X\cap(Y\cap Z)$ (iv) $(X\cap Y)\cap Z$
(v) $X\cup\cap Y\cap Z)$ (vi) $(X\cup Y)\cap(X\cup Z)$
(vii) $X\cap(Y\cup Z)$ (viii) $(X\cap Y)\cup(X\cap Z)$

Solution:

(i) $X\cup(Y\cup Z)$

$= X \cup (\{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$

$= \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\} \cup \{0,2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 23\}$

$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,13, 14,15,16,17,18,19,20,23\}$

(ii) $(X\cup Y)\cup Z$

$= (\{1,3,5,7,\ldots,19\}\cup\{0,2,4,6,8,\ldots,20\})\cup Z$

$= \{0, 1, 2, 3, 4, 5, 6, 7, \ldots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

$= \{0, 1, 2, 3, 4, 5, 6, 7, \ldots, 20, 23\}$

(iii) $X\cap (Y\cap Z)$

$= X \cap (\{0,2,4,6, 8, \ldots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$

$= \{1, 3, 5, 7, \ldots, 19\} \cap \{2\}$

$= \phi$

(iv) $(X\cap Y)\cap Z$

$= (\{1,3,5,7, \ldots,19\}\cap\{0,2,4,6, 8,\ldots,20\}) \cap Z$

$= \{\ \} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

$= \phi$

(v) $X\cup(Y\cap Z)$

$= X \cup (\{0,2,4,6,8, \ldots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$

$= \{1, 3, 5, 7, \ldots, 19\} \cup \{2\}$

$= \{1, 2, 3 , 5, 7, \ldots, 19\}$

(vi) $(X\cup Y)\cap(X\cup Z)$

$X\cup Y = \{1,3,5, 7,\ldots, 19\}\cup\{0,2,4,6,8, \ldots, 20\}$

$= \{0, 1, 2, 3, 4, 5, \ldots, 20\}$

$X\cup Z = \{1,3,5,7, \ldots, 19\}\cup\{2,3,5,7,11,13,17, 19, 23\}$

$= \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 23\}$

$(X\cup Y)\cap(X\cup Z)$

$= \{0,1,2,3,4, \ldots, 20\}\cap\{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19,23\}$

$= \{1,2, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

(vii) $X\cap(Y\cup Z)$

$X\cap(Y\cup Z)$

$= X\cap(\{0,2,4,6,8, \ldots,20\}\cup\{2,3,5,7,11,13, 17,19,23\})$

$= \{1,3,5,7, \ldots, 19\}\cap\{0,2,3,4,5,6,7,8, 10,11, 12,13,14,16,17,18,19,20,23\}$

$= \{3, 5, 7, 11, 13, 17, 19\}$

(viii) $(X\cap Y)\cup(X\cap Z)$

$X\cap Y = \{1,3,5,7, \ldots, 19\}\cap\{0,2,4,6,8, \ldots, 20\}$

$= \{\ \}$

$X\cap Z=\{1,3,5,7, \ldots, 19\}\cap\{2,3,5,7,11,13,17, 19,23\}$

$= \{3, 5, 7, 11, 13, 17, 19\}$

$(X\cap Y) \cup(X\cap Z)=\{\ \}\cup \{3,5,7,11,13, 17,19\}$

$= \{3, 5, 7, 11, 13, 17, 19\}$

Q. 2. If A = {1, 2, 3, 4, 5, 6}
B = {2, 4, 6, 8} C = {1, 4, 8} Prove the following identities:

(i) $A\cap B = B\cap A$
(ii) $A\cup B = B\cup A$
(iii) $A\cap (B\cap C) = (A\cap B) \cup (A\cap C)$
(iv) $A\cup (B\cap C) = (A\cup B) \cap (A\cup C)$

Solution:

(i) $A\cap B = B\cap A$

L.H.S = $A\cap B$

= {1, 2, 3, 4, 5, 6} ∩ {2, 4, 6, 8}

= {2, 4, 6}

R.H.S = $B\cap A$

={2, 4, 6, 8} ∩ {1, 2, 3, 4, 5, 6}

= {2, 4, 6}

L.H.S = R.H.S, so

$A\cap B = B\cap A$

(ii) $A\cup B = B\cup A$

L.H.S = $A\cup B$

= {1, 2, 3, 4, 5, 6} ∪ {2, 4, 6, 8}

= {1, 2, 3, 4, 5, 6, 8}

R.H.S = $B\cup A$

= {2, 4, 6, 8} ∪ {1, 2, 3, 4, 5, 6}

= {1, 2, 3, 4, 5, 6, 8}

L.H.S = R.H.S,

So, $A\cup B = B\cup A$

(iii) $A\cap (B\cup C) = (A\cap B) \cup (A\cap C)$

L.H.S = $A\cap (B\cup C)$

= A ∩ ({2, 4, 6, 8} ∪ {1, 4, 8})

= {1, 2, 3, 4, 5, 6} ∩ {1, 2, 4, 6, 8}

= {1, 2, 4, 6}

R.H.S = $(A\cap B) \cup (A\cap C)$

$A\cap B$ = {1,2,3,4,5,6} ∩ {2,4,6,8}

= {2, 4, 6}

$A\cap C$ = {1, 2, 3, 4, 5, 6} ∩ {1, 4, 8}

= {1, 4}

$(A\cap B) \cup (A\cap C)$ = {2, 4, 6} ∪ {1, 4}

= {1, 2, 4, 6}

L.H.S = R.H.S

So, $A\cap (B\cup C) = (A\cap B) \cup (A\cap C)$

(iv) $A\cup (B\cap C) = (A\cup B) \cap (A\cup C)$

L.H.S = $A\cap (B\cap C)$

= A ∪ ({2, 4, 6, 8} ∩ {1, 4, 8})

= {1, 2, 3, 4, 5, 6} ∪ {4, 8}

= {1, 2, 3, 4, 5, 6, 8}

R.H.S = $(A\cup B) \cap (A\cup C)$

$A\cup B$ ={1, 2, 3, 4, 5, 6}∪{2, 4, 6, 8}

={1, 2, 3, 4, 5, 6, 8}

$A\cup C$ = {1, 2, 3, 4, 5, 6} ∪ {1, 4, 8}

= {1, 2, 3, 4, 5, 6, 8}

$(A\cup B) \cap (A\cup C)$

= {1,2,3,4,5,6,8}∩{1,2,3,4,5,6,8}

= {1, 2, 3, 4, 5, 6, 8}

L.H.S = R.H.S,

So, $A\cup (B\cap C) = (A\cup B) \cap (A\cup C)$

Q.3 If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {1, 3, 5, 7, 9}, B = {2, 3, 5, 7} then verify the De Morgan's laws i.e.,
$(A\cup B)'=A'\cap B'$ and $(A\cap B)' = A'\cup B'$

Solution:

(i) $(A\cup B)'=A'\cap B'$

L.H.S = $(A\cup B)'$

$A\cup B$ = {1, 3, 5, 7, 9} ∪ {2, 3, 5, 7}

= {1, 2, 3, 5, 7, 9}

$(A\cup B)' = U - (A\cup B)$

={1,2,3,4, …,10} – {1,2,3,5,7,9}

= {4, 6, 8, 10}…………(i)

R.H.S = $A'\cap B'$

$A' = U - A$

= {1, 2, 3, 4, …, 10} – {1,3, 5, 7, 9}

= {2, 4, 6, 8, 10}

$B' = U - B$

$= \{1, 2, 3, 4, 5, \ldots, 10\} - \{2, 3, 5, 7\}$

$= \{1, 4, 6, 8, 9, 10\}$

$A' \cap B' = \{2, 4, 6, 8, 10\} \cap \{1, 4, 6, 8, 9, 10\}$

$= \{4, 6, 8, 10\}$ ....(ii)

From (i) and (ii)

L.H.S = R.H.S

$(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

L.H.S = $(A \cap B)'$

$A \cap B = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\}$

$= \{3, 5, 7\}$

$(A \cap B)' = U - (A \cap B)$

$= \{1, 2, 3, 4, \ldots, 10\} - \{3, 5, 7\}$

$= \{1, 2, 4, 6, 8, 9, 10\}$ ....(i)

R.H.S = $A' \cup B'$

$A' = U - A$

$= \{1,2,3,4,5,\ldots,10\} - \{1,3,5,7,9\}$

$= \{2, 4, 6, 8, 10\}$

$B' = U - B$

$= \{1, 2, 3, 4, 5, \ldots, 10\} - \{2, 3, 5, 7\}$

$= \{1, 4, 6, 8, 9, 10\}$

$A' \cup B' = \{2, 4, 6, 8, 10\} \cup \{1, 4, 6, 8, 9, 10\}$

$= \{1, 2, 4, 6, 8, 9, 10\}$ ....(ii)

From (i) and (ii)

L.H.S = R.H.S

So $(A \cap B)' = A' \cup B'$

Q.4 If $U = \{1, 2, 3, \ldots, 20\}$

$X = \{1, 3, 7, 9, 15, 18, 20\}$

$Y = \{1, 3, 5, \ldots, 17\}$ then show that,

(i) $X - Y = X \cap Y'$ (ii) $Y - X = Y \cap X'$

Solution:

(i) $X - Y = X \cap Y'$

L.H.S = $X - Y$

$= \{1, 3, 7, 9, 15, 18, 20\}$

$\{1, 3, 5, 7, 9, 11, 13, 15, 17\}$

$= \{18, 20\}$ ....(i)

R.H.S = $X \cap Y'$

$Y' = U - Y$

$= \{1,2,3,\ldots,20\} - \{1,3,5,\ldots,17\}$

$= \{2,4,6,8,10,12,14,16,18,19,20\}$

$X \cap Y' = \{1, 3, 7, 9, 15, 18, 20\} \cap$
$\{2, 4, 6, 8, 10, 12, 14, 16, 18, 19, 20\}$

$= \{18, 20\}$ ....(ii)

From (i) and (ii)

L.H.S = R.H.S,

So, $X - Y = X \cap Y'$

(ii) $Y - X = Y \cap X'$

L.H.S = $Y - X$

$= \{1,3,5,\ldots,17\} - \{1,3,7,9,15,18,20\}$

$= \{5, 11, 13, 17\}$ ....(i)

R.H.S = $Y \cap X'$

$X' = U - X$

$= \{1,2,3,\ldots,20\} - \{1,3,7,9,15,18,20\}$

$= \{2, 4, 5, 6, 8, 10, 11, 12, 13, 14, 16, 17, 19\}$

$Y \cap X' = \{1, 3, 5, 7, 9, 11, 13, 15, 17\} \cap$
$\{2, 4, 5, 6, 8, 10, 11, 12, 13, 14, 16, 17, 19\}$

$= \{5, 11, 13, 17\}$ ....(ii)

From (i) and (ii)

L.H.S = R.H.S

So, $Y - X = Y \cap X'$

EXERCISE 5.3

Q.1 If U = {1, 2, 3, 4, ….., 10}
A = {1, 3, 5, 7, 9} , B = {1, 4, 7, 10}
then verify the following questions:
Solution:

(i) A – B = A∩B′

L.H.S = A – B = {1,3,5,7,9} – {1,4,7,10}
= {3, 5, 9} …………… (i)
R.H.S = A∩B′
B′=U – B={1, 2, 3, …., 10} – {1, 4, 7, 10}
= {2, 3, 5, 6, 8, 9}
A ∩ B′ = {1, 3, 5, 7, 9} ∩ {2, 3, 5, 6, 8, 9}
= {3, 5, 9} …………… (ii)
From (i) and (ii)
L.H.S = R.H.S
A–B = A∩B′

(ii) B–A =B∩A′

L.H.S = B – A = {1,4,7,10} – {1, 3, 5, 7, 9}
= {4, 10}…………… (i)
R.H.S = B∩A′
A′= U – A = {1,2,3, …., 10} – {1,3,5,7,9}
= {2, 4, 6, 8, 10}
B∩A′ ={1, 4, 7, 10} ∩ {2, 4, 6, 8, 10}
= {4, 10}…………… (ii)
From (i) and (ii)
L.H.S = R.H.S
B – A = B∩A′

(iii) (A∪B)′ = A′∩B′

L.H.S = (A∪B)′
A∪B = {1, 3, 5, 7, 9} ∪ {1, 4, 7, 10}
= {1, 3, 4, 5, 7, 9, 10}
(A∪B)′ = U – (A∪B)
= {1,2,3,….,10}–{1,3,4,5,7,9,10}
= {2, 6, 8} …………… (i)
R.H.S = A′∩B′
A′= U – A = {1,2,3,….., 10}– {1,3,5,7,9}
= {2, 4, 6, 8, 10}
B′= U – B = {1,2,3,…., 10}– {1,4,7,10}
= {2, 3, 5, 6, 8, 9}
A′ ∩ B′={2, 4, 6, 8, 10}∩{2, 3, 5, 6, 8, 9}
= {2, 6, 8} …………… (ii)
From (i) and (ii)
L.H.S = R.H.S

(iv) (A∩B)′=A′∪B′

L.H.S = (A∩B)′
A∩B ={1, 3, 5, 7, 9} ∩ {1, 4, 7, 10}
= {1, 7}
(A∩B)′ =U – (A∩B)
= {1, 2, 3, …, 10} – {1, 7}
= {2, 3, 4, 5, 6, 8, 9, 10} …… (i)
R.H.S = A′∪B′
A′= U – A = {1,2,3,….,10} – {1,3,5,7,9}
= {2, 4, 6, 8, 10}
B′= U–B ={1,2,3,…., 10} – {1, 4,7,10}
= {2, 3, 5, 6, 8, 9}
A′∪B′ ={2, 4, 6, 8, 10} ∪ {2, 3, 5, 6, 8, 9}
= {2, 3, 4, 5, 6, 8, 9, 10}……. (ii)
From (i) and (ii)
L.H.S = R.H.S

(v) (A–B)′ = A′∪B

L.H.S = (A–B)′
A – B = {1, 3, 5, 7, 9} – {1, 4, 7, 10}
= {3, 5, 9}
(A – B)′ = U– (A–B)
= {1, 2, 3, ….., 10} – {3, 5, 9}
= {1, 2, 4, 6, 7, 8, 10}……. (i)
R.H.S = A′∪B
A′ = U – A= {1,2,3, ….,10} – {1,3,5,7,9}
= {2, 4, 6, 8, 10}
A′∪B = {2, 4, 6, 8, 10} ∪ {1, 4, 7, 10}
= {1, 2, 4, 6, 7, 8, 10}……. (ii)
From (i) and (ii)
L.H.S = R.H.S

(vi) (B–A)′ = B′∪A

L.H.S = (B–A)′
B – A = {1, 4, 7, 10} – {1, 3, 5, 7, 9}
= {4, 10}
(B–A)′ = U – (B–A)
= {1, 2, 3, ….., 10} – {4, 10}
= {1, 2, 3, 5, 6, 7, 8, 9}……. (i)
R.H.S = B′∪A
B′= U – B ={1,2,3,4,….,10} – {1, 4, 7, 10}
= {2, 3, 5, 6, 8, 9}
B′∪A = {2,3,5,6,8,9}∪ {1, 3, 5, 7, 9}
= {1, 2, 3, 5, 6, 7, 8, 9}……. (ii)
From (i) and (ii)
L.H.S = R.H.S

Q.2 If A = {1, 3, 5, 7, 9} B = {1, 4, 7, 10} C = {1, 5, 8, 10}
Then verily the following:

Solution:

(i) (A∪B) ∪C = A∪(B∪C)

L.H.S = (A∪B)∪C
= ({1,3,5,7,9}∪{1,4,7,10}) ∪ {1,5,8,10}
= {1, 3, 4, 5, 7, 9, 10} ∪ {1, 5, 8, 10}
= {1, 3, 4, 5, 7, 8, 9, 10} .... (i)
R.H.S = A∪(B∪C)
= {1,3,5,7,9}∪({1,4,7,10} ∪ {1,5,8,10})
= {1,3,5,7,9} ∪ {1,4,5,7,8,10}
= {1, 3, 4, 5, 7, 8, 9, 10} .... (ii)
From (i) and (ii)
L.H.S = R.H.S

(ii) (A∩B)∩C = A∩(B∩C)

L.H.S = (A∩B)∩C
= ({1,3,5,7,9}∩{1,4,7,10}) ∩ {1,5,8,10}
= {1, 7} ∩ {1, 5, 8, 10}
= {1} .... (i)
R.H.S = A∩(B∩C)
= {1,3,5,7,9}∩({1,4,7,10} ∩ {1,5,8,10})
= {1, 3, 5, 7, 9} ∩ {1, 10}
= {1} .... (ii)
From (i) and (ii)
L.H.S = R.H.S

(iii) A∪(B∩C) = (A∪B)∩(A∪C)

L.H.S = A∪(B∩C)
= {1,3,5,7,9}∪({1,4,7,10} ∩ {1,5,8,10})
= {1, 3, 5, 7, 9} ∪ {1, 10}
= {1, 3, 5, 7, 9, 10} .... (i)
R.H.S = (A∪B)∩(A∪C)
A∪B ={1, 3, 5, 7, 9} ∪ {1, 4, 7, 10}
={1, 3, 4, 5, 7, 9, 10}
A∪C ={1, 3, 5, 7, 9} ∪ {1, 5, 8, 10}
= {1, 3, 5, 7, 8, 9, 10}
Now (A∪B)∩(A∪C)
= {1,3,4,5,7,9,10} ∩ {1,3,5,7,8,9,10}
= {1, 3, 5, 7, 9, 10} .... (ii)
From (i) and (ii)
L.H.S = R.H.S

(iv) A∩(B∪C) = (A∩B)∪(A∩C)

L.H.S = A∩(B∪C)
= {1,3,5,7,9}∩({1,4,7,10}∪{1, 5, 8, 10})
= {1, 3, 5, 7, 9} ∩ {1, 4, 5, 7, 8, 10}
= {1, 5, 7} .... (i)
R.H.S = (A∩B)∪(A∩C)
A∩B ={1,3,5,7,9} ∩ {1, 4, 7, 10}
= {1, 7}
A∩C ={1,3, 5, 7, 9} ∩ {1, 5, 8, 10}
= {1, 5}
Now (A∩B)∪(A∩C) ={1,7}∪{1,5}
= {1,5,7}....... (ii)
From (i) and (ii)
L.H.S = R.H.S

Q.3 If U = N, then verify De-Morgan's laws by using:
A= ϕ, B = P

Solution:

A = { }
B = {2, 3, 5, 7, ...}
U = {1, 2, 3, 4, 5, 6, 7....}

(i) (A∪B)′ = A′∩B′

L.H.S = (A∪B)′
A∪B = { } ∪ {2, 3, 5, 7, ...}
= {2, 3, 5, 7, ...}
(A∪B)′ = U – (A∪B)
={1, 2, 3, 4,5,6,7 ....} – {2, 3, 5, 7, ...}
= {1, 4, 6, ...} .... (i)
R.H.S = A′∩B′
A′= U – A ={1, 2, 3, 4, 5, 6, 7.....} – { }
= {1, 2, 3, 4, 5, 6, 7....}
B′=U – B={1,2,3,4,5,6,7....}–{2,3,5,7,...}
= {1, 4, 6, ...}
A′∩B′={1,2,3,4,5,6,7 ....} ∩ {1, 4, 6, ...}
={1, 4, 6, ...} .... (ii)
From (i) and (ii)
L.H.S = R.H.S

(ii) (A∩B)′ = A′∪B′

L.H.S = (A∩B)′

(A∩B) = { } ∩ {2, 3, 5, 7, ...}

= { }

(A∩B)′= U – (A∩B)

= {1, 2, 3, 4, 5, 6, 7 ....} – { }

= {1, 2, 3, 4, 5, 6, 7...}....... (i)

R.H.S = A′∪B′

A′=U – A={1, 2, 3, 4, ...} – { }

= {1, 2, 3, 4, ...}

B′=U – B={1,2,3,4,5,6,7....} – {2,3,5,7,...}

= {1, 4, 6, ...}

A′∪B′= {1,2,3,4,5,6,7 ...} ∪ {1, 4, 6, ...}

= {1, 2, 3, 4, ...}....... (ii)

From (i) and (ii)

L.H.S = R.H.S

Q.4 If ∪ = {1, 2, 3, 4, ...10}, A = {1, 3, 5, 7, 9} and B = {2, 3, 4, 5, 8} then prove the following questions by Venn Diagram.

Solution: A ∩B = {1, 3, 5, 7, 9}∩{2, 3, 4, 5, 8} = {3, 5}

So given sets A and B are overlapping sets

(i) **A – B = A∩B′**

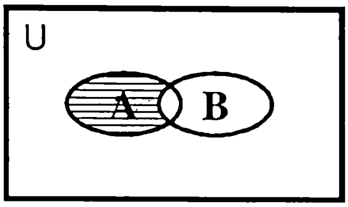


Fig: 1 (A-B)

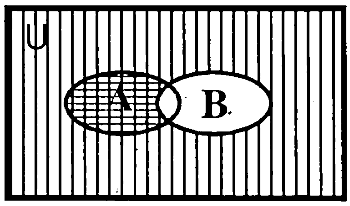


Fig: 2)A∩B′(

- A – B is shown by horizontal line segments in fig. 1.
- B′ is shown by vertical line segments and squares in fig. 2.
- A∩B′ is shown by squares in fig. 2.

Regions shown in fig. 1 and fig. 2 are equal, thus A – B = A∩B′

(ii) **B – A = B∩A′**

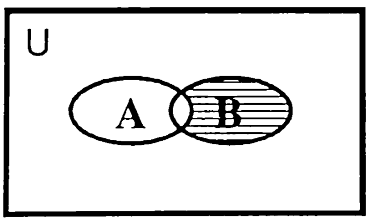


Fig. 1 (B - A)

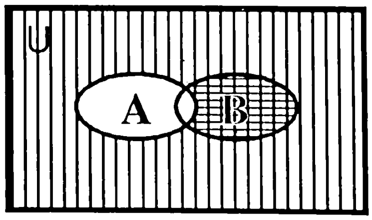


Fig: 2 (B∩A′)

- B – A is shown by horizontal line segments in fig. 1.
- A′ is shown by vertical line segments and squares in fig. 2.
- B∩A′ is shown by squares in fig. 2.

Regions shown in fig. 1 and fig. 2 are equal, thus B – A = B∩A′.

(iii) (A∪B)′ = A′∩B′ (De-Morgan's Law)

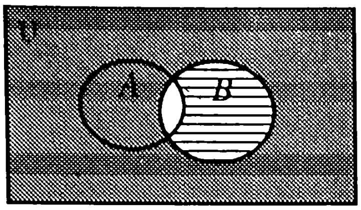


*Fig.*1

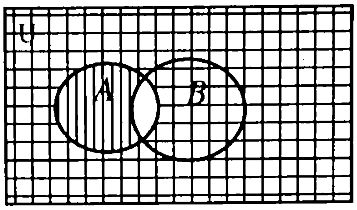


*Fig.*2

- A∪B is shown by horizontal line segments in Fig. 1.
- (A∪B)′ is shown by shaded area in Fig. 1.
- A′is shown by horizontal line segments and squares in Fig. 2.
- B′ is shown by vertical line segments and squares in Fig. 2.
- A′∩B′ is shown by squares in Fig. 2.

Shaded area shown in Fig. 1 and square area shown in Fig. 2 are equal.
thus (A∪B)′ = A′∩B′

(iv) (A∩B)′ = A′∪B′

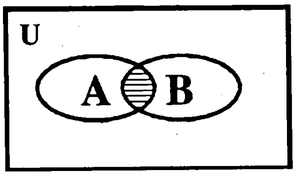


Fig. 1

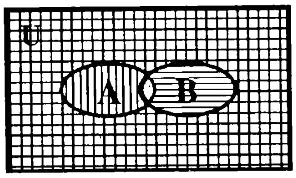


Fig. 2

- A∩B is shown by horizontal line segments in Fig. 1.
- (A∩B)′ is shown by shaded area in Fig. 1.
- A′ is shown by horizontal line segments and squares in Fig. 2.
- B′ is shown by vertical line segments and squares in Fig. 2.
- A′∪B′ is shown by squares, horizontal and vertical line segments in Fig. 2.

Shaded area shown in Fig. 1 and area of squares, vertical and horizontal line segments shown in Fig. 2 are equal. thus (A∩B)′ = A′∪B′

(v) (A–B)′ = A′∪B

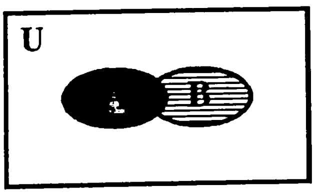


Fig. 1

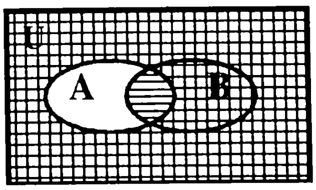


Fig. 2

- A–B is shown by horizontal line segments in Fig. 1.
- (A–B)′ is shown by shaded area in Fig.1.
- A′ is shown by squares Fig. 2.
- A′∪B is shown by squares and horizontal line segments in Fig. 2.

Shaded area in Fig. 1 and area of squares & horizontal line segments in Fig. 2 are equal.
thus (A – B)′ = A′∪ B

(vi) $(B-A)' = B' \cup A$

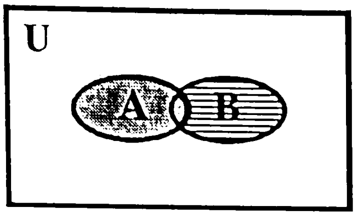


Fig. 1

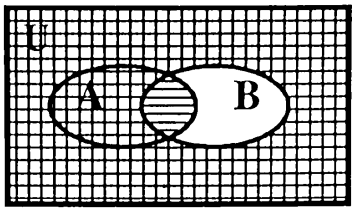


Fig. 2

- $B-A$ is shown by horizontal line segments in Fig. 1.
- $(B-A)'$ is shown by shaded area in Fig.1.
- B' is shown by squares in Fig. 2.
- $B' \cup A$ is shown by squares and horizontal line segments in Fig. 2.

Shaded area in Fig. 1 and area of squares & vertical line segments in Fig. 2 are equal.
thus $(B-A)' = B' \cup A$

EXERCISE 5.4

Q.1 If $A = \{a, b\}$ and $B = \{c, d\}$, then find $A \times B$ and $B \times A$

Solution:

$A \times B = \{a, b\} \times \{c, d\}$
$= \{(a, c), (a, d), (b, c), (b, d)\}$

$B \times A = \{c, d\} \times \{a, b\}$
$= \{(c, a), (c, b), (d, a), (d, b)\}$

Q.2 If $A = \{0, 2, 4\}$, $B = \{-1,3\}$, then find $A\times B$, $B \times A$, $A \times A$, $B \times B$

Solution:

$A \times B = \{0, 2, 4\} \times \{-1, 3\}$
$=\{(0,-1),(0,3),(2,-1),(2,3),(4,-1),(4,3)\}$

$B \times A = \{-1, 3\} \times \{0, 2, 4\}$
$= \{(-1,0),(-1,2),(-1,4),(3,0),(3,2),(3,4)\}$

$A\times A = \{0, 2, 4\} \times \{0, 2, 4\}$
$=\{(0,0),(0,2),(0,4),(2,0),(2,2),(2,4),(4,0)(4,2),(4,4)\}$

$B \times B = \{-1, 3\} \times \{-1, 3\}$
$= \{(-1, -1), (-1, 3), (3, -1), (3, 3)\}$

Q.3 Find a and b if

Solution:

(i) $(a - 4, b - 2) = (2, 1)$

$a - 4 = 2$, $b - 2 = 1$
$a = 2+ 4$, $b = 1 + 2$
$\boxed{a = 6}$, $\boxed{b = 3}$

(ii) $(2a + 5, 3) = (7, b - 4)$

$2a + 5 = 7$, $3 = b - 4$
$2a = 7 - 5$, $3 + 4 = b$
$2a = 2$, $7 = b$
$a = \frac{2}{2} = 1$, $\boxed{b = 7}$
$\boxed{a = 1}$

(iii) $(3 - 2a, b - 1) = (a - 7, 2b + 5)$

$3 - 2a = a - 7$, $b - 1 = 2b + 5$
$3 + 7 = a + 2a$, $-1 - 5 = 2b - b$
$10 = 3a$, $-6 = b$
$\frac{10}{3} = a$, $\boxed{b = -6}$
$\boxed{a = \frac{10}{3}}$

Q.4 Find the sets X and Y if $X\times Y = \{(a, a), (b, a), (c, a), (d, a)\}$

Solution:

$X\times Y = \{(a, a), (b, a), (c, a), (d, a)\}$
$X\times Y = \{a, b, c, d\} \times \{a\}$
$X = \{a, b, c, d\}$
$Y= \{a\}$

Q.5 If $X = \{a, b, c\}$ and $Y = \{d, e\}$, then find the number of elements in (i) $X \times Y$ (ii) $Y \times X$ (iii) $X \times X$

Solution:

No. of elements in $X = 3$
No. of elements in $Y = 2$
(i) No. of Elements in $X\times Y = 3\times 2= 6$
(ii) No. of Elements in $Y\times X = 2\times 3= 6$
(iii) No. of Elements in $X\times X = 3\times 3= 9$

Miscellaneous Exercises - 5

Q.1 Multiple choice questions. Four possible answers are given for the following questions. Tick mark ( ) the correct answer.

1. The different number of ways to describe a set are
   (a) 1 (b) 2
   (c) 3 (d) 4
2. The set $\{x / x \in W \wedge x \leq 101\}$ is
   (a) infinite set (b) subset
   (c) Null set (d) finite set
3. A collection of well-defined distinct objects is called
   (a) subset (b) power set
   (c) set (d) none of these
4. If $A \subseteq B$ then $A \cup B$ is equal to
   (a) A (b) B
   ϕ
5. A set $Q = \frac{a}{b} | a, b \in Z \wedge b \neq 0$ is called a set of
   (a) Whole numbers
   (b) Natural numbers
   (c) Irrational numbers
   (d) Rational numbers
6. If $A \subseteq B$ then $A - B$ is equal to
   (a) A (b) B
   (c) ϕ (d) None of these
7. If A and B are disjoint sets, then $A \cup B$ is equal to
   (a) A (b) B
   (c) ϕ (d) $B \cup A$
8. The number of elements in power set $\{1,2,3\}$ is
   (a) 4 (b) 6
   (c) 8 (d) 9
9. A set with no element is called
   (a) subset (b) empty set
   (c) singleton set (d) super set
10. If $A \subseteq B$ then $A \cap B$ is equal to
    (a) A (b) B
    (c) ϕ (d) None of these
11. The set having only one element is called
    (a) Null set (b) power set
    (c) singleton set (d) subset
12. The relation $\{(1,2),(2,3),(3,3)(3,4)\}$ is
    (a) onto function
    (b) into function
    (c) not a function
    (d) one-one function
13. If $A \subseteq B$ and $B \subseteq A$, then
    (a) A = B (b) $A \neq B$
    (c) $A \cap B = \phi$ (d) $A \cup B = \phi$
14. Power set of an empty set is
    (a) ϕ (b) $\{a\}$
    (c) $\{\phi, \{a\}\}$ (d) $\{\phi\}$
15. If number of elements in set A is 3 and in set B is 4, then number of elements in A×B is
    (a) 3 (b) 4
    (c) 12 (d) 7
16. Point (-1,4) lies in the quadrant
    (a) I (b) II
17. The domain of R $=\{(0,2),(2,3),(3,3)(3,4)\}$ is
    (a) $\{0,3,4\}$ (b) $\{0,2,3\}$
    (c) $\{0,2,4\}$ (d) $\{2,3,4\}$
18. The point $(-5, -7)$ lies in … quadrant
    (a) I (b) II
    (c) III (d) IV

19. If number of elements in se A is 3 and in set B is 2, then number of binary relations in A×B is
(a) 2^3 (b) 2^6
(c) 2^8 (d) 2^2

20. $(A\cup B)\cup C$ is equal to
(a) $A\cap(B\cup C)$ (b) $(A\cup B)\cap C)$
(c) $A\cup(B\cup C)$ (d) $A\cap(B\cap C)$

21. If $A\cap B=\phi$, then set A and B are ....sets.
(a) sub (b) over lapping
(c) disjoint (d) power

22. $A\cup(B\cap C)$ is equal to
(a) $(A\cup B)\cap(A\cup C)$
(b) $A\cap(B\cap C)$
(c) $(A\cap B)\cap(A\cap C)$
(d) $A\cup(B\cup C)$

23. The range of {(a,a), (b,b), (c,c)} is......
(a) {a,b} (b) {a,b,c}
(c) {a} (d) ϕ

24. The complement of ϕ is ......
(a) U (b) ϕ
(c) impossible (d) union

25. $A\cap A^c=$ ......
(a) U (b) A
(c) A^c (d) ϕ

26. Venn diagram was first used by.......
(a) John Venn
(b) Netwon
(c) Arthur Cayley
(d) John Napier

27. The set { x | x ∈ A and x ∉ B } is ......
(a) $A\cup B$ (b) $A\cap B$
(c) A – B (d) B – A

28. The Range of R ={(1,3),(2,2),(3,1)(4,4)} is
(a) {1,2,4} (b) {3,2,4}
(c) {1,2,3,4} (d) {1,3,4}

29. $N\cup W=$ ....
(a) ϕ (b) {0}
(c) N (d) W

30. y co-ordinate of every pint on x – axis is..
(a) +ve (b) –ve
(c) zero (d) 1

31. By definition, which of the following is a set?
(a) {a,b,c,a} (b) {1,2,3,2}
{ ,m,n,o} {0,1,2,3,1}

32. The domain of {(a,b), (b,c), (c,d)} is...
(a) {a,b,c} (b) {b,c,d}
(c) {a,b} (d) {a,b,c,d,}

33. The complement of U is ....
(a) U (b) ϕ
(c) impossible (d) union

34. $A\cup A^c=$ ......
(a) U (b) A
(c) A^c (d) ϕ

35. Which of the following is true?
(a) $P\subseteq N\subseteq Z\subseteq W$
(b) $P\subseteq N\subseteq W\subseteq Z$
(c) $P\subseteq W\subseteq N\subseteq Z$
(d) $P\subseteq Z\subseteq N\subseteq W$

36. W – N = ...
(a) ϕ (b) {0}
(c) N (d) W

37. N – W = ...
(a) ϕ (b) {0}
(c) N (d) W

38. The relation {(a,b),(b,c),(a,d),} is....
(a) a function (b) not a function
(c) range (d) domain 05(091)

39. x co-ordinate of every pint on y – axis is..
(a) +ve (b) –ve
(c) zero (d) 1

40. Which of the following is true?
(a) $W \subseteq N$
(b) $Z \subseteq W$
(c) $N \subseteq P$
(d) $P \subseteq W$

41. A subset of $A \times A$ is called……in A.
(a) set (b) relation
(c) function (d) into function

42. Which of the following is true?
(a) N and $W \subseteq Z$
(b) P and $O \subseteq W$
(c) O and $E \subseteq W$
(d) P and $E \subseteq N$

43. If $x \in A$ and $x \in B$, then $\{x\}$ is equal to
(a) $A - B$ (b) A^c
(c) $A \cap B$ (d) B^c

44. The point (4, −6) lies in …. Quadrant
(a) I (b) II
(c) III (d) IV

45. If $f: A \to B$ and range of $f \neq B$, then f is an ………..
(a) into function
(b) onto function
(c) bijective function
(d) function

46. If $f: A \to B$ and range of $f = B$, then f is an……
(a) into function
(b) onto function
(c) bijective function
(d) function

47. If set A has all its elements common with set B then set A is called….set.
(a) sub (b) overlapping
(c) disjoint (d) super

48. $O \cup E =$ ……….
(a) ϕ (b) O
(c) E (d) Z

49. Which of the following is commutative law?
(a) $A \cup (B \cup C) = (A \cup B) \cup C$
(b) $A \cap (B \cap C) = (A \cap B) \cap C$
(c) $A \cap B = B \cap A$
(d) $)A \cup B(^C = A^C \cap B^C$

50. Which of the following is associative law of intersection?
(a) $A \cup (B \cup C) = (A \cup B) \cup C$
(b) $A \cap (B \cap C) = (A \cap B) \cap C$
(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

51. Which of the following is distributive property of intersection over union?
(a) $A \cup (B \cup C) = A \cup (B \cup C)$
(b) $A \cap (B \cap C) = (A \cap B) \cap C$
(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

52. $N \cap W =$ ………
(a) ϕ (b) $\{0\}$
(c) N (d) W

53. If A is subset of U, then $(A^C)^C =$ ……
(a) A (b) A^c
(c) ϕ^C (d) U^c

54. If two sets have some elements common but not all are called….sets.
(a) sub (b) overlapping
(c) disjoint (d) super

55. Which of the following is De-Morgan's law?
(a) $(A \cup B) \cup C = A \cup (B \cup C)$
(b) $)A \cap B(^C = A^C \cup B^C$
(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

56. Which of the following is associative law of union?

(a) $A\cup(B\cup C)=(A\cup B)\cup C$

(b) $A\cap(B\cap C)=(A\cap B)\cap C$

(c) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

(d) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

57. $O-E=$..........

(a) ϕ (b) O

(c) E (d) Z

58. If union and intersection of two sets are equal then sets are ......sets.

(a) disjoint (b) overlapping

(c) equal (d) super

59. Which of the following is complete description of Real numbers?

(a) $N\cup W=R$ (b) $O\cup E=R$

(c) $P\cup Q=R$ (d) $Q\cup Q'=R$

60. $E-O=$...........

(a) ϕ (b) O

(c) E (d) Z

61. Two sets having no common element, are called .... sets.

(a) subet (b) overlapping

(c) disjoint (d) super

62. If $x\in U$ and $x\notin A$, then $\{x\}$ is equal to.............

(a) U^C (b) A^C

(c) ϕ^C (d) $A-U$

63. $O\cap E=$...........

(a) ϕ (b) O

(c) E (d) Z

64. A and A^C are ....sets.

(a) universal (b) overlapping

(c) disjoint (d) super

65. Which of the following is distributive property of union over intersection?

(a) $A\cup(B\cup C)=A\cup(B\cup C)$

(b) $A\cap(B\cap C)=(A\cap B)\cap C$

(c) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

(d) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

66. If $x\in A$ and $x\notin B$, then $\{x\}$ is equal to..........

(a) $A-B$ (b) $B-A$

(c) $A\cap B$ (d) A^c

ANSWER KEY

1	c	2	d	3	c	4	b	5	d	6	c	7	d	8	c
9	b	10	a	11	c	12	c	13	a	14	d	15	c	16	b
17	b	18	c	19	b	20	c	21	c	22	a	23	b	24	a
25	d	26	a	27	c	28	c	29	d	30	c	31	c	32	a
33	b	34	a	35	b	36	b	37	a	38	b	39	c	40	d
41	b	42	a	43	c	44	d	45	a	46	b	47	a	48	d
49	c	50	b	51	d	52	c	53	a	54	b	55	b	56	a
57	b	58	c	59	d	60	c	61	c	62	b	63	a	64	c
65	c	66	a												

Q.2 Write short answers of the following questions.

(i) Define a subset and give one example.

Subset: If A and B are two sets and every element of A is an element of B then set A is called subset of set B. It is denoted by $A \subseteq B$.

Example: $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$

As all elements of Set A are also present in Set B. Therefore $A \subseteq B$.

(ii) Write all subsets of the set {a, b}

Solution:

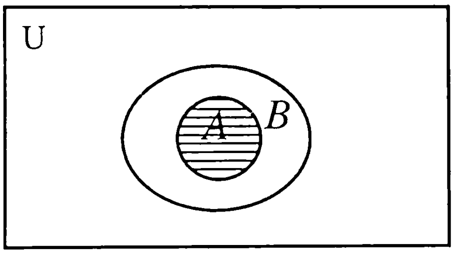
All subsets ($2^n = 2^2 = 4$)

{ }, {a}, {b}, {a, b}

(iii) Show $A \cap B$ by Venn Diagram when $A \subseteq B$.

Solution:

If $A \subseteq B$ then $A \cap B = A$



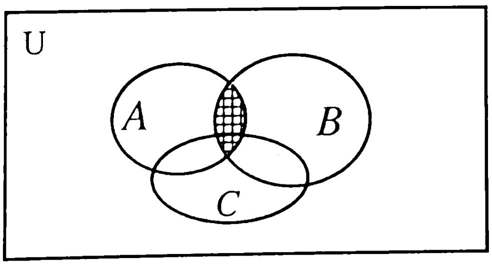
$A \cap B$

Horizontal line segments show $A \cap B$

(iv) Show by Venn Diagram $A \cap (B \cup C)$

Solution:

$A \cap (B \cup C)$ by Venn diagram



- Horizontal line segments and squares show $B \cup C$.
- $A \cap (B \cup C)$ is shown by squares.

(v) Define intersection of two sets.

Intersection of two sets:

The intersection of two sets A and B, written as $A \cap B$ (read as A intersection B) is the set consisting of all the common elements of A and B.

(vi) Define a function.

Function: Suppose A and B are two non-empty sets, then relation $f : A \rightarrow B$ is called a function if

(i) Dom f = A

(ii) Every $x \in A$ appears in one and only and ordered pair in f.

(vii) Define an one – one function.

One – one function:

A function $f : A \rightarrow B$ is called one – one function, if all distinct elements of A have distinct images in B, i.e:

$f(x_1) = f(x_2) \quad x_1 = x_2 \in A$ or

$\forall\, x_1 \neq x_2 \in A \quad f(x_1) \neq f(x_2)$

(viii) Define an onto function.

Onto function:

A function $f : A \rightarrow B$ is called an onto function, if every element of set B is an image of at least one element of set A.

i.e. Range of f = B

(ix) Define a Bijective function.

Bijective function:

A function $f : A \rightarrow B$ is called bijective function iff function f is one-one and onto.

(x) Write De Morgan's law.

De Morgan's Law:

If two sets A and B are the sub sets of U then De-Morgan's laws are expressed as

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Q.3 Fill in the Blanks

i. If $A \subseteq B$ then $A \cup B =$ ______.

ii. If $A \cap B = \phi$ then A and B are ______.

iii. If $A \subseteq B$ and $B \subseteq A$ then________.

iv. $A \cap (B \cup C) =$ ______.

v. $A \cup (B \cap C) =$ ______.

vi. The complement of U is _________.

vii. The complement of ϕ is________.

viii. $A \cap A^c =$ ______.

ix. $A \cup A^c =$ ______.

x. The set $\{x | x \in A \text{ and } x \notin B\} =$ _____.

xi. The point (-5, -7) lies in _______ quadrant.

xii. The point (4,-6) lies in ______quadrant.

xiii. The y co-ordinate of every point is __________on x-axis.

xiv. The x co-ordinate of every point is __________on y-axis.

xv. The domain of {(a,b), (b,c), (c,d)} is _______.

xvi. The range of {(a,a), (b,b), (c,c)} is _______.

xvii. Venn-diagram was first used by_______.

xviii. A subset of A x A is called the _____in A.

xix. If $f : A \longrightarrow B$ and range of f=B, then f is an ______function.

xx. The relation of {(a,b), (b,c), (a,d)} is _______ function.

Answers:

(i)	B	(ii)	Disjoint sets
(iii)	A = B	(iv)	$(A \cap B) \cup (A \cap C)$
(v)	$(A \cup B) \cap (A \cup C)$		
(vi)	ϕ	(vii)	U
(viii)	ϕ	(ix)	U
(x)	$A \setminus B$	(xi)	IIIrd
(xii)	IVth		
(xiii)	Zero	(xiv)	Zero
(xv)	{a, b, c}		
(xvi)	{a, b, c}	(xvii)	John venn
(xviii)	Binary relation		
(xix)	onto	(xx)	not

EXERCISE 5.5

Q.1 If L = {a, b, c}, M = {3, 4}, then find two binary relations of L×M and M×L.

Solution: L = {a, b, c}, M = {3, 4}

L×M = {a, b, c} × {3, 4}

= {(a, 3),(a, 4), (b, 3), (b, 4), (c, 3), (c, 4)}

Two binary Relations:

R_1 = {(a, 3), (a, 4)}

R_2 = {(b, 4), (c, 3), (c, 4)}

M×L = {3, 4} × {a, b, c}

={(3, a),(3, b),(3, c),(4, a),(4, b),(4, c)}

Two binary Relations:

R_1 = {(3, a), (3, b)}

R_2 = {(4, b), (4, c)}

Q.2 If Y = {-2, 1, 2}, then make two binary relations for Y × Y. Also find their domain and range.

Solution: Y = {-2, 1, 2}

Y×Y = {-2, 1, 2}×{-2, 1, 2}

={(-2,-2),(-2,1),(-2,2),(1,-2),(1,1),(1,2),
(2,-2,) (2,1),(2,2)}

R_1 = {(-2, 1), (-2, 2), (1, -2)}

Domain R_1 = {-2, 1}

Range R_1 = {1, 2, -2}

R_2 = {(1, 1), (2, -2), (2, 2)}

Domain R_2 = {1, 2}

Range R_2 = {1, -2, 2}

Q.3 If L = {a, b, c} and M = {d, e, f, g}, then find two binary relations in each.

(i) L×L (ii) L×M (iii) M×M

Solution: L = {a, b, c} , M = {d, e, f, g}

(i) $L \times L = \{a, b, c\} \times \{a, b, c\}$

$L \times L = \{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c)\}$

Two binary Relations:

$R_1 = \{(a, a), (a, b), (a, c)\}$

$R_2 = \{(b, c), (c, a), (c, b)\}$

(ii) L×M

$L \times M = \{a, b, c\} \times \{d, e, f, g\}$

$= \{(a,d),(a,e),(a,f),(a,g),(b,d),(b,e),(b,f),(b,g),(c,d),(c,e),(c,f),(c,g)\}$

Two binary Relations:

$R_1 = \{(a, d), (a, e), (a, f)\}$

$R_2 = \{(b, d), (b, e), (b, f)\}$

(iii) M×M

$M \times M = \{d, e, f, g\} \times \{d, e, f, g\}$

$= \{(d,d),(d,e),(d,f),(d,g),(e,d),(e,e),(e,f),(e,g),(f,d),(f,e),(f,f),(f,g),(g,d),(g,e),(g,f),(g,g)\}$

Two binary Relations:

$R_1 = \{(d, e), (d, f), (d, g)\}$

$R_2 = \{(f, d), (g, d)\}$

Q.4 If set M has 5 elements then find the number of binary relations in M.

No. of binary relations in $M = 2^{5\times5} = 2^{25}$

Q.5 If $L = \{x | x \in N \wedge x \leq 5\}$,

$M = \{y | y \in P \wedge y < 10\}$, then make the following relations from L to M. Also write Domain and Range of each Relation.

(i) $R_1 = \{(x, y) \mid y < x\}$,

(ii) $R_2 = \{(x, y) \mid y = x\}$

(iii) $R_3 = \{(x, y) \mid x + y = 6\}$

(iv) $R_4 = \{(x, y) \mid y - x = 2\}$

Solution

$L = \{1, 2, 3, 4, 5\}$,

$M = \{2, 3, 5, 7\}$

$L \times M = \{1, 2, 3, 4, 5\} \times \{2, 3, 5, 7\}$

$= \{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),(2,7),(3,2),(3,3),(3,5),(3,7),(4,2),(4,3),(4,5),(4,7),(5,2),(5,3),(5,5),(5,7)\}$

(i) $R_1 = \{(x, y) \mid y < x\}$

$R_1 = \{(3, 2), (4, 2), (4, 3), (5, 2), (5, 3)\}$

Domain $R_1 = \{3, 4, 5\}$

Range $R_1 = \{2, 3\}$

(ii) $R_2 = (x, y) \mid y = x\}$

$R_2 = \{(2, 2), (3, 3), (5, 5)\}$

Domain $R_2 = \{2, 3, 5\}$

Range $R_2 = \{2, 3, 5\}$

(iii) $R_3 = \{(x, y) \mid x + y = 6\}$

$R_3 = \{(1, 5), (3, 3), (4, 2)\}$

Domain $R_3 = \{1, 3, 4\}$

Range $R_3 = \{5, 3, 2\}$

(iv) $R_4 = \{(x, y) \mid y - x = 2\}$

$R_4 = \{(1, 3), (3, 5), (5, 7)\}$

Domain $R_4 = \{1, 3, 5\}$

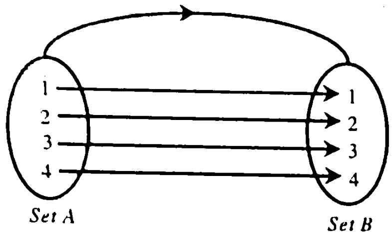
Range $R_4 = \{3, 5, 7\}$

Q.6 Indicate relations, into function, one-one function, onto function, and bijective function from the following. Also find their domain and the range.

$R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

Domain $R_1 = \{1, 2, 3, 4\}$

Range $R_1 = \{1, 2, 3, 4\}$

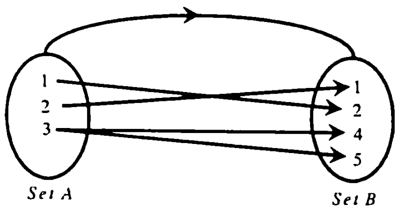


It is a bijective function.

(ii) $R_2 = \{(1, 2), (2, 1), (3, 4), (3, 5)\}$

Domain $R_2 = \{1, 2, 3\}$

Range $R_2 = \{1, 2, 4, 5\}$

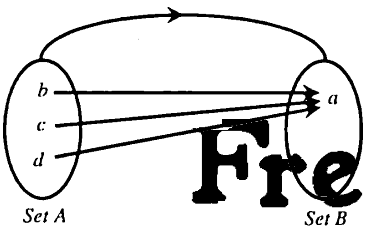


It is a relation. As 3 has no distinct image.

(iii) $R_3 = \{(b, a), (c, a), (d, a)\}$

Domain $R_3 = \{b, c, d\}$

Range $R_3 = \{a\}$

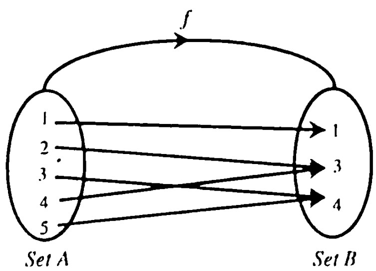


It is an onto function.

(iv) $R_4 = \{(1, 1), (2, 3), (3, 4), (4, 3), (5, 4)\}$

Domain $R_4 = \{1, 2, 3, 4, 5\}$

Range $R_4 = \{1, 3, 4\}$



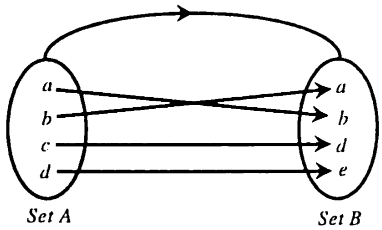
It is an onto function.

(v) $R_5 = \{(a, b), (b, a), (c, d), (d, e)\}$

Domain $R_5 = \{a, b, c, d\}$

Range $R_5 = \{a, b, d, e\}$

It is a bijective function.

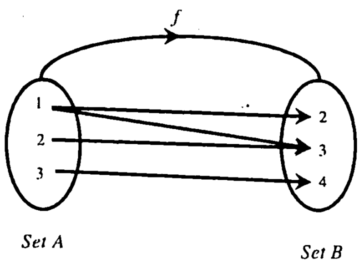


(vi) $R_6 = \{(1, 2), (2, 3), (1, 3), (3, 4)\}$

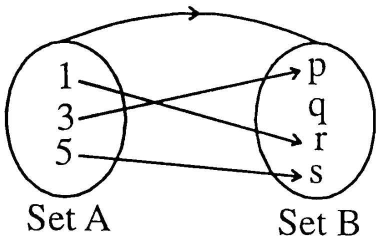
Domain $R_6 = \{1, 2, 3\}$

Range $R_6 = \{2, 3, 4\}$

It is a relation. As 1 has no distinct image.



(vii) $R_7 =$

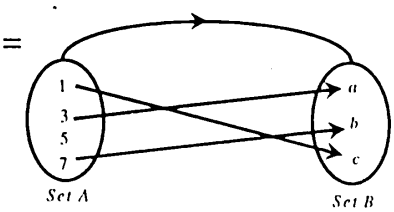


Domain $R_7 = \{1, 3, 5\}$

Range $R_7 = \{p, r, s\}$

R_7 is one-one function.

(viii) $R_8 =$



Domain $R_8 = \{1, 3, 7\}$

Range $R_8 = \{a, b, c\}$

It is a relation. As 5 has no distinct image.

Miscellaneous Exercises - 5

Q.1 Multiple choice questions. Four possible answers are given for the following questions. Tick mark ( ) the correct answer.

1. The different number of ways to describe a set are
   (a) 1 (b) 2
   (c) 3 (d) 4
2. The set $\{x / x \in W \wedge x \leq 101\}$ is
   (a) infinite set (b) subset
   (c) Null set (d) finite set
3. A collection of well-defined distinct objects is called
   (a) subset (b) power set
   (c) set (d) none of these
4. If $A \subseteq B$ then $A \cup B$ is equal to
   (a) A (b) B
   ϕ
5. A set $Q = \frac{a}{b} | a, b \in Z \wedge b \neq 0$ is called a set of
   (a) Whole numbers
   (b) Natural numbers
   (c) Irrational numbers
   (d) Rational numbers
6. If $A \subseteq B$ then $A - B$ is equal to
   (a) A (b) B
   (c) ϕ (d) None of these
7. If A and B are disjoint sets, then $A \cup B$ is equal to
   (a) A (b) B
   (c) ϕ (d) $B \cup A$
8. The number of elements in power set $\{1, 2, 3\}$ is
   (a) 4 (b) 6
   (c) 8 (d) 9
9. A set with no element is called
   (a) subset (b) empty set
   (c) singleton set (d) super set
10. If $A \subseteq B$ then $A \cap B$ is equal to
    (a) A (b) B
    (c) ϕ (d) None of these
11. The set having only one element is called
    (a) Null set (b) power set
    (c) singleton set (d) subset
12. The relation {(1,2),(2,3),(3,3)(3,4)} is
    (a) onto function
    (b) into function
    (c) not a function
    (d) one-one function
13. If $A \subseteq B$ and $B \subseteq A$, then
    (a) A = B (b) $A \neq B$
    (c) $A \cap B = \phi$ (d) $A \cup B = \phi$
14. Power set of an empty set is
    (a) ϕ (b) $\{a\}$
    (c) $\{\phi, \{a\}\}$ (d) $\{\phi\}$
15. If number of elements in set A is 3 and in set B is 4, then number of elements in A×B is
    (a) 3 (b) 4
    (c) 12 (d) 7
16. Point (-1,4) lies in the quadrant
    (a) I (b) II
17. The domain of R ={(0,2),(2,3),(3,3)(3,4)} is
    (a) {0,3,4} (b) {0,2,3}
    (c) {0,2,4} (d) {2,3,4}
18. The point (–5 , –7) lies in ... quadrant
    (a) I (b) II
    (c) III (d) IV

19. If number of elements in se A is 3 and in set B is 2, then number of binary relations in A×B is
(a) 2^3 (b) 2^6
(c) 2^8 (d) 2^2

20. $(A\cup B)\cup C$ is equal to
(a) $A\cap(B\cup C)$ (b) $(A\cup B)\cap C)$
(c) $A\cup(B\cup C)$ (d) $A\cap(B\cap C)$

21. If $A\cap B=\phi$, then set A and B are ....sets.
(a) sub (b) over lapping
(c) disjoint (d) power

22. $A\cup(B\cap C)$ is equal to
(a) $(A\cup B)\cap(A\cup C)$
(b) $A\cap(B\cap C)$
(c) $(A\cap B)\cap(A\cap C)$
(d) $A\cup(B\cup C)$

23. The range of {(a,a), (b,b), (c,c)} is......
(a) {a,b} (b) {a,b,c}
(c) {a} (d) ϕ

24. The complement of ϕ is ......
(a) U (b) ϕ
(c) impossible (d) union

25. $A\cap A^c$ = ...
(a) U (b) A
(c) A^c (d) ϕ

26. Venn diagram was first used by.......
(a) John Venn
(b) Netwon
(c) Arthur Cayley
(d) John Napier

27. The set { x | x ∈ A and x ∉ B } is ......
(a) $A\cup B$ (b) $A\cap B$
(c) A – B (d) B – A

28. The Range of R ={(1,3),(2,2),(3,1)(4,4)} is
(a) {1,2,4} (b) {3,2,4}
(c) {1,2,3,4} (d) {1,3,4}

29. $N\cup W$ = ...
(a) ϕ (b) {0}
(c) N (d) W

30. y co-ordinate of every pint on x – axis is..
(a) +ve (b) –ve
(c) zero (d) 1

31. By definition, which of the following is a set?
(a) {a,b,c,a} (b) {1,2,3,2}
{ ,m,n,o} {0,1,2,3,1}

32. The domain of {(a,b), (b,c), (c,d)} is...
(a) {a,b,c} (b) {b,c,d}
(c) {a,b} (d) {a,b,c,d,}

33. The complement of U is ....
(a) U (b) ϕ
(c) impossible (d) union

34. $A\cup A^c$ = ...
(a) U (b) A
(c) A^c (d) ϕ

35. Which of the following is true?
(a) $P\subseteq N\subseteq Z\subseteq W$
(b) $P\subseteq N\subseteq W\subseteq Z$
(c) $P\subseteq W\subseteq N\subseteq Z$
(d) $P\subseteq Z\subseteq N\subseteq W$

36. W – N = ...
(a) ϕ (b) {0}
(c) N (d) W

37. N – W = ...
(a) ϕ (b) {0}
(c) N (d) W

38. The relation {(a,b),(b,c),(a,d),} is....
(a) a function (b) not a function
(c) range (d) domain 05(091)

39. x co-ordinate of every pint on y – axis is..
(a) +ve (b) –ve
(c) zero (d) 1

40. Which of the following is true?
(a) $W \subseteq N$
(b) $Z \subseteq W$
(c) $N \subseteq P$
(d) $P \subseteq W$

41. A subset of $A \times A$ is called……in A.
(a) set (b) relation
(c) function (d) into function

42. Which of the following is true?
(a) N and $W \subseteq Z$
(b) P and $O \subseteq W$
(c) O and $E \subseteq W$
(d) P and $E \subseteq N$

43. If $x \in A$ and $x \in B$, then $\{x\}$ is equal to
(a) $A - B$ (b) A^c
(c) $A \cap B$ (d) B^c

44. The point (4, −6) lies in …. Quadrant
(a) I (b) II
(c) III (d) IV

45. If $f: A \to B$ and range of $f \neq B$, then f is an ………..
(a) into function
(b) onto function
(c) bijective function
(d) function

46. If $f: A \to B$ and range of $f = B$, then f is an……
(a) into function
(b) onto function
(c) bijective function
(d) function

47. If set A has all its elements common with set B then set A is called….set.
(a) sub (b) overlapping
(c) disjoint (d) super

48. $O \cup E = $ ……….
(a) ϕ (b) O
(c) E (d) Z

49. Which of the following is commutative law?
(a) $A \cup (B \cup C) = (A \cup B) \cup C$
(b) $A \cap (B \cap C) = (A \cap B) \cap C$
(c) $A \cap B = B \cap A$
(d) $)A \cup B(^C = A^C \cap B^C$

50. Which of the following is associative law of intersection?
(a) $A \cup (B \cup C) = (A \cup B) \cup C$
(b) $A \cap (B \cap C) = (A \cap B) \cap C$
(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

51. Which of the following is distributive property of intersection over union?
(a) $A \cup (B \cup C) = A \cup (B \cup C)$
(b) $A \cap (B \cap C) = (A \cap B) \cap C$
(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

52. $N \cap W = $ ………
(a) ϕ (b) $\{0\}$
(c) N (d) W

53. If A is subset of U, then $(A^C)^C = $ ……
(a) A (b) A^c
(c) ϕ^C (d) U^c

54. If two sets have some elements common but not all are called….sets.
(a) sub (b) overlapping
(c) disjoint (d) super

55. Which of the following is De-Morgan's law?
(a) $(A \cup B) \cup C = A \cup (B \cup C)$
(b) $)A \cap B(^C = A^C \cup B^C$
(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

56. Which of the following is associative law of union?
(a) $A\cup(B\cup C)=(A\cup B)\cup C$
(b) $A\cap(B\cap C)=(A\cap B)\cap C$
(c) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$
(d) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

57. $O-E=$..........
(a) ϕ (b) O
(c) E (d) Z

58. If union and intersection of two sets are equal then sets are ......sets.
(a) disjoint (b) overlapping
(c) equal (d) super

59. Which of the following is complete description of Real numbers?
(a) $N\cup W=R$ (b) $O\cup E=R$
(c) $P\cup Q=R$ (d) $Q\cup Q'=R$

60. $E-O=$...........
(a) ϕ (b) O
(c) E (d) Z

61. Two sets having no common element, are called .... sets.
(a) subet (b) overlapping
(c) disjoint (d) super

62. If $x\in U$ and $x\notin A$, then $\{x\}$ is equal to.............
(a) U^C (b) A^C
(c) ϕ^C (d) $A-U$

63. $O\cap E=$...........
(a) ϕ (b) O
(c) E (d) Z

64. A and A^C are ....sets.
(a) universal (b) overlapping
(c) disjoint (d) super

65. Which of the following is distributive property of union over intersection?
(a) $A\cup(B\cup C)=A\cup(B\cup C)$
(b) $A\cap(B\cap C)=(A\cap B)\cap C$
(c) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$
(d) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

66. If $x\in A$ and $x\notin B$, then { x } is equal to..........
(a) $A-B$ (b) $B-A$
(c) $A\cap B$ (d) A^c

ANSWER KEY

1	c	**2**	d	**3**	c	**4**	b	**5**	d	**6**	c	**7**	d	**8**	c
9	b	**10**	a	**11**	c	**12**	c	**13**	a	**14**	d	**15**	c	**16**	b
17	b	**18**	c	**19**	b	**20**	c	**21**	c	**22**	a	**23**	b	**24**	a
25	d	**26**	a	**27**	c	**28**	c	**29**	d	**30**	c	**31**	c	**32**	a
33	b	**34**	a	**35**	b	**36**	b	**37**	a	**38**	b	**39**	c	**40**	d
41	b	**42**	a	**43**	c	**44**	d	**45**	a	**46**	b	**47**	a	**48**	d
49	c	**50**	b	**51**	d	**52**	c	**53**	a	**54**	b	**55**	b	**56**	a
57	b	**58**	c	**59**	d	**60**	c	**61**	c	**62**	b	**63**	a	**64**	c
65	c	**66**	a												

Q.2 Write short answers of the following questions.

(i) Define a subset and give one example.

Subset: If A and B are two sets and every element of A is an element of B then set A is called subset of set B. It is denoted by $A \subseteq B$.

Example: $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$

As all elements of Set A are also present in Set B. Therefore $A \subseteq B$.

(ii) Write all subsets of the set {a, b}

Solution:

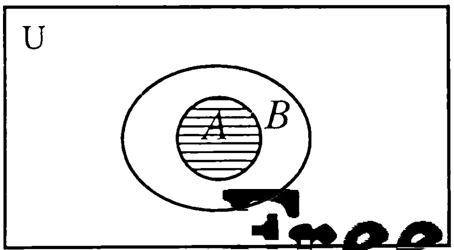
All subsets ($2^n = 2^2 = 4$)

$\{\ \}, \{a\}, \{b\}, \{a, b\}$

(iii) Show $A \cap B$ by Venn Diagram when $A \subseteq B$.

Solution:

If $A \subseteq B$ then $A \cap B = A$



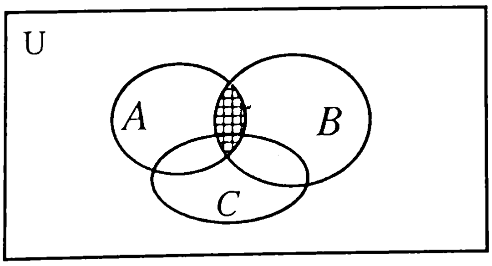
$A \cap B$

Horizontal line segments show $A \cap B$

(iv) Show by Venn Diagram $A \cap (B \cup C)$

Solution:

$A \cap (B \cup C)$ by Venn diagram



- Horizontal line segments and squares show $B \cup C$.
- $A \cap (B \cup C)$ is shown by squares.

(v) Define intersection of two sets.

Intersection of two sets:

The intersection of two sets A and B, written as $A \cap B$ (read as A intersection B) is the set consisting of all the common elements of A and B.

(vi) Define a function.

Function: Suppose A and B are two non-empty sets, then relation $f : A \to B$ is called a function if

(i) Dom f = A

(ii) Every $x \in A$ appears in one and only and ordered pair in f.

(vii) Define an one – one function.

One – one function:

A function $f : A \to B$ is called one – one function, if all distinct elements of A have distinct images in B, i.e:

$f(x_1) = f(x_2) \quad x_1 = x_2 \in A$ or

$\forall\ x_1 \neq x_2 \in A \quad f(x_1) \neq f(x_2)$

(viii) Define an onto function.

Onto function:

A function $f : A \to B$ is called an onto function, if every element of set B is an image of at least one element of set A.

i.e. Range of f = B

(ix) Define a Bijective function.

Bijective function:

A function $f : A \to B$ is called bijective function iff function f is one-one and onto.

(x) Write De Morgan's law.

De Morgan's Law:

If two sets A and B are the sub sets of U then De-Morgan's laws are expressed as

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Q.3 Fill in the Blanks

i. If $A \subseteq B$ then $A \cup B =$ ______.

ii. If $A \cap B = \phi$ then A and B are ______.

iii. If $A \subseteq B$ *and* $B \subseteq A$ then________.

iv. $A \cap (B \cup C) =$ ______ .

v. $A \cup (B \cap C) =$ ______ .

vi. The complement of U is _________.

vii. The complement of ϕ is________.

viii. $A \cap A^c =$ ______.

ix. $A \cup A^c =$ ______.

x. The set $\{x | x \in A \text{ and } x \notin B\} =$ _____.

xi. The point (-5, -7) lies in _______ quadrant.

xii. The point (4,-6) lies in ______quadrant.

xiii. The y co-ordinate of every point is __________on x-axis.

xiv. The x co-ordinate of every point is __________on y-axis.

xv. The domain of {(a,b), (b,c), (c,d)} is _______.

xvi. The range of {(a,a), (b,b), (c,c)} is _______.

xvii. Venn-diagram was first used by_______.

xviii. A subset of A x A is called the _____in A.

xix. If $f : A \longrightarrow B$ and range of f=B, then f is an ______function.

xx. The relation of {(a,b), (b,c), (a,d)} is _______ function.

Answers:

(i)	B	(ii)	Disjoint sets
(iii)	A = B	(iv)	$(A \cap B) \cup (A \cap C)$
(v)	$(A \cup B) \cap (A \cup C)$		
(vi)	ϕ	(vii)	U
(viii)	ϕ	(ix)	U
(x)	$A \setminus B$	(xi)	IIIrd
(xii)	IVth		
(xiii)	Zero	(xiv)	Zero
(xv)	{a, b, c}		
(xvi)	{a, b, c}	(xvii)	John venn
(xviii)	Binary relation		
(xix)	onto	(xx)	not

EXERCISE 6.1

Q. No. 1 The following data shows the number of members in various families. Construct frequency distribution. Also find cumulative frequencies.

9, 11, 4, 5, 6, 8, 4, 3, 7, 8, 5, 5, 8, 3, 4, 9, 12, 8, 9, 10, 6, 7, 7, 11, 4, 4, 8, 4, 3, 2, 7, 9, 10, 9, 7, 6, 9, 5, 7

Solution:

Let number of family members = X

X: 9, 11, 4, 5, 6, 8, 4, 3, 7, 8, 5, 5, 8, 3, 4, 9, 12, 8, 9, 10, 6, 7, 7, 11, 4, 4, 8, 4, 3, 2, 7, 9, 10, 9, 7, 6, 9, 5, 7

Minimum Value = $X_{min} = 2$

Maximum value = $X_{max} = 12$

Number of members = n = 39

X	Tally Marks	f	Cumulative Frequency (C.F)
2	I	1	1
3	III	3	1+3=4
4	~~IIII~~ I	6	4+6=10
5	IIII	4	10+4=14
6	III	3	14+3=17
7	~~IIII~~ I	6	17+6=23
8	~~IIII~~	5	23+5=28
9	~~IIII~~ I	6	28+6=34
10	II	2	34+2=36
11	II	2	36+2=38
12	I	1	38+1=39

Q. 2 The following data has been obtained after weighting 40 students of class V.

Make a frequency distribution taking class interval size as 5. Also find the class boundaries and midpoints.

34, 26, 33, 32, 24, 21, 37, 40, 41, 28, 28, 31, 33, 34, 37, 23, 27, 31, 31, 36, 29, 35, 36, 37, 38, 22, 27, 28, 29, 31, 35, 35, 40, 21, 32, 33, 27, 29, 30, 23

Also make a less than cumulative frequency distribution (Hint: Make Classes 20-24, 25-29,..........,)

Solution:

Let weights of students = X

X: 34, 26, 33, 32, 24, 21, 37, 40, 41, 28, 28, 31 33, 34, 37, 23, 27, 31, 31, 36, 29, 35, 36, 37, 38, 22, 27, 28, 29, 31, 35, 35, 40, 21, 32, 33, 27, 29, 30, 23

Minimum value = $X_{min} = 21$

Maximum value = $X_{max} = 41$

Number of observations = n = 40

Size of class intervals = 5

Classes	Tally Marks	f	Mid points (x)	Class Boundaries
20–24	𝍸 I	6	22	19.5–24.5
25–29	𝍸 𝍸	10	27	24.5–29.5
30–34	𝍸 𝍸 II	12	32	29.5–34.5
35–39	𝍸 IIII	9	34	34.5–39.5
40–44	III	3	39	39.5–44.5
	$\sum f = 40$			

Less than Cumulative Frequency Distribution

C.B	f	C.F	Class Boundaries	F
14.5–19.5	0	0	Less than 19.5	0
19.5–24.5	6	0+6=6	Less than 24.5	6
24.5–29.5	10	6+10=16	Less than 29.5	16
29.5–34.5	12	16+12=28	Less than 34.5	28
34.5–39.5	9	28+9=37	Less than 39.5	37
39.5–44.5	3	37+3=40	Less than 44.5	40

Q. 3 From the following data representing the salaries of 30 teachers of a school. Make a frequency distribution taking class interval size of Rs. 100, 450, 500, 550, 580, 670, 1200, 1150, 1120, 950, 1130, 1230, 890, 780, 760, 670, 880, 890, 1050, 980, 970, 1020, 1130, 1220, 760, 690, 710, 750, 1120, 760, 1240 06(003)

(**Hints:** Make classes 450—549, 550–649,..).

Solution: Let Salaries of teachers are X.

X: 450, 500, 550, 580, 670, 1200, 1150, 1120, 950, 1130, 1230, 890, 780, 760, 670, 880, 890, 1050, 980, 970, 1020, 1130, 1220, 760, 690, 710, 750, 1120, 760, 1240

Minimum value = X_{min} = 450

Maximum value = X_{max}= 1240

Number of observations = n = 30

Size of class intervals = Rs.100

Classes	Tally Marks	Frequency (f)
450–549	II	2
550–649	II	2
650–749	IIII	4
750–849	𝍸	5
850–949	III	3
950–1049	IIII	4
1050–1149	𝍸	5
1150–1249	𝍸	5
		Σf = 30

Q.4 The following data shows the daily load shedding duration in hours in 30 localities of a certain city. Make a frequency distribution of the load shedding duration taking 2 hours as class intervals size and answer the following questions.

6, 12, 5, 7, 8, 3, 6, 7, 10, 2, 14, 11, 12, 8, 6, 8, 9, 7, 11, 6, 9, 12, 13, 10, 14, 7, 6, 10, 11, 14,12.

(a) Find the most frequent load shedding hours

(b) Find least load shedding intervals

(Hint: Make classes 2–3,4–5,6–7,.....)

Solution:

Let load shedding hours are X.

X: 6, 12, 5, 7, 8, 3, 6, 7, 10, 2, 14, 11, 12, 8, 6, 8, 9, 7, 11, 6, 9, 12, 13, 10, 14, 7, 6, 10, 11, 14, 12

Minimum value = X_{min} = 2

Maximum value = X_{max}= 14

Classes	Tally marks	Frequency (f)
2–3	II	2
4–5	I	1
6–7	𝍸 IIII	9
8–9	𝍸	5
10–11	𝍸 I	6
12–13	𝍸	5
14–15	III	3
Total		**Σf = 31**

(a) The most frequent load shedding hours are (6–7)

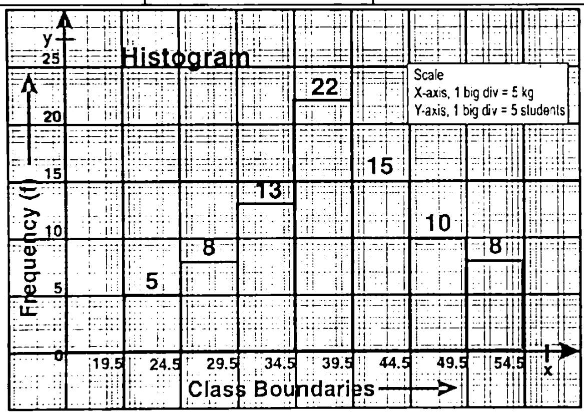
(b) The least load shedding interval is (4–5)

Q.5 Construct a Histogram and frequency Polygon for the following data showing weights of students in Kg.

Weights	Frequency / No. of students
20–24	5
25–29	8
30–34	13
35–39	22
40–44	15
45–49	10
50–54	8

(i) Solution: Histogram

Weights	Frequency (f)	Class Boundaries
20–24	5	19.5–24.5
25–29	8	24.5–29.5
30–34	13	29.5–34.5
35–39	22	34.5–39.5
40–44	15	39.5–44.5
45–49	10	44.5–49.5
50–54	8	49.5–54.5

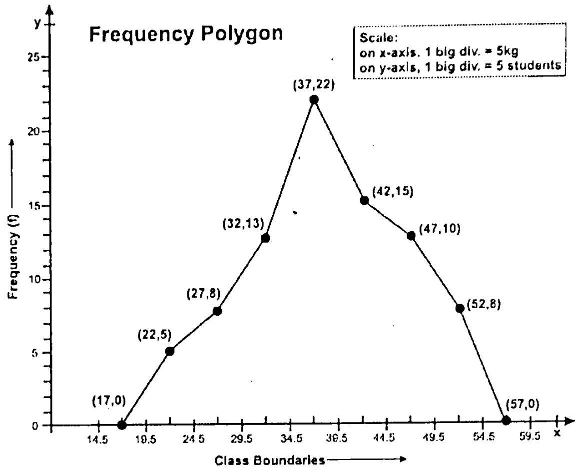


(ii) Frequency Polygon:

Weights (kg)	(f)	Class Boundaries	Midpoint
15–19	0	14.5–19.5	17
20–24	5	19.5–24.5	22
25–29	8	24.5–29.5	27
30–34	13	29.5–34.5	32
35–39	22	34.5–39.5	37
40–44	15	39.5–44.5	42
45–49	10	44.5–49.5	47
50–54	8	49.5–54.5	52
55–59	0	54.5–59.5	57

Note: Two additional groups with same size of class interval are taken. One before the very first group and the second after the very last group. These two groups will have frequency "0".

Frequency Polygon:



EXERCISE 6.2

Q. 1. What do you mean by measures of central tendency?

Ans. The Specific value of the variable around which the majority of the observations of the data tend to concentrate is called average or central value. The measures or techniques that are used to determine the central value are called measure of central tendency.

Examples.

i. Arithmetic mean
ii. Median
iii. Mode
iv. Geometric mean
v. Harmonic mean

Q.2. Define Arithmetic mean, geometric mean, Harmonic mean, mode and median.

Answer

(i) Arithmetic Mean:

Arithmetic mean is a measure that determines a value of the variable under study by dividing the sum of all values of the variable by their number of observations. We denote Arithmetic mean by $\overline{X}$. In symbols it is defined as.

$$\overline{X} = \frac{\sum X}{n} = \frac{\text{Sum of all values of observations}}{\text{No. of observations}}$$

(ii) Mode:

The most frequent occurring observation in the data or the observation that occurs maximum number of times in the given data is called mode

Mode = the most frequent or common value in the ungrouped data.

(iii) Median:

Median is the middle most observation in an arranged data set. It divides the data set into two equal parts. Median is

represented by $\tilde{x}$. For ungrouped data median is calculated by the following formula.

- **When n is odd**

$$\tilde{x}=\left(\frac{n+1}{2}\right)\text{th term in arranged data}$$

- **When n is even**

$$\tilde{x}=\frac{1}{2}\left[\frac{n}{2}\text{th term}+\frac{n+2}{2}\text{ th term}\right]$$

(iv) Geometric Mean:

Geometric mean of a variable x is the n^{th} positive root of the product of the $x_1,x_2,x_3,\ldots\ldots\ldots\ldots, x_n$ observation. In symbols we write, (Basic Formula)

$$G.M=(X_1.X_2.X_3.\ldots\ldots.X_n)^{1/n}$$

The Geometric Mean can also be calculated by using logarithm.

- **For Ungrouped data:**

$$G.M = Anti\log\left(\frac{\sum \log X}{n}\right)$$

- **For Grouped data:**

$$G.M = Anti\log\left(\frac{\sum f\log X}{\sum f}\right)$$

(v) Harmonic Mean:

Harmonic mean refers to the value obtained by reciprocating the mean of the reciprocal of x_1, $x_2,x_3,\ldots\ldots,x_n$ observations. In symbols,

- **For Ungrouped data:**

$$H.M=\frac{n}{\sum\frac{1}{X}}$$

- **For Grouped data:**

$$H.M=\frac{n}{\sum\frac{f}{X}}$$

Q.3. Find arithmetic mean by direct method for the following set of data.

(i) 12, 14, 17, 20, 24, 29, 35, 45
(ii) 200, 225, 350, 375, 270, 320, 290

(i) Solution:

Arithmetic mean by direct formula

X = 12, 14, 17, 20, 24, 29, 35, 45

$$\overline{X}=\frac{\sum X}{n}$$

$$\overline{X}=\frac{12+14+17+20+24+29+35+45}{8}$$

$$\overline{X}=\frac{196}{8}$$

$$\overline{X}=24.5$$

(ii) Solution:

Arithmetic mean by direct formula

x = 200, 225, 350, 375, 270, 320, 290

$$\overline{X}=\frac{\sum X}{n}$$

$$\overline{X}=\frac{200+225+350+375+270+320+290}{7}$$

$$\overline{X}=\frac{2030}{7}$$

$$\overline{X}=290$$

Q.4. For each of the data in Q. No. 3. Compute arithmetic mean using indirect method.

(i) Solution:

Arithmetic Mean by indirect (short) formula

X = 12, 14, 17, 20, 24, 29, 35, 45

Let Assumed mean = A = 20

Number of Observations = n = 8

X	D = X – A
12	12 – 20 = –8
14	14 – 20= –6
17	17 – 20 = –3
20	20 – 20 = 0
24	24 – 20 = 4
29	29 – 20 = 9
35	35 – 20 = 15
45	45 – 20 = 25
	$\sum D = 36$

$$\overline{X}=A+\frac{\sum D}{n}$$

$$\overline{X}=20+\frac{36}{8}$$

$$\overline{X}=20+4.5$$

$$\overline{X}=24.5$$

(ii) Solution:

Arithmetic mean by indirect (short) formula

Let assumed mean = A = 225

Number of observations = n = 7

X	D = X – A
200	200 – 225 = –25
225	225 – 225 = 0
270	270 – 225 = 45
290	290 – 225 = 65
320	320 – 225 = 95
350	350 – 225 = 125
375	375 – 225 = 150
	$\sum D = 455$

$$\overline{X} = A + \frac{\sum D}{n}$$

$$\overline{X} = 225 + \frac{455}{7}$$

$$\overline{X} = 225 + 65$$

$$\overline{X} = 290$$

Q.5. The marks obtained by students of Class XI in mathematics are given below. Compute arithmetic mean by direct and indirect methods.

Classes /Groups	Frequency
0 – 9	2
10 – 19	10
20 – 29	5
30 – 39	9
40 – 49	6
50 – 59	7
60 – 69	1

Solution:

(i) Arithmetic mean by Direct Method

Classes / Groups	f	Class Marks or Mid Points (x)	fx
0 – 9	2	4.5	9
10 – 19	10	14.5	145
20 – 29	5	24.5	122.5
30 – 39	9	34.5	310.5
40 – 49	6	44.5	267
50 – 59	7	54.5	381.5
60 – 69	1	64.5	64.5
Total	$\sum f = 40$		$\sum fx = 1300$

$$\overline{X} = \frac{\sum fx}{\sum f} = \frac{1300}{40}$$

$$\overline{X} = 32.5$$

$\overline{X}$ = 32.5 Marks

(ii) Arithmetic mean by indirect (short) method

Let assumed mean = A= 34.5

Class /Groups	f	Class Mark (x)	D= X–A	fD
0 – 9	2	4.5	4.5 – 34.5 = –30	–60
10 – 19	10	14.5	14.5 – 34.5 = –20	–200
20 – 29	5	24.5	24.5 – 34.5= –10	–50
30 – 39	9	34.5	34.5 – 34.5 = 0	0
40 – 49	6	44.5	44.5 – 34.5 = 10	60
50 – 59	7	54.5	54.5 – 34.5 = 20	140
60 – 69	1	64.5	64.5 – 34.5 = 30	30
Total	$\sum f = 40$		$\sum fD = -80$	

$$\overline{X} = A + \frac{\sum fD}{\sum f}$$

$$\overline{X} = 34.5 + \frac{-80}{40}$$

$$\overline{X} = 34.5 + (-2)$$

$$\overline{X} = 34.5 - 2$$

$\overline{X} = 32.5$ Marks

Q.6. The following data relates to the ages of children in a school. Compute the mean age by direct and short cut method taking any provincial mean. (Hint: Take A= 8)

Class Limits	Frequency
4 – 6	10
7 – 9	20
10 – 12	13
13 – 15	7
Total	50

Also compute Geometric mean and Harmonic mean

Solution:

(i) Arithmetic mean by direct formula.

Class Limits	Frequency (f)	Class Marks (x)	fx
4 – 6	10	5	50
7 – 9	20	8	160
10 – 12	13	11	143
13 – 15	7	14	98
Total	$\sum f = 50$		$\sum fx = 451$

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{451}{50}$$

$$\bar{X} = 9.02$$

(ii) Arithmetic Mean by Indirect (Short) method

Here A = 8 Assumed mean (A) = 8

Class Limits	f	Class Marks (x)	D= x–A	fD
4 – 6	10	5	5 – 8= –3	– 30
7 – 9	20	8	8 – 8= 0	0
10 – 12	13	11	11 – 8=3	39
13 – 15	7	14	14 – 8=6	42
Total	50			$\sum fD = 51$

$$\bar{X} = A + \frac{\sum fD}{\sum f} \Rightarrow \bar{X} = 8 + \frac{51}{50}$$

$$\bar{X} = 8 + 1.02 \Rightarrow \bar{X} = 9.02$$

(iii) Geometric Mean:

Class Limit	f	Marks (x)	log x	f log x
4 – 6	10	5	0.6990	6.990
7 – 9	20	8	0.9031	18.062
10 – 12	13	11	1.0414	13.5382
13 – 15	7	14	1.1461	8.0227
Total	50		$\sum f \log x = 46.6129$	

$$G.M = \text{Anti log}\left(\frac{\sum f \log x}{\sum f}\right)$$

$$G.M = \text{Anti log}\left(\frac{46.6129}{50}\right)$$

$$G.M = \text{Anti log}(0.932258)$$

or $$G.M = \text{Anti log}(0.9323)$$

$$G.M = 8.557$$

(iv) Harmonic Mean:

Class Limits	f	Class Marks (x)	f/x
4–6	10	5	2
7–9	20	8	2.5
10–12	13	11	1.1818
13–15	7	14	0.5
Total	$\sum f = 50$		$\sum f/x = 6.1818$

$$H.M = \frac{\sum f}{\sum\left(\frac{f}{x}\right)}$$

$$H.M = \frac{50}{6.1818} = 8.088$$

or $H.M = 8.089$

Q. 7 The following data shows the number of children in various families. Find mode and median.

9, 11, 4, 5, 6, 8, 4, 3, 7, 8, 5, 5, 8, 3, 4, 9, 12, 8, 9, 10, 6, 7, 7, 11, 4, 4, 8, 4, 3, 2, 7, 9, 10, 9, 7, 6, 9, 5

Solution:

Arranged Data:

2, 3, 3, 3, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 9, 9, 9, 9, 9, 9, 10, 10, 11, 11, 12

Mode:

From the arranged data we see that 4 and 9 both are repeated 6 times i.e. the most frequent values are 4 and 9. So,

Mode = 4 and 9

Median:

As n is even (n=38). So,

$$\text{Median} = \frac{1}{2}\left[\frac{n}{2}\text{th item} + \frac{n+2}{2}\text{th item}\right]$$

$$\text{Median} = \frac{1}{2}\left[\frac{38}{2}\text{th item} + \frac{38+2}{2}\text{th item}\right]$$

$$\text{Median} = \frac{1}{2}[19\text{th item} + 20\text{th item}]$$

$$\text{Median} = \frac{1}{2}[7+7]$$

$$\text{Median} = \frac{1}{2}[14]$$

$$\text{Median} = 7$$

Q. 8 Find Modal number of heads for the following distribution showing the number of heads when 5 coins are tossed. Also determine median.

X (number of heads)	Frequency (f)
1	3
2	8
3	5
4	3
5	1

Solution: we first make cumulative frequency column as given below:

X	Frequency	Cumulative frequency
1	3	3
2	8	3 + 8 = 11
3	5	11 + 5 =16
4	3	16 + 3 =19
5	1	19 + 1 = 20

(i) Mode: As most frequent number is 2 i.e., 2 is repeated 8 times. So, Mode = 2 Ans.

(ii) Median:

Median = the class containing $\left[\frac{n}{2}\right]^{th}$ observation

Median = the class containing $\left[\frac{20}{2}\right]^{th}$ observation

Median = the class containing $(10)^{th}$ observation

Median = 2

Q. 9 The following frequency distribution shows the weights of boys in kilogram. Compute mean, median, mode.

Class Interval	Frequency
1 – 3	2
4 – 6	3
7 – 9	5
10 – 12	4
13 – 15	6
16 – 18	2
19 – 21	1

Solution:

(i) Arithmetic Mean

Class Intervals	Frequency (f)	Class marks (x)	fx
1 – 3	2	2	4
4 – 6	3	5	15
7 – 9	5	8	40
10 – 12	4	11	44
13 – 15	6	14	84
16 – 18	2	17	34
19 – 21	1	20	20
	$\Sigma f = 23$		**$\Sigma fx = 241$**

$$\overline{X} = \frac{\Sigma fx}{\Sigma f} = \frac{241}{23} = 10.478kg$$

(ii) Median:

Class Intervals	(f)	Class Boundaries	Cumulative Frequency
1 – 3	2	0.5 – 3.5	2
4 – 6	3	3.5 – 6.5	2 + 3 = 5
7 – 9	5	6.5 – 9.5	5 + 5 = 10
10 – 12	**4**	**9.5 – 12.5**	**10 + 4 = 14**
13 – 15	6	12.5 – 15.5	14 + 6 = 20
16 – 18	2	15.5 – 18.5	20 + 2 = 22
19 – 21	1	18.5 – 21.5	22 + 1 = 23

Median = class containing

$$\left(\frac{n}{2}\right)^{th} = \left(\frac{23}{2}\right)^{th} = 11.5^{th} \text{ item}$$

As 11.5^{th} item is present in the group (9.5 – 12.5) so median group is (9.5 – 12.5).

$$\text{Median } = l + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

$$\text{Median} = 9.5 + \frac{3}{4}\left(\frac{23}{2} - 10\right)$$

$$\text{Median} = 9.5 + \frac{3}{4}(11.5 - 10)$$

$$\text{Median} = 9.5 + \frac{3}{4}(1.5)$$

$$\text{Median} = 9.5 + \frac{4.5}{4}$$

Median = 9.5 + 1.125

Median = 10.625 kg

(iii) Mode:

Class Intervals	Frequency	Class Boundaries
1 – 3	2	0.5 – 3.5
4 – 6	3	3.5 – 6.5
7 – 9	5	6.5 – 9.5
10 – 12	$f_1 \rightarrow$ 4	9.5 – 12.5
13 – 15	$f_m \rightarrow$ 6	12.5 – 15.5
16 – 18	$f_2 \rightarrow$ 2	15.5 – 18.5
19 – 21	1	18.5 – 21.5

As the group (12.5–15.5) has maximum frequency (i.e. 6) so modal group is (12.5–15.5).

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

$$\text{Mode} = 12.5 + \frac{(6-4)}{2(6)-4-2} \times 3$$

$$\text{Mode} = 12.5 + \frac{(2\times 3)}{12-6}$$

$$\text{Mode} = 12.5 + \frac{6}{6}$$

$$\text{Mode} = 12.5 + 1$$

$$\text{Mode} = 13.5 \text{ kg}$$

Q.10 A student obtained the following marks at a certain examination: English 73, Urdu 82, Mathematics 80, History 67 and Science 62.

(i) If the weights accorded these marks are 4, 3, 3, 2 and 2, respectively, what is an appropriate average mark?

(ii) What is the average mark if equal weights are used?

Solution:

Subjects	Marks (x)	Weights (w)	Wx
English	73	4	292
Urdu	82	3	246
Maths	80	3	240
History	67	2	134
Science	62	2	124
		$\sum w = 14$	$\sum wx = 1036$

$$\text{Weighted mean } \overline{X}_w = \frac{\sum wx}{\sum w}$$

$$\overline{X}_w = \frac{1036}{14}$$

$$\overline{X}_w = 74 \text{ marks}$$

(ii) Solution: Let equal weight is 2, then

Subjects	Marks	Weight	Wx
English	73	2	146
Urdu	82	2	164
Maths	80	2	160
History	67	2	134
Science	62	2	124
		$\sum w = 10$	$\sum wx = 728$

$$\text{Equal weighted mean} = \frac{\sum wx}{\sum w} = \frac{728}{10}$$

$$= 72.8 \text{ marks}$$

Q.11 On a vacation trip a family bought 21.3 liters of petrol at 39.90 rupees per liter, 18.7 liters at 42.90 rupees per liter, and 23.5 liters at 40.90 rupees per liter. Find the mean price paid per liter.

Solution:

Liters (w)	Price per liters (x)	Payments (Rs) (wx)
21.3	39.90	849.87
18.7	42.90	802.23
23.5	40.90	961.15
Total liters Σw = 63.5		**Total Payments Σwx = 2613.25**

$$\text{Average price} = \frac{\text{Total payment}}{\text{Total liters}}$$

$$\overline{X}_w = \frac{\sum wx}{\sum w}$$

$$\overline{X}_w = \frac{2613.25}{63.5}$$

$$\overline{X}_w = 41.15 \text{ rupees per liter}$$

Q. 12 Calculate simple moving average of 3 year from the following data.

Years	Values
2001	102
2002	108
2003	130
2004	140
2005	158
2006	180
2007	196
2008	210
2009	220
2010	230

Solution:

Years	Values	3 years Moving Total	3 years Moving Average
2001	102	–	–
2002	108	340	$\frac{340}{3}=113.33$
2003	130	378	$\frac{378}{3}=126$
2004	140	428	$\frac{428}{3}=142.66$
2005	158	478	$\frac{478}{3}=159.33$
2006	180	534	$\frac{534}{3}=178$
2007	196	586	$\frac{586}{3}=195.33$
2008	210	626	$\frac{626}{3}=208.67$
2009	220	660	$\frac{660}{3}=220$
2010	230	–	–

Q.13 Determine graphically for the following data and check your answer by using formulae.

(i) Median and Quartiles using cumulative frequency polygon.

(ii) Mode using Histogram

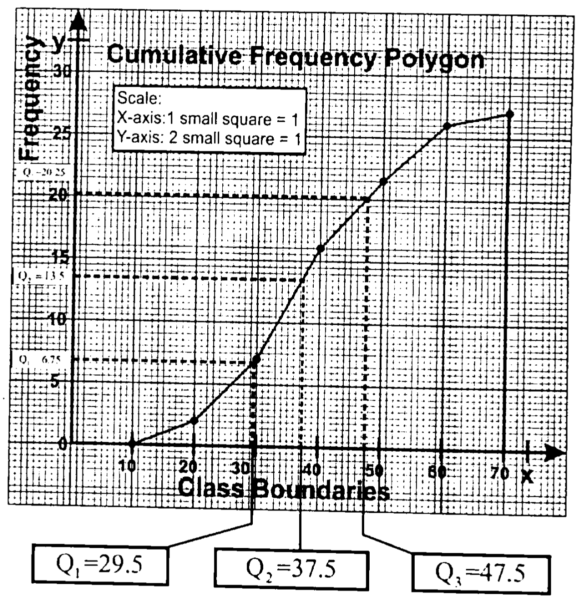
Class Boundaries	Frequency
10 – 20	2
20 – 30	5
30 – 40	9
40 – 50	6
50 – 60	4
60 – 70	1

Solution:

(i) Less than Frequency Table:

Class Boundaries	C. F
Less than 10	0
Less than 20	2
Less than 30	7
Less than 40	16
Less than 50	22
Less than 60	26
Less than 70	27

Cumulative Frequency Polygon:



Finding Q_1: Take the value of Q_1 on the x-axis against the $\frac{27}{4}$th observation (6.75) on the y-axis which is 29.5.

Finding Q_2: Take the value of Q_2 on the x-axis against the $2\left(\frac{27}{4}\right)$th observation (13.5) on the y-axis which is 37.5.

Finding Q_3: Take the value of Q_3 on the x-axis against the $3\left(\frac{27}{4}\right)$th observation (20.25) on the y-axis which is 47.5.

Verification by Formulae:

Class Boundaries	f	C. F
10–20	2	2
20–30	5	2+5=7
30–40	9	7+9=16
40–50	6	16+6=22
50–60	4	22+4=26
60–70	1	26+1=27

Verification of Q_1 by Formula:

Q_1 Class = Class Containing $(\frac{n}{4})$th item

Q_1 Class = Class Containing $(\frac{27}{4})$th item

Q_1 Class = Class Containing (6.75)th item

As 6.75 item is present in the group (20–30) so Q_1 Class is (20–30)

$$\text{Value of } Q_1 = l + \frac{h}{f}(\frac{n}{4} - c)$$

$$= 20 + \frac{10}{5}\left(\frac{27}{4} - 2\right)$$

$$= 20 + \frac{10}{5}(6.75 - 2)$$

$$= 20 + \frac{10}{5}(4.75)$$

$$= 20 + \frac{47.5}{5}$$

$$= 20 + 9.5$$

$$Q_1 = 29.5$$

This result (29.5) is exactly same as obtained from the graph.

Verification of Median (Q_2) by Formula:

Median Class = Class Containing $2\left(\frac{n}{4}\right)$th item

Median Class = Class Containing $(\frac{27}{2})$th item

Median Class = Class Containing (13.5)th item

As 13.5 item is present in the group (30–40) so, Median Group is (30–40)

$$\text{Median} = l + \frac{h}{f}(\frac{n}{2} - c)$$

$$= 30 + \frac{10}{9}\left(\frac{27}{2} - 7\right)$$

$$= 30 + \frac{10}{9}(13.5 - 7)$$

$$= 30 + \frac{10}{9}(6.5)$$

$$= 30 + \frac{65}{9}$$

$$= 30 + 7.22 \Rightarrow Q_2 = 37.22$$

The result is very close to the value (37.5) which is obtained from graph.

Verification of Q_3 by Formula:

Q_3 Class = Class Containing $3(\frac{n}{4})$th item

Q_3 Class = Class Containing $3(\frac{27}{4})$th item

Q_3 Class = Class Containing 3(6.75)th item

Q_3 Class = Class containing 20.25^{th} item

As 20.25^{th} item is present in the group (40–50) so, Q_3 Class is (40–50).

$$\text{Value of } Q_3 = l + \frac{h}{f}(\frac{3n}{4} - c)$$

$$= 40 + \frac{10}{6}\left(\frac{3 \times 27}{4} - 16\right)$$

$$= 40 + \frac{10}{6}(20.25\text{-}16)$$

$$= 40 + \frac{10}{6}(4.25)$$

$$= 40 + \frac{42.5}{6}$$

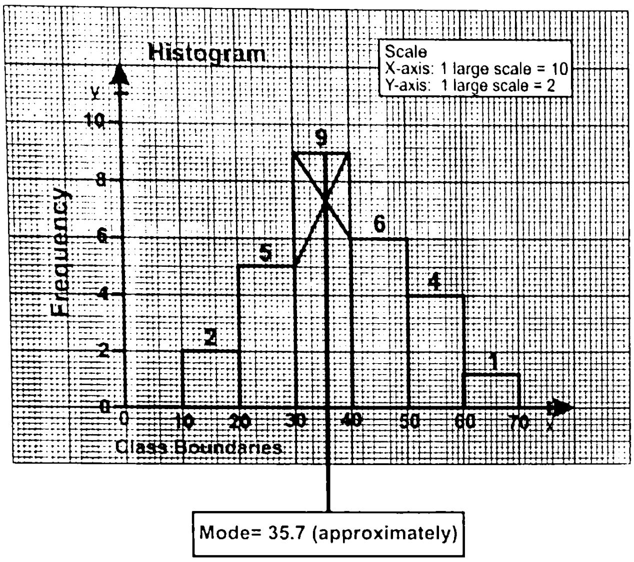
$$= 40 + 7.08 \Rightarrow Q_3 = 47.08$$

The result is very close to the value (47.5) which is obtained from graph.

(ii) Solution:

Class Boundaries	Frequency
10–20	2
20–30	5
30–40	9
40–50	6
50–60	4
60–70	1

Histogram:



From the Graph

Mode = 35.75

Verification of Mode by Formula

Class Boundaries	Frequency
10–20	2
20–30	$f_1 \rightarrow$ 5
30–40	$f_m \rightarrow$ 9
40–50	$f_2 \rightarrow$ 6
50–60	4
60–70	1

As the group (30–40) has maximum frequency (9).

So modal group is (30–40)

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

$$= 30 + \frac{(9-5)\times 10}{2(9)-5-6}$$

$$= 30 + \frac{4\times 10}{18-11}$$

$$= 30 + \frac{40}{7}$$

$$= 30 + 5.71$$

$$\textbf{Mode} = \textbf{35.71}$$

This result is very close to the value (35.5) which is obtained from the graph.

EXERCISE 6.3

Q.1 What do you understand by Dispersion?

Dispersion: Statistically, dispersion means the spread or scatterness of observations in a data set.

Q.2 How do you define measures of dispersion?

Measures of Dispersion:

The measures or techniques that are used to determine the degree or extent of variation in a data set are called Measure of Dispersion. For example Range, Variance and Standard Deviation etc.

Q.3 Define range, standard deviation and variance?

Definitions:

Range:

Range measures the extent of variation between two extreme observations of a data set. It is given by the formula.

Range = $X_{max} - X_{min} = X_m - X_o$ Where,

$X_{max} = X_m$: the maximum, observation.

$X_{min} = X_o$: the minimum observation.

Variance:

Variance is defined as the mean of the squared deviation of X_i,(i=1,2,3,......., n) observations from their arithmetic mean.

Variance from Ungrouped Data

- **Definitional Formula**

$$\text{Variance of } x = \text{var}(X) = S^2 = \frac{\sum (x - \bar{x})^2}{n}$$

- **Computational formula:**

$$\text{Var}(x) = S^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$$

Variance from Grouped Data

- **Definitional Formula**

$$\text{Var}(x) = S^2 = \frac{\sum f(x - \bar{x})^2}{\sum f}$$

- **Computational Formula**

$$\text{Var}(x) = S^2 = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2$$

Standard Deviation:
Standard deviation is defined as the positive square root of mean of the squared deviations of X_i, (i= 1,2,3….., n) observations from their arithmetic mean.

Standard Deviation from Un-Grouped Data

- **Definitional Formula**

$$S.D(X) = S = \sqrt{\frac{\sum (x-\bar{x})^2}{n}}$$

- **Computational Formula**

$$S.D(x) = S = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

Standard Deviation from Grouped Data

- **Definitional Formula**

$$S.D(x) = S = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$$

- **Computational Formula**

$$S.D(x) = S = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Q.4 The salaries of five teachers in Rupees are as follows. 11500, 12400, 15000, 14500, 14800
Find range and standard deviation

Solution

Maximum value = X_{max} = Rs. 15000
Minimum value = X_{min} = Rs. 11500
Range (R)= ?
Range $= X_{max} - X_{min}$
R = 15000–11500
Range = Rs. 3500

Standard Deviation:

Let X = 11500, 12400, 15000, 14500, 14800

We know that $\bar{X} = \frac{\sum X}{n} = \frac{68200}{5} = 13640$

X	$x-\bar{x}$	$(x-\bar{x})^2$
11500	11500 – 13640 = –2140	4579600
12400	12400 – 13640 = –1240	1537600
15000	15000 – 13640 = 1360	1849600
14500	14500 – 13640 = 860	739600
14800	14800 – 13640 = 1160	1345600
$\sum x = 68200$	$\sum (x-\bar{x})^2 = 10052000$	

Standard Deviation:

$$S = \sqrt{\frac{\sum (x-\bar{x})^2}{n}}$$

$$S = \sqrt{\frac{10052000}{5}}$$

$$S = \sqrt{2010400}$$

$$S = 1417.88575$$

Q. 5

a. Find the standard deviation "S" of each set of numbers.

(i) 12, 6, 7, 3, 15, 10, 18, 5

(ii) 9, 3, 8, 8, 9, 8, 9, 18

b. Calculate variance for the data:
10, 8, 9, 7, 5, 12, 8, 6, 8, 2

(i) Solution:

X = 12, 6, 7, 3, 15, 10, 18, 5
Number of observations = n = 8

X	X^2
3	9
5	25
6	36
7	49
10	100
12	144
15	225
18	324
$\sum x = 76$	$\sum x^2 = 912$

$$S = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$S = \sqrt{\frac{912}{8} - \left(\frac{76}{8}\right)^2}$$

$$S = \sqrt{114 - 90.25}$$

$$S = \sqrt{23.75}$$

$$S = 4.87$$

(ii) Solution:

X = 9, 3, 8, 8, 9, 8, 9, 18

Number of observations = n = 8

X	X^2
3	9
8	64
8	64
8	64
9	81
9	81
9	81
18	324
$\sum x = 72$	$\sum x^2 = 768$

We know that,

$$S = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$S = \sqrt{\frac{768}{8} - \left(\frac{72}{8}\right)^2}$$

$$S = \sqrt{96 - 81}$$

$$S = \sqrt{15} \quad \Rightarrow \quad S = 3.87$$

(b). Calculate variance for the data

10, 8, 9, 7, 5, 12, 8, 6, 8, 2

Solution:

Number of Observations = n = 10

X	X^2
10	100
8	64
9	81
7	49
5	25
12	144
8	64
6	36
8	64
2	4
$\sum x = 75$	$\sum x^2 = 631$

We know that,

$$S^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$$

$$S^2 = \frac{631}{10} - \left(\frac{75}{10}\right)^2$$

$$S^2 = 63.1 - (7.5)^2$$

$$S^2 = 63.1 - 56.25 \Rightarrow S^2 = 6.85$$

Q. 6. The lengths of 32 items are given below. Find the mean length and standard deviation of the distribution.

Length	20–22	23–25	26–28	29–31	32–34
f	3	6	12	9	2

Solution:

Length	(f)	Class Marks (x)	fx	x^2	fx^2
20–22	3	21	63	441	1323
23–25	6	24	144	576	3456
26–28	12	27	324	729	8748
29–31	9	30	270	900	8100
32–34	2	33	66	1089	2178
$\sum f = 32$		$\sum fx = 867$		$\sum fx^2 = 23805$	

(i) Arithmetic Mean

$$\overline{X} = \frac{\sum fx}{\sum f} = \frac{867}{32} = 27.093$$

(ii) Standard Deviation

$$S = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$S = \sqrt{\frac{23805}{32} - \left(\frac{867}{32}\right)^2}$$

$$S = \sqrt{743.90 - (27.093)^2}$$

$$S = \sqrt{743.90 - 734.03065}$$

$$S = \sqrt{9.86935}$$

$$S = 3.1415$$

Q. 7 For the following distribution of marks calculate range.

Marks in percentage	Frequency / No of students
31 – 40	28
41 – 50	31
51 – 60	12
61 – 70	9
71 – 75	5

Solution:

Marks in percentage	Class Boundaries	Frequency / No of students
31 – 40	30.5 – 40.5	28
41 – 50	40.5 – 50.5	31
51 – 60	50.5 – 60.5	12
61 – 70	60.5 – 70.5	9
71 – 75	70.5 – 75.5	5

Upper class boundary of last class = 75.5

Lower class boundary of 1^{st} class = 30.5

Range = ?

Range (R) = 75.5 – 30.5

R = 45

MISCELLANEOUS EXERCISE – 6

1. A grouped frequency table is also called……
   (a) data
   (b) frequency distribution
   (c) frequency polygon
   (d) Histogram

2. Mean of a variable with similar observations say constant k is…
   (a) negative (b) k itself
   (c) zero (d) one

3. A deviation is defined as a difference of any value of the variable from a……
   (a) constant (b) histogram
   (c) sum (d) frequency

4. Mean is affected by change in…
   (a) value (b) ratio
   (c) origin (d) none of these

5. Mean is affected by change in…
   (a) place (b) scale
   (c) rate (d) none of these

6. Arithmetic mean is a measure that determines a value of the variable under study by dividing the sum of all values of the variable by their…
   (a) number (b) group
   (c) denominator (d) numerator

7. A frequency polygon is a many sided…
   (a) closed figure (b) rectangle
   (c) square (d) Circles

8. A data in the form of frequency distribution is called……
   (a) Grouped data
   (b) Ungrouped data
   (c) Histogram
   (d) Dispersion

9. A histogram is a set of adjacent…
   (a) squares (b) rectangles
   (c) circles (d) Dots

10. A cumulative frequency table is also called……
    (a) frequency distribution
    (b) data
    (c) less than cumulative frequency distribution
    (d) Histogram

11. In a cumulative frequency polygon frequencies are plotted against
    (a) midpoints
    (b) upper class boundaries
    (c) class limits (d) frequencies

12. The measures that are used to determine the degree or extent of variation in a data set are called measures of…….
    (a) dispersion
    (b) central tendency
    (c) average
    (d) quartile

13. The positive square root of mean of the squared deviations of x_i (i – 1,2,.... n) observations from their arithmetic mean is called......
(a) harmonic mean (b) range
(c) S.D (d) variance

14. The value obtained by reciprocating the mean of the reciprocal of $x_1,x_2,x_3..x_n$ observations is called......
(a) Geometric mean
(b) Median
(c) Harmonic mean
(d) S.D

15. The most frequent occurring observation in a data set is called......
(a) Mode (b) Median
(c) Harmonic mean
(d) Mean

16. The measure which determines the middlemost observation in a data set is called......
(a) median (b) mode
(c) mean (d) variance

17. The size of class interval (6–10) is.
(a) 4 (b) 5
(c) 8 (d) 10

18. The arrangement of data is necessary to find the value of.
(a) Mean (b) Median
(c) Mode (d) Range

19. Sum of the deviations of the variable x from its mean is always......
(a) zero (b) one
(c) same (d) negative

20. The extent of variation between two extreme observations of a data set is measured by......
(a) average (b) range
(c) quartiles (d) mode

21. The mean of the squared deviations of x_i (i = 1, 2......,n) observations from their arithmetic mean is called...
(a) variance
(b) standard deviation
(c) range (d) mode

22. The n^{th} positive root of the product of the $x_1,x_2,x_3,\ldots\ldots x_n$ observations is called.....
(a) Mode (b) Mean
(c) Geometric mean
(d) median

23. The observation that divide a data set into four equal part, are called
(a) deciles (b) quartiles
(c) percentiles (d) mode

24. The spread or scatterness of observations in a data set is called ......
(a) average
(b) dispersion
(c) central tendency
(d) quartile

25. The class having maximum frequency is called ....class.
(a) Modal (b) Median
(c) Lower (d) Upper

26. The class containing $\frac{n}{2}$th observation is called.
(a) Modal (b) Median
(c) Boundary of (d) Size of

27. During frequency distribution number of groups should be between....
(a) 5 and 10 (b) 10 and 15
(c) 10 and 20 (d) 5 and 15

28. Direct formula to find mean from ungrouped data.
(a) $\overline{X}=\frac{\sum x}{n}$
(b) $\overline{X}=\frac{\sum fx}{\sum f}$
(c) $\overline{X}=A+\frac{\sum D}{n}$
(d) $\overline{X}=A+\frac{\sum fD}{\sum f}$

29. Formula to find Harmonic mean from grouped data is......
(a) $\overline{X}=\frac{n}{\sum \frac{1}{x}}$

(b) $\overline{X}=\frac{n}{\sum\frac{f}{x}}$

(c) $\overline{X}=A+\frac{\sum fu}{n}\times h$

(d) $\overline{X}=A+\frac{\sum fu}{\sum f}\times h$

30. Variance is denoted by……
(a) V (b) S
(c) S^2 (d) $\overline{X}$

31. Short formula to find mean from grouped data is……

(a) $\overline{X}=\frac{\sum x}{n}$ (b) $\overline{X}=\frac{\sum fx}{\sum f}$

(c) $\overline{X}=A+\frac{\sum D}{n}$ (d) $\overline{X}=A+\frac{\sum fD}{\sum f}$

32. In class (30–39), lower class limit is……
(a) 39 (b) 9
(c) 30 (d) 34.5

33. Coding formula to find mean from grouped data is……

(a) $\overline{X}=\frac{n}{\sum\frac{1}{x}}$

(b) $\overline{X}=\frac{n}{\sum\frac{f}{x}}$

(c) $\overline{X}=A+\frac{\sum u}{n}\times h$

(d) $\overline{X}=A+\frac{\sum fu}{\sum f}\times h$

34. Formula to find Harmonic mean from ungrouped data is……

(a) $\overline{X}=\frac{n}{\sum\frac{1}{x}}$

(b) $\overline{X}=\frac{n}{\sum\frac{f}{x}}$

(c) $\overline{X}=A+\frac{\sum fu}{n}\times h$

(d) $\overline{X}=A+\frac{\sum fu}{\sum f}\times h$

35. Direct formula to find mean from grouped data is……

(a) $\overline{X}=\frac{\sum x}{n}$ (b) $\overline{X}=\frac{\sum fx}{\sum f}$

(c) $\overline{X}=A+\frac{\sum D}{n}$ (d) $\overline{X}=A+\frac{\sum fD}{\sum f}$

36. The concept of antilogarithm is used to find the value of……
(a) A.M (b) G. M
(c) H. M (d) Mode

37. Short formula to find mean from ungrouped data is……

(a) $\overline{X}=\frac{\sum x}{n}$ (b) $\overline{X}=\frac{\sum fx}{\sum f}$

(c) $\overline{X}=A+\frac{\sum D}{n}$ (d) $\overline{X}=A+\frac{\sum fD}{\sum f}$

38. In class (20–29), Midpoint or class mark is……
(a) 20.5 (b) 24.5
(c) 29 (d) 49

39. Median from the data 82,93,86,92 and 79 is……
(a) 82 (b) 86
(c) 92 (d) 93

40. Harmonic mean for 12,5,8,4 is…
(a) 6.08 (b) 5.08
(c) 7.08 (d) 4.08

41. Geometric Mean of 2,4,8 is.….
(a) 2 (b) 4
(c) 8 (d) 3

42. Coding formula to find mean from ungrouped data is……

(a) $\overline{X}=\frac{n}{\sum\frac{1}{x}}$

(b) $\overline{X}=\frac{n}{\sum\frac{f}{x}}$

(c) $\overline{X}=A+\frac{\sum u}{n}\times h$

(d) $\overline{X}=A+\frac{\sum fu}{\sum f}\times h$

43. Standard deviation is denoted by......

(a) $\widetilde{X}$ (b) S

(c) S^2 (d) $\overline{X}$

44. Types of measures of central tendency are.......

(a) 3 (b) 4

(c) 5 (d) 6

45. Median is denoted by......

(a) $\overline{X}$ (b) $\widetilde{X}$

(c) S (d) S^2

46. If y = 10X then $\overline{y}$ =.......

(a) 10 (b) $10\overline{X}$

(c) $\overline{X}$ (d) $10+\overline{X}$

47. If Y = X + 5 then $\overline{Y}$ =

(a) $\overline{X}$ (b) 5

(c) $\overline{X}+5$ (d) $5\overline{X}$

48. In class (10 – 19), upper class limit is......

(a) 10 (b) 19

(c) 29 (d) 14.5

49. On the basis of types of variable or data, the types of frequency distribution are.

(a) 2 (b) 3

(c) 4 (d) 5

50. Types of dispersion are.......

(a) 4 (b) 5

(c) 6 (d) 3

51. Range for the data 110, 109, 84, 89, 77, 104, 74, 97, 49, 59, 103, 62 is......

(a) 41 (b) 51

(c) 61 (d) 71

52. If standard deviation is 6 then its variance is......

(a) $\sqrt{6}$ (b) 36

(c) 3 (d) 6

53. $\sum(X-\overline{X})=$ ......

(a) 0 (b) 1

(c) −1 (d) 2

54. Arithmetic mean of 34,34,34,34,34,34 is

(a) 0 (b) 34

(c) 6 (d) 170

55. Mode from the following data 4,4,5,5,6,6,6,7,7,5,8,8,8,6,5,6,5,7 is......

(a) 4 (b) 5

(c) 5, 6 (d) 5, 7

56. Median from the data 2.3, 2.7, 2.5, 3.1 and 1.9 is......

(a) 2.3 (b) 2.5

(c) 2.7 (d) 2.9

57. Which one is formula for weighted Arithmetic mean?

(a) $\frac{\sum w}{\sum wx}$ (b) $\sum wx$

(c) $\sum x$ (d) $\frac{\sum wx}{\sum w}$

58. Range = ...

(a) $X_m + X_o$ (b) $X_m - X_o$

(c) $\frac{X_m}{X_o}$ (d) $\frac{X_o}{X_m}$

ANSWER KEY

1.	b	**2.**	b	**3.**	a	**4.**	c	**5.**	b
6.	a	**7.**	a	**8.**	a	**9.**	b	**10.**	c
11.	b	**12.**	a	**13.**	c	**14.**	c	**15.**	a
16.	a	**17.**	b	**18.**	b	**19.**	a	**20.**	b
21.	a	**22.**	c	**23**	b	**24**	b	**25**	a
26	b	**27**	d	**28**	a	**29**	b	**30**	c
31	d	**32**	c	**33**	d	**34**	a	**35**	b
36	b	**37**	c	**38**	b	**39**	b	**40**	a
41	b	**42**	c	**43**	b	**44**	c	**45**	b
46	b	**47**	c	**48**	b	**49**	a	**50**	b
51	c	**52**	b	**53**	a	**54**	b	**55**	c
56	b	**57**	d	**58**	b				

Q. 2 Write short answer of the following questions.

(i) Define class limits.

Class Limits: The minimum and the maximum values defined for a class or group are called class limits. The minimum value is called the lower class limit and the maximum value is called the upper class limit of that class. For example in the group (5–10), 5 is lower class limit and 10 is called upper class limit.

(ii) Define class mark

Midpoint or Class Mark:

For a given class the average of that class obtained by dividing the sum of upper and lower class limits by 2, is called the midpoint or class mark of that class

(iii) What is cumulative frequency?

Cumulative Frequency: The total of frequency up to an upper class limit or boundary is called the cumulative frequency.

(iv) Define a frequency distribution.

Frequency Distribution

A frequency distribution is a tabular arrangement for classifying data into different groups and the number of observation falling in each group corresponds to the respective group. In fact a frequency distribution is a method to summarize data.

(v) What is Histogram?

Histogram

A Histogram is a graph of adjacent rectangles constructed on XY–plane. It is a graph of frequency distribution. In practice both discrete and continuous frequency distribution are represented by means of histogram.

(vi) Name two measures of central tendency.

1– Arithmetic Mean 2– Median 3– Mode

(vii) Define Arithmetic mean?

i. Arithmetic Mean:

Arithmetic mean is a measure that determines a value of the variable under study by dividing the sum of all values (observations) of the variable by their number of observations. We denote Arithmetic mean by $\overline{X}$. In symbols we define.

Arithmetic mean of n observations

$$\overline{X}=\frac{\sum X}{n}=\frac{\text{Sum of all values of observations}}{\text{No. of observations}}$$

(viii) Write three properties of Arithmetic mean.

Properties of Arithmetic Mean:

Following are the properties of Arithmetic Mean.

(a) Mean of a variable with similar observations say constant k is the constant k itself.

(b) Mean is affected by change in origin.

(c) Mean is affected by change in scale.

(d) Sum of the deviations of the variable X from its mean is always zero.

(ix) Define Median.

Median:

Median is the middle most observations in an arranged data set. It divides the data set into two equal parts. $\tilde{x}$ is used to represent median. We determine Median by using the following formulae.

Median From Ungrouped Data

Case 1:

When the number of observations (n) is odd.

$$\tilde{x}=\left(\frac{n+1}{2}\right)\text{th term in arranged data}$$

Case 2:

When the number of observations (n) is even.

$$\tilde{x}=\frac{1}{2}\left[\frac{n}{2}\text{th term}+\frac{n+2}{2}\text{th term}\right]$$

(x) Define mode.

Mode:

Mode is defined as the most frequent occurring observation is the data. It is the observation that occurs maximum number of times in given data. The following formula is used to determine Mode.

(i)Ungrouped Data and Discrete Grouped Data

Mode = The Most frequent observation

(ii) Grouped Data (Continuous)

The following Formulae are used to determine mode for grouped data.

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

(xi) What do you mean by Harmonic mean?

Harmonic Mean:

Harmonic mean refers to the value obtained by reciprocating the mean of the reciprocal of $x_1, x_2, x_3, \ldots\ldots, x_n$ observations. In symbols,

- **For Ungrouped data.**

$$H.M = \frac{n}{\sum \frac{1}{X}}$$

- **For Grouped data.**

$$H.M = \frac{n}{\sum \frac{f}{X}}$$

(xii) Define Geometric mean.

Geometric Mean:

Geometric mean of a variable x is the n^{th} positive root of the product of the $x_1, x_2, x_3, \ldots\ldots\ldots\ldots, x_n$ observation. In symbols we write,

$$G.M = (X_1 . X_2 . X_3 . \ldots\ldots . X_n)^{1/n}$$

The Geometric mean can also be calculated by using logarithm.

- **For Ungrouped data:**

$$\text{G.M} = \text{Anti log}\left(\frac{\sum \log X}{n}\right)$$

- **For Grouped data:**

$$\text{G.M} = \text{Anti log}\left(\frac{\sum f \log X}{\sum f}\right)$$

(xiii) What is Range?

Range:

Range measures the extent of variation between two extreme observations of a data set. It is given by the formula.

Range = $X_{max} - X_{min} = X_m - X_o$ Where,

$X_{max} = X_m$: the maximum observation.

$X_{min} = X_o$: the minimum observation.

(xiv) Define Standard deviation?

Standard Deviation:

Standard deviation is defined as the positive square root of mean of the squared deviations of Xi, (i = 1,2,3….., n) observations from their arithmetic mean.

- **Standard Deviation from Un–Grouped Data**

Definitional Formula

$$\text{S. D(X)} = S = \sqrt{\frac{\sum (X - \overline{X})^2}{n}}$$

Computational Formula

$$\text{S. D (X)} = S^2 = \sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2}$$

- **Standard Deviation from Grouped Data**

Definitional Formula

$$\text{S.D (X)} = S = \sqrt{\frac{\sum f (X - \overline{X})^2}{\sum f}}$$

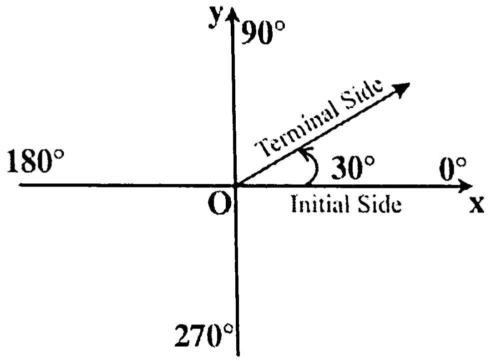
Computational Formula

$$\text{S.D (X)} = S = \sqrt{\frac{\sum fX^2}{\sum f} - \left(\frac{\sum fX}{\sum f}\right)^2}$$

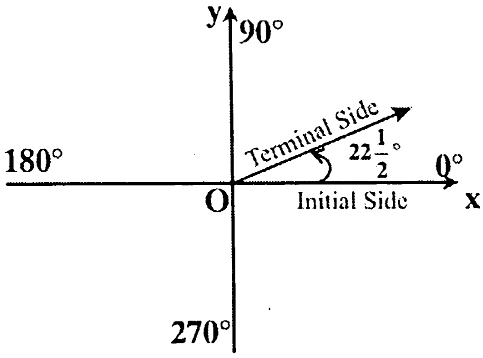
EXERCISE 7.1

Q.1. Locate the following angles:

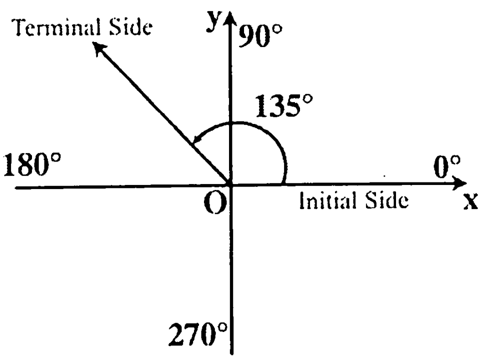
(i) 30°



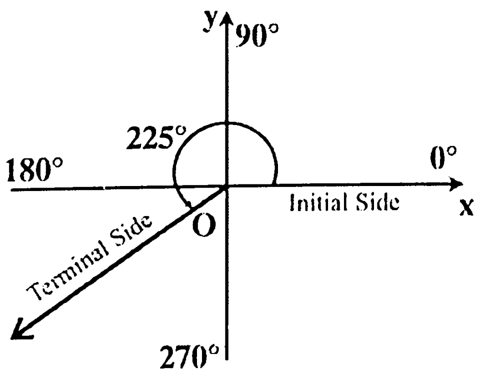
(ii) $22\frac{1}{2}°$



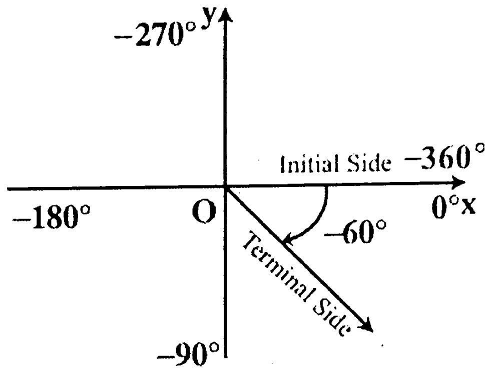
(iii) 135°



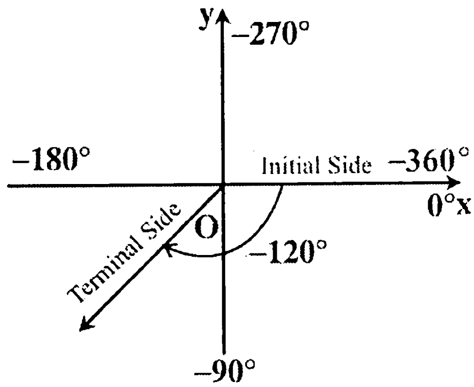
(iv) 225°



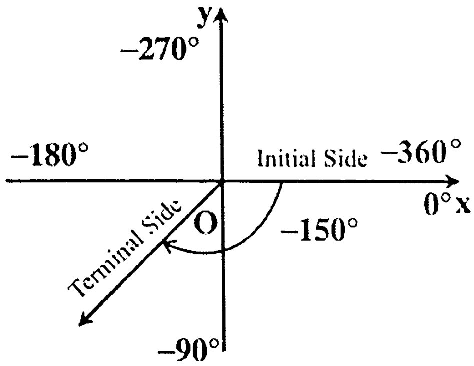
(v) −60°



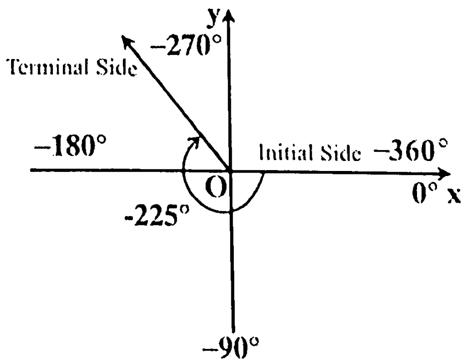
(vi) −120°



(vii) −150°



(viii) − 225°



Q.2. Express the following sexagesiml measures of angles in decimal form.

(i) 45°30′

Solution: 45°30′

$$= 45° + \left(\frac{30}{60}\right)^{\circ}$$
$$= 45° + 0.5°$$
$$= 45.5°$$

(ii) 60°30′30″

Solution: 60°30′30″

$$= 60° + \left(\frac{30}{60}\right)^{\circ} + \left(\frac{30}{60\times 60}\right)^{\circ}$$
$$= 60° + 0.5° + 0.008°$$
$$= 60.508°$$

(iii) 125°22′50″

Solution: 125°22′50″

$$= 125° + \left(\frac{22}{60}\right)^{\circ} + \left(\frac{50}{60\times 60}\right)^{\circ}$$
$$= 125° + 0.367° + 0.0139°$$
$$= 125.3808°$$

Q.3. Express the following in D°M′S″:

(i) 47.36°

Solution: 47.36°

$$= 47° + 0.36°$$
$$= 47° + (0.36 \times 60)'$$
$$= 47° + 21.6'$$
$$= 47° + 21' + (0.6 \times 60)''$$
$$= 47° + 21' + 36''$$
$$= 47°\,21'36''$$

(ii) 125.45°

Solution: 125.45°

$$= 125° + 0.45°$$
$$= 125° + (0.45 \times 60)'$$
$$= 125° + 27'$$
$$= 125°27'0''$$

(iii) 225.75°

Solution: 225.75°

$$= 225° + 0.75°$$
$$= 125° + (0.75 \times 60)'$$
$$= 225° + 45'$$
$$= 225°45'0''$$

(iv) – 22.5°

Solution: – 22.5°

$$= -[22° + 0.5°]$$
$$= -[22° + (0.5 \times 60)']$$
$$= -[22° + 30']$$
$$= -22°30'$$

(v) – 67.58°

Solution: – (67° + 0.58°)

$$= -[67° + (0.58 \times 60)']$$
$$= -[67° + 34.8']$$
$$= -[67° + 34' + 0.8']$$
$$= -[67° + 34' + (0.8 \times 60)'']$$
$$= -[67° + 34' + 48'']$$
$$= -67°34'48''$$

(vi) 315.18°

Solution: 315.18°

$$= 315° + 0.18°$$
$$= 315° + (0.18 \times 60)'$$
$$= 315° + 10.8'$$
$$= 315° + 10' + 0.8'$$
$$= 315° + 10' + (0.8 \times 60)''$$
$$= 315° + 10' + 48''$$
$$= 315°\,10'\,48''$$

Q.4. Express the following angles into radians.

(i) 30°

Solution: 30°

$$= 30\,\frac{\pi}{180}\,radians$$
$$= \cancel{30}\,\frac{\pi}{\cancel{30}\times 6}\,radians$$
$$= \frac{\pi}{6}\text{ radians}$$

(ii) 60°

Solution: 60°

$$= 60\,\frac{\pi}{180}\,radians$$
$$= \cancel{60}\,\frac{\pi}{\cancel{60}\times 3}\,radians$$
$$= \frac{\pi}{3}\text{ radians}$$

(iii) 135°

Solution: 135°

$= 135 \frac{\pi}{180} radians$

$= \cancel{45}\times 3 \frac{\pi}{\cancel{45}\times 4} radians$

$= \frac{3\pi}{4}$ radians

(iv) 225°

Solution: 225°

$= 225 \frac{\pi}{180} radians$

$= \cancel{45}\times 5 \frac{\pi}{\cancel{45}\times 4} radians$

$= \frac{5\pi}{4} radains$

(v) – 150°

Solution: – 150°

$= -150 \frac{\pi}{180} radians$

$= -5\times\cancel{30} \frac{\pi}{\cancel{30}\times 6} radians$

$= \frac{-5\pi}{6}$ radians

(vi) – 225°

Solution: – 225°

$= -225 \frac{\pi}{180}$ radians

$= -5\times\cancel{45} \frac{\pi}{\cancel{45}\times 4}$ radians

$= \frac{-5\pi}{4}$ radians

(vii) 300°

Solution: 300°

$= 300 \frac{\pi}{180} radians$

$= \cancel{60}\times 5 \frac{\pi}{\cancel{60}\times 3} radians$

$= \frac{5\pi}{3}$ radians

(viii) 315°

Solution: 315°

$= 315 \frac{\pi}{180} radians$

$= \cancel{45}\times 7 \frac{\pi}{\cancel{45}\times 4} radians$

$= \frac{7\pi}{4}$ radians

Q.5. Convert each of the following to degrees.

(i) $\frac{3\pi}{4}$

Solution: $\frac{3\pi}{4}$ radians

$= \frac{3\pi}{4} \frac{180}{\pi}$ degrees

$= \frac{3\cancel{\pi}}{\cancel{4}} \frac{\overset{45}{\cancel{180}}}{\cancel{\pi}}$ degrees

$= 3\times 45$ degrees

$= 135°$

(ii) $\frac{5\pi}{6}$

Solution: $\frac{5\pi}{6}$

$= \frac{5\pi}{6} \frac{180}{\pi}$ degrees

$= \frac{5\cancel{\pi}}{\cancel{6}} \frac{\overset{30}{\cancel{180}}}{\cancel{\pi}}$ degrees

$= 5\times 30$ degrees

$= 150°$

(iii) $\frac{7\pi}{8}$

Solution: $\frac{7\pi}{8}$ radians

$= \frac{7\pi}{8} \frac{180}{\pi}$ degrees

$= \frac{7\cancel{\pi}}{8} \frac{180}{\cancel{\pi}}$ degrees

$= \frac{7\times 180}{8}$ degrees

$= \frac{1260}{8}$ degrees

$= 157.5°$

(iv) $\frac{13\pi}{16}$

Solution: $\frac{13\pi}{16}$ radians

$= \frac{13\pi}{16} \frac{180}{\pi}$ degrees

$= \frac{13\not{\pi}}{16} \frac{180}{\not{\pi}}$ degrees

$= \frac{13\times180}{16}$ degrees

$= \frac{2340}{16}$ degrees

$= 146.25°$

(v) 3 radians

Solution: 3 radians

$= 3 \frac{180}{\pi}$ degrees

$= \frac{540}{\pi}$ degrees

$= 171.887°$

(vi) **4.5**

Solution: 4.5 radians

$= 4.5 \frac{180}{\pi}$ degrees

$= \frac{810}{\pi}$ degrees

$= 257.831°$

(vii) $-\frac{7\pi}{8}$

Solution: $-\frac{7\pi}{8}$ radians

$= -\frac{7\not{\pi}}{8} \frac{180}{\not{\pi}}$ degrees

$= \frac{-1260}{8}$ degrees

$= -157.5°$

(viii) $-\frac{13}{16}\pi$

Solution: $-\frac{13}{16}\pi$ radians

$= -\frac{13\not{\pi}}{16} \frac{180}{\not{\pi}}$ degrees

$= \frac{-2340}{16}$

$= -146.25°$

EXERCISE 7.2

Q.1. Find θ, when:

(i) $l = 2cm$, $r = 3.5$ cm

Solution : Using rule

$$l = r\theta,$$
$$2 = 3.5\,\theta$$
$$\frac{2}{3.5} = \theta$$
$$\theta = 0.57 \text{ radian}$$

(ii) $l = 4.5$ m, $r = 2.5$ m

Solution : Using rule

$$l = r\theta,$$
$$\frac{\;}{r} = \theta$$
$$\frac{4.5}{2.5} = \theta$$
$$\theta = 1.8 \text{ radian}$$

Q.2. Find l, when

(i) $\theta = 180°$, $r = 4.9$ cm

Solution: As θ should be in radians so

$$\theta = 180°$$
$$= \cancel{180}\,\frac{\pi}{\cancel{180}} \text{ radian}$$
$$= \pi \text{ radian}$$

Using rule $l = r\theta$

$$= 4.9 \text{ cm} \times \pi$$
$$= 15.4 \text{ cm}$$

(ii) $\theta = 60°30'$, $r = 15$ mm 07(036)

Solution : As 'θ' should be in radians, so

$$\theta = 60°30'$$
$$= 60° + \frac{30}{60}^{o}$$
$$= 60° + 0.5°$$
$$= 60.5°$$
$$= 60.5\,\frac{\pi}{180} \text{ radian}$$
$$\theta = 1.056 \text{ radian}$$

Using rule $l = r\theta$

$$= 15 \text{ mm} \times 1.056$$
$$= 15.84 \text{ mm}$$

Q.3. Find r, when

(i) $l = 4$ cm, $\theta = \frac{1}{4}$ radian

Solution: Using rule $l = r\theta$

$$4\text{cm} = r\,\frac{1}{4}$$
$$4\text{cm} \times 4 = r$$
$$r = 16 \text{ cm}$$

(ii) $l = 52$ cm , $\theta = 45°$

Solution : As θ should be in radians.

$$\theta = 45°$$
$$= 45\,\frac{\pi}{180} \text{ radian}$$
$$= \frac{\pi}{4} \text{ radian}$$

Now using rule $l = r\theta$

$$52 \text{ cm} = r\,\frac{\pi}{4}$$
$$\frac{52\text{cm} \times 4}{\pi} = r$$
$$r = 66.21 \text{ cm}$$

Q.4. In a circle of radius 12m, find the length of an arc which subtends a central angle θ = 1.5 radian.

Solution : Radius = $r = 12$m

Arc length = l ? =

Central angle = $\theta = 1.5$ radian

Using rule $l = r\theta$

$$l = 12\text{m} \times 1.5$$
$$l = 18\text{m}$$

Q.5. In a circle of radius 10m, find the distance travelled by a point moving on this circle if the point makes 3.5 revolution.

Solution: Radius = r = 10m

Number of revolutions = 3.5

Angle of one revolution = 2π radian

Angle of 3.5 revolution = θ

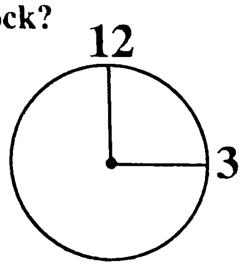
$$= 3.5 \times 2\pi \text{ radian}$$
$$\theta = 7\pi \text{ radian}$$

Distance travelled = l ? =

Using rule $l = r\theta$

$$l = 10 \text{ m} \times 7\pi$$
$$l = 220 \text{ m}$$

Q.6. What is the circular measure of the angle between the hands of the watch at 3 O' clock?



Solution:

At 3 O' clock the minute hand will be at 12 and hour hand will be at 3 i.e the angle between the hands of watch will be one quarter of the central angle of full circle

i.e $= \frac{1}{4}$ of $360°$

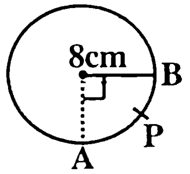
$= \frac{1}{4} \times 360°$

$= 90°$

$= 90 \frac{\pi}{180}$ radian

$= \frac{\pi}{2}$ radian.

Q.7. What is the length of arc APB?



Solution: From the figure we see that

Radius $= r = 8$cm

Central angle $= \theta$

$= 90°$

$= \frac{\pi}{2}$ radian

Arc length $= l \;\; ? =$

By rule $l = r\,\theta$

$l = {}^{4}\cancel{8}\text{cm} \times \frac{\pi}{\cancel{2}}$

$l = 4\text{cm} \times \pi$

$l = 12.57$ cm

So, length of arc APB is 12.57 cm

Q.8. In a circle of radius 12 cm, how long an arc subtends a central angle of 84°?

Solution: Radius $= r = 12$cm

Arc length $= l \;\; ? =$

Central angle $= \theta = 84°$

$= 84 \frac{\pi}{180}$ radian

$= 1.466$ radian

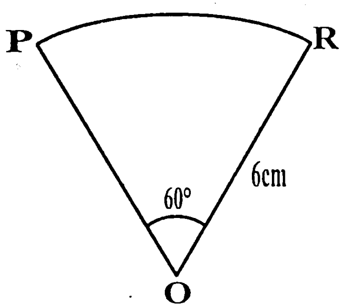
Now by rule $l = r\,\theta$

$= 12\text{cm} \times 1.466$

$= 17.6$ cm

Q.9. Find the area of sector OPR.

(a)



Radius $= r = 6$cm

Central angle $= \theta = 60°$

$= 60 \frac{\pi}{180}$ radian

$= \frac{\pi}{3}$ radian

Area of sector $= ?$

As Area of sector $= \frac{1}{2} r^2 \theta$

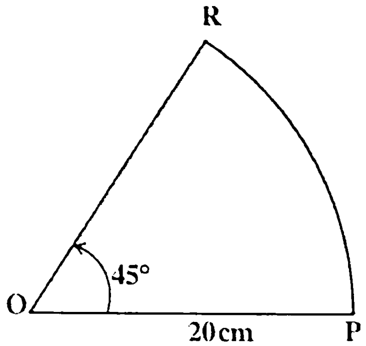
$= \frac{1}{2} \times (6\text{cm})^2 \times \frac{\pi}{3}$

$= \frac{1}{\cancel{6}} \times {}^{6}\cancel{36}\text{cm}^2 \times \pi$

$= 6\pi \text{ cm}^2$

$= 18.85 \text{ cm}^2$

(b)



Radius $= r = 20cm$

Central angle $= \theta = 45°$

$= 45 \frac{\pi}{180}$ radian

$= \frac{\pi}{4}$ radian

Area of sector = ?

Area of sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(20cm)^2 \times \frac{\pi}{4}$

$= \frac{400cm^2}{8} \times \pi$

$= 50 \pi \, cm^2$

$= 157.1 \, cm^2$

Q.10. Find area of sector inside a central angle of 20° in a circle of radius 7 m.

Solution: Area of sector = ?

Radius $= r = 7m$

Central angle $= \theta = 20°$

$= 20 \frac{\pi}{180}$ radian

$= \frac{\pi}{9}$ radian

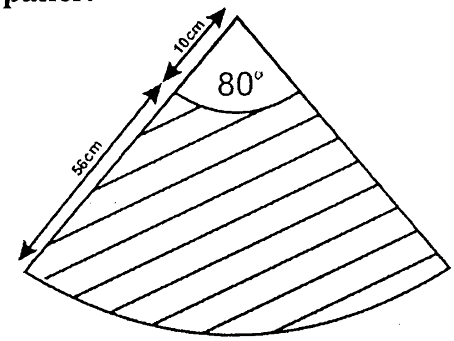
Area of sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times (7m)^2 \times \frac{\pi}{9}$

$= \frac{49\pi}{18} m^2$

$= 8.55 \, m^2$

Q.11. Sehar is making skirt. Each panel of this skirt is of the shape shown shaded in the diagram. How much material (cloth) is required for each panel?



Solution: Central angle $= \theta = 80°$

$= 80 \frac{\pi}{180}$ radian

$= \frac{4\pi}{9}$ radian

Radius of bigger sector $= R = (56 + 10)cm$

$R = 66 \, cm$

Radius of smaller sector $= r = 10 \, cm$

Shaded area = ?

Area of bigger sector $= \frac{1}{2} R^2 \theta$

$= \frac{1}{\cancel{2}} \times (66cm)^2 \times \frac{2\cancel{4}\pi}{9}$

$= 484 \cancel{4356} cm^2 \times \frac{2\pi}{\cancel{9}}$

$= 968 \pi \, cm^2$

Area of smaller sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{\cancel{2}} (10cm)^2 \times \frac{2\cancel{4}\pi}{9}$

$= \frac{200}{9} \pi \, cm^2$

Shaded area $= 968 \pi - \frac{200}{9} \pi$

$= \frac{8712\pi - 200\pi}{9}$

$= \frac{8512}{9} \pi \, cm^2$

$= 2971.25 \, cm^2$

Q.12. Find the area of a sector with central angle of $\frac{\pi}{5}$ radian in a circle of radius 10 cm.

Solution: Area of sector = ?

Central angle = θ = $\frac{\pi}{5}$ radian

Radius = r = 10cm

Area of sector = $\frac{1}{2}r^2\theta$

$= \frac{1}{2}(10\text{cm})^2 \times \frac{\pi}{5}$

$= \frac{1}{10} \times 100\text{cm}^2 \times \pi$

$= 10\pi \text{ cm}^2$

$= 31.43 \text{ cm}^2$

Q.13. The area of sector with central angle θ in a circle of radius 2m is 10 square meter. Find θ in radians.

Solution: Area of sector = 10 m²

Radius = r = 2m

Central angle = θ ? =

As Area of sector = $\frac{1}{2}r^2\theta$

$10\text{m}^2 = \frac{1}{2}(2m)^2 \theta$

$10 \text{ m}^2 = \frac{1}{2}(4m^2)\theta$

$10\text{m}^2 = 2\theta\text{m}^2$

$\theta = \frac{10\text{m}^2}{2\text{m}^2}$

$\theta = 5$ radian

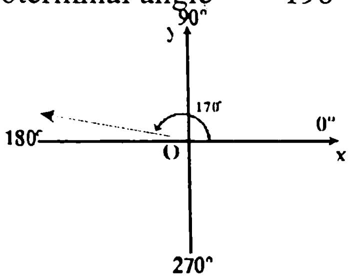
EXERCISE 7.3

Q.1. Locate each of the following angles in standard position using a protractor or fair free hand guess. Also find a positive and a negative angle coterminal with each given angle:

(i) **170°**

Positive coterminal angle $= 360° + 170°$
$= 530°$

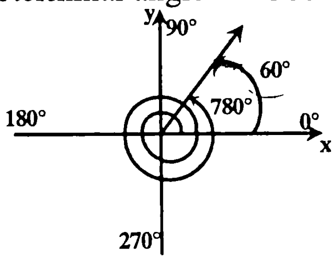
Negative coterminal angle $= -190°$



(ii) **780°**

Positive coterminal angle $780° - 2(360°) = 60° = 60°$

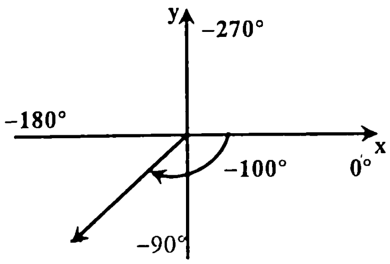
Negative coterminal angle $= -300°$



(iii) **– 100°**

Positive coterminal angle $= 260°$

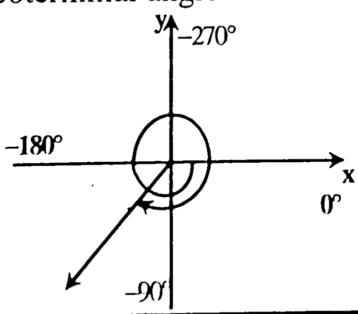
Negative coterminal angle $= -360° - 100°$
$= -460°$



(iv) **– 500°**

Positive coterminal angle $= 220°$

Negative coterminal angle $= -140°$



Q.2. Identify closest quadrantal angles between which the following angles lie.

(i) **156°**

Ans: 90° and 180°

(ii) **318°**

Ans: 270° and 360°

(iii) **572°**

Ans: 540° and 630°

(iv) **–330°**

Ans: 0° and 90°

Q.3. Write the closest quadrantal angles between which the angles lie. Write your answer in radian measure.

(i) $\frac{\pi}{3}$

Ans: 0 and $\frac{\pi}{2}$

(ii) $\frac{3\pi}{4}$

Ans: $\frac{\pi}{2}$ and π

(iii) $\frac{-\pi}{4}$

Ans: 0 and $-\frac{\pi}{2}$

(iv) $-\frac{3\pi}{4}$

Ans: $-\frac{\pi}{2}$ and $-\pi$

Q.4. In which quadrant θ lies, when

(i) **$\sin\theta > 0, \tan\theta < 0$**

Ans: II quadrant

(ii) **$\cos\theta < 0, \sin\theta < 0$**

Ans: III quadrant

(iii) **$\sec\theta > 0, \sin\theta < 0$**

Ans: IV quadrant

(iv) **$\cos\theta < 0, \tan\theta < 0$**

Ans: II quadrant

(v) **$\text{cosec}\,\theta > 0, \cos\theta > 0$**

Ans: I quadrant

(vi) **$\sin\theta < 0, \sec\theta < 0$**

Ans: III quadrant

Q.5. Fill in the blanks:

(i) cos(–150°) = ———— cos 150°
(ii) sin(– 310°) = ———— sin 310°
(iii) tan(– 210°) = ———— tan 210°
(iv) cot(– 45°) = ———— cot 45°
(v) sec(– 60°) = ———— sec 60°
(vi) cosec(– 137°)= ———— cosec137°

Answers:

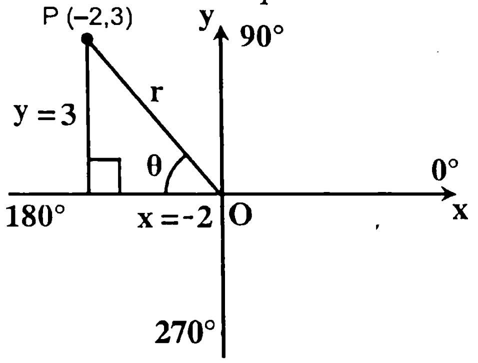
(i) +ve (ii) –ve (iii) –ve
(iv) –ve (v) +ve (vi) –ve

Q.6. The given point P lies on the terminal side of θ. Find quadrant of θ and all six trigonometric ratios.

(i) (–2, 3)

Solution: P(x, y) = P(–2, 3)

As x –coordinate is negative and y– coordinate is positive so P lies in II quadrant



The point P can be shown in II quadrant.

By Pythagorean theorem

$$r^2 = x^2 + y^2$$
$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{(-2)^2 + (3)^2}$$
$$r = \sqrt{4+9}$$
$$r = \sqrt{13}$$

Now $\sin\theta = \frac{y}{r} = \frac{3}{\sqrt{13}}$

$\text{cosec}\ \theta = \frac{r}{y} = \frac{\sqrt{13}}{3}$

$\cos\theta = \frac{x}{r} = -\frac{2}{\sqrt{13}}$

$\sec\theta = \frac{r}{x} = \frac{-\sqrt{13}}{2}$

$\tan\theta = \frac{y}{x} = -\frac{3}{2}$

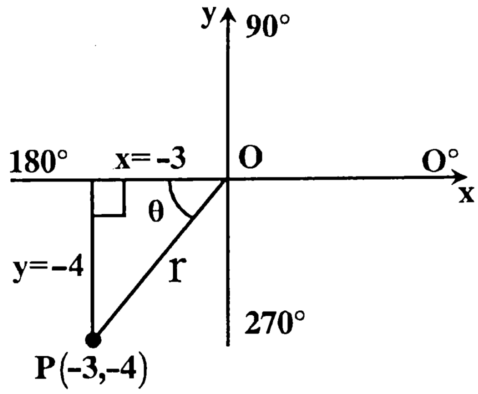
$\cot\theta = \frac{x}{y} = -\frac{2}{3}$

(ii) (– 3, – 4)

Solution: P(x, y) = P(– 3, –4)

As x and y both coordinates are negative , so ‘P’ lies in III quadrant.

The point P can be shown in III quadrant.



By Pythagorean, theorem

$$r^2 = x^2 + y^2$$
$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{(-3)^2 + (-4)^2}$$
$$r = \sqrt{9+16}$$
$$r = \sqrt{25}$$
$$r = 5$$

Now $\sin\theta = \frac{y}{r} = \frac{-4}{5}$

$\text{cosec}\ \theta = \frac{r}{y} = \frac{-5}{4}$

$\cos\theta = \frac{x}{r} = \frac{-3}{5}$

$\sec\theta = \frac{r}{x} = \frac{-5}{3}$

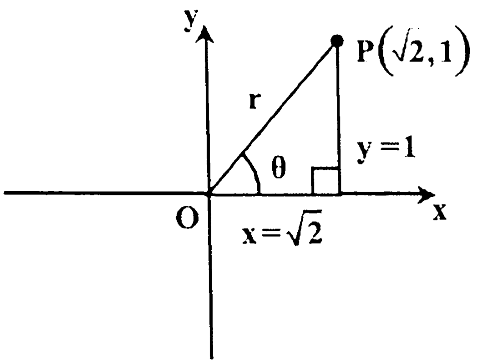
$\tan\theta = \frac{y}{x} = \frac{-4}{-3} = \frac{4}{3}$

$\cot\theta = \frac{x}{y} = \frac{-3}{-4} = \frac{3}{4}$

(iii) $(\sqrt{2},1)$

Solution: $P(x, y) = P(\sqrt{2},1)$

As x and y both coordinates are positive, so P lies in I quadrant.



The point 'P' can be shown in quadrant I.

By Pythagorean theorem

$$r^2 = x^2 + y^2$$
$$r = \sqrt{x^2+y^2}$$
$$r = \sqrt{(\sqrt{2})^2+(1)^2}$$
$$r = \sqrt{2+1}$$
$$r = \sqrt{3}$$

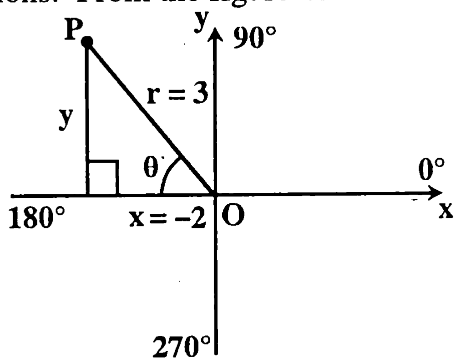
Now $\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{3}}$

$$\operatorname{cosec}\theta = \frac{r}{y} = \frac{\sqrt{3}}{1} = \sqrt{3}$$
$$\cos\theta = \frac{x}{r} = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$$
$$\sec\theta = \frac{r}{x} = \frac{\sqrt{3}}{\sqrt{2}} = \sqrt{\frac{3}{2}}$$
$$\tan\theta = \frac{y}{x} = \frac{1}{\sqrt{2}}$$
$$\cot\theta = \frac{x}{y} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

Q.7. If $\cos\theta = \frac{-2}{3}$ and terminal arm of the angle θ is in quadrant II, find the values of remaining trigonometric functions.

Solution:

As $\cos\theta = \frac{-2}{3}$ and θ is in quadrant II, so we complete the figure according to conditions. From the figure $x = -2$ and $r = 3$



By Pythagorean theorem

$$r^2 = x^2 + y^2$$
$$y^2 = r^2 - x^2$$
$$y = \sqrt{r^2 - x^2}$$
$$y = \sqrt{(3)^2 - (-2)^2}$$
$$y = \sqrt{9-4}$$
$$y = \sqrt{5}$$

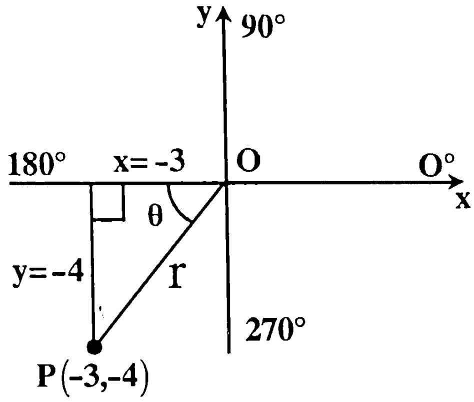
Now

$$\sin\theta = \frac{y}{r} = \frac{\sqrt{5}}{3}$$
$$\operatorname{cosec}\theta = \frac{r}{y} = \frac{3}{\sqrt{5}}$$
$$\cos\theta = \frac{x}{r} = \frac{-2}{3}$$
$$\sec\theta = \frac{r}{x} = \frac{-3}{2}$$
$$\tan\theta = \frac{y}{x} = \frac{-\sqrt{5}}{2}$$
$$\cot\theta = \frac{x}{y} = \frac{-2}{\sqrt{5}}$$

Q.8. If $\tan\theta = \frac{4}{3}$ and $\sin\theta < 0$, find the values of other trigonometric functions at θ.

Solution:

As $\tan\theta = \frac{4}{3}$ and $\sin\theta$ is –ve, which is possible in quadrant III only. We complete the figure.



From the figure $x = -3$ and $y = -4$

By pathagorean theorem

$r^2 = x^2 + y^2$

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-3)^2 + (-4)^2}$

$r = \sqrt{9+6}$

$r = \sqrt{25}$

$r = 5$

Now

$\sin\theta = \frac{y}{r} = \frac{-4}{5}$

$\text{cosec}\theta = \frac{r}{y} = \frac{-5}{4}$

$\cos\theta = \frac{x}{r} = \frac{-3}{5}$

$\sec\theta = \frac{r}{x} = \frac{-5}{3}$

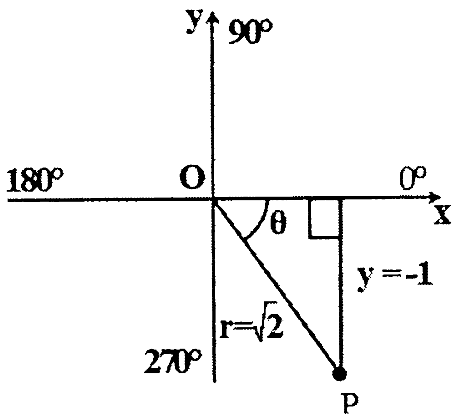
$\tan\theta = \frac{y}{x} = \frac{4}{3}$

$\cot\theta = \frac{x}{y} = \frac{3}{4}$

Q.9. If $\sin\theta = -\frac{1}{\sqrt{2}}$ and terminal side of the angle is not in quadrant III, find the values of $\tan\theta$, $\sec\theta$ and $\text{cosec}\theta$.

Solution:

As $\sin\theta = \frac{-1}{\sqrt{2}}$ and terminal side of angle is not in III quadrant, so it lies in quadrant IV.



From the figure $y = -1$ and $r = \sqrt{2}$

By Pathagorean theorem

$r^2 = x^2 + y^2$

$x^2 = r^2 - y^2$

$x = \sqrt{r^2 - y^2}$

$x = \sqrt{(\sqrt{2})^2 - (-1)^2}$

$x = \sqrt{2-1}$

$x = \sqrt{1}$

$x = 1$

Now

$\tan\theta = \frac{y}{x} = \frac{-1}{1} = -1$

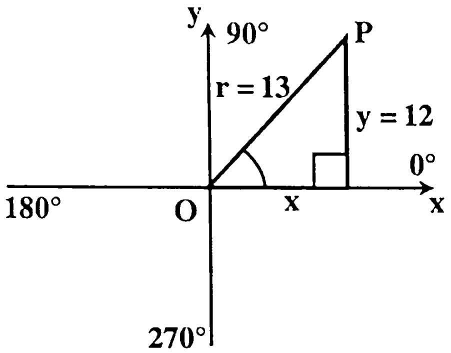
$\sec\theta = \frac{r}{x} = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\text{cosec}\,\theta = \frac{r}{y} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$

Q.10. If $\text{cosec}\theta = \frac{13}{12}$ and $\sec\theta > 0$, find the remaining trigonometric functions.

Solution:

As $\text{cosec}\theta = \frac{13}{12}$ and also $\sec\theta$ is +ve, which is only possible in quadrant I.



From the figure $y = 12$ and $r = 13$

By Pythagorean theorem

$r^2 = x^2 + y^2$

$x^2 = r^2 - y^2$

$x = \sqrt{r^2 - y^2}$

$x = \sqrt{(13)^2 - (12)^2}$

$x = \sqrt{169 - 144}$

$x = \sqrt{25}$

$x = 5$

Now

$\sin\theta = \frac{y}{r} = \frac{12}{13}$

$\text{cosec}\theta = \frac{r}{y} = \frac{13}{12}$

$\cos\theta = \frac{x}{r} = \frac{5}{13}$

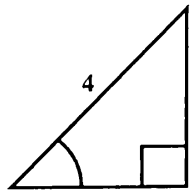
$\sec\theta = \frac{r}{x} = \frac{13}{5}$

$\tan\theta = \frac{y}{x} = \frac{12}{5}$

$\cot\theta = \frac{x}{y} = \frac{5}{12}$

Q.11. Find the values of trigonometric functions at the indicated angle θ in the right triangle.

(i)



From the figure Hypotenuse = 4 and Base = 3

By Pythagorean theorem we can find perpendicular

$(\text{Per.})^2 + (\text{Base})^2 = (\text{Hyp.})^2$

$(\text{Per.})^2 + (3)^2 = (4)^2$

$(\text{Per.})^2 = 16 - 9$

$(\text{Per.})^2 = 7$

$\text{Perpendicular} = \sqrt{7}$

Now $\sin\theta = \frac{\text{Per.}}{\text{Hyp.}} = \frac{\sqrt{7}}{4}$

$\text{cosec}\,\theta = \frac{\text{Hyp.}}{\text{Per.}} = \frac{4}{\sqrt{7}}$

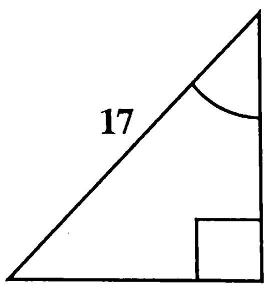
$\cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{3}{4}$

$\sec\theta = \frac{\text{Hyp.}}{\text{Base}} = \frac{4}{3}$

$\tan\theta = \frac{\text{Per.}}{\text{Base}} = \frac{\sqrt{7}}{3}$

$\cot\theta = \frac{\text{Base}}{\text{Per.}} = \frac{3}{\sqrt{7}}$

(ii)



From the figure

Hypotenuse = 17

Perpendicular = 8

Base = 15

Now

$$\sin\theta = \frac{\text{Per.}}{\text{Hyp.}} = \frac{8}{17}$$

$$\operatorname{cosec}\theta = \frac{\text{Hyp.}}{\text{Per.}} = \frac{17}{8}$$

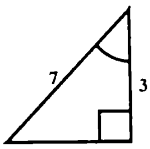
$$\cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{15}{17}$$

$$\sec\theta = \frac{\text{Hyp.}}{\text{Base}} = \frac{17}{15}$$

$$\tan = \frac{\text{Per.}}{\text{Base}} = \frac{8}{15}$$

$$\cot\theta = \frac{\text{Base}}{\text{Per.}} = \frac{15}{8}$$

(iii)



From the figure Hypotenuse = 7, Base = 3
We can find perpendicular by Pythagorean theorem.

$$(\text{Base})^2 + (\text{Per.})^2 = (\text{Hyp.})^2$$
$$(\text{Per.})^2 + (3)^2 = (7)^2$$
$$(\text{Per.})^2 = 49 - 9$$
$$(\text{Per.})^2 = 40$$
$$\text{Per.} = \sqrt{40}$$
$$\text{Per.} = \sqrt{4\times10}$$
$$\text{Per.} = 2\sqrt{10}$$

Now, $$\sin\theta = \frac{\text{Per.}}{\text{Hyp.}} = \frac{2\sqrt{10}}{7}$$

$$\operatorname{cosec}\theta = \frac{\text{Hyp.}}{\text{Per.}} = \frac{7}{2\sqrt{10}}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{3}{7}$$

$$\sec\theta = \frac{\text{Hyp.}}{\text{Base}} = \frac{7}{3}$$

$$\tan\theta = \frac{\text{Per.}}{\text{Base}} = \frac{2\sqrt{10}}{3}$$

$$\cot\theta = \frac{\text{Base}}{\text{Per.}} = \frac{3}{2\sqrt{10}}$$

Q.12. Find the values of the trigonometric functions. Do not use trigonometric table or calculator.

Solution:

We know that $2k\pi + \theta = \theta$, where $k \in Z$

(i) tan 30°

$$30^\circ = 30\ \frac{\pi}{180}\ \text{radian} = \frac{\pi}{6}\ \text{radian}$$

$$\tan 30^\circ = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

(ii) tan 330°

$$\tan 330^\circ = \tan(360^\circ - 30^\circ)$$
$$= \tan\ 2\pi - \frac{\pi}{6}$$
$$= \tan\ -\frac{\pi}{6}$$
$$= -\tan\frac{\pi}{6}$$
$$= -\frac{1}{\sqrt{3}}$$

(iii) sec 330°

$$\sec 330^\circ = \sec(360^\circ - 30^\circ)$$
$$= \sec\ 2\pi - \frac{\pi}{6}$$
$$= \sec\ -\frac{\pi}{6}$$
$$= \sec\frac{\pi}{6}$$
$$= \frac{2}{\sqrt{3}}$$

(iv) $\cot\frac{\ }{4}$

$$\cot\frac{\pi}{4} = 1$$

(v) $\cos\frac{2}{3}$

$$\cos\frac{2\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

(vi) $\textbf{cosec}\ \dfrac{2\pi}{3}$

$\text{cosec}\dfrac{2\pi}{3} = \text{cosec}\ \dfrac{\pi}{3} = \dfrac{2}{\sqrt{3}}$

(vii) **cos (– 450°)**

$\cos(-450°) = \cos(-360° - 90°)$

$= \cos\ -2\pi - \dfrac{\pi}{2}$

$= \cos\ 2(-1)\pi - \dfrac{\pi}{2}$

$= \cos\ -\dfrac{\pi}{2}$

$= \cos\ \dfrac{\pi}{2}\ = 0$

(viii) **tan (– 9π)**

$\tan(-9\pi) = \tan(-8\pi - \pi)$

$= \tan[2(-4)\pi - \pi]$

$= \tan(-\pi)$

$= -\tan\pi$

$= -(0) = 0$

(ix) $\textbf{cos}\ \dfrac{-5\pi}{6}$

$\cos\ -\dfrac{5\pi}{6} = -\cos\ \dfrac{\pi}{6}$

$= -\dfrac{\sqrt{3}}{2}$

(x) $\textbf{sin}\ 7\dfrac{}{6}$

$\sin\dfrac{7\pi}{6} = \sin\quad + \dfrac{}{6}$

$= -\sin\dfrac{\pi}{6} = -\dfrac{1}{2}$

(xi) $\textbf{cot}\ \dfrac{7}{6}$

$\cot\dfrac{7\pi}{6} = \cot\ \pi + \dfrac{\pi}{6}$

$= \cot\dfrac{\pi}{6} = \sqrt{3}$

(xii) **cos 225°**

$\cos 225° = \cos(180° + 45°)$

$= \cos\ \pi + \dfrac{\pi}{4}$

$= -\cos\dfrac{\pi}{4} = -\dfrac{1}{\sqrt{2}}$

EXERCISE 7.4

In problem 1— 6, simply each expression to single trigonometric function:

Q.1. $\dfrac{\sin^2 x}{\cos^2 x}$

Solution: $\dfrac{\sin^2 x}{\cos^2 x} = \tan x^2$

Q.2. $\tan x \sin x \sec x$

Solution: $\tan x \sin x \sec x$

$= \dfrac{\sin x}{\cos x} . \sin x . \dfrac{1}{\cos x}$

$= \dfrac{\sin^2 x}{\cos^2 x}$

$= \tan^2 x$

Q.3. $\dfrac{\tan x}{\sec x}$

Solution: $\dfrac{\tan x}{\sec x} = \tan x \div \sec x$

$= \dfrac{\sin x}{\cos x} \div \dfrac{1}{\cos x}$

$= \dfrac{\sin x}{\cancel{\cos x}} \times \cancel{\cos x}$

$= \sin x$

Q.4. $1 - \cos^2 x$

Solution: $1 - \cos^2 x$

$= \sin^2 x + \cancel{\cos^2} x - \cancel{\cos}^2 x$

$= \sin^2 x$

Q.5. $\sec^2 x - 1$

Solution: $\sec^2 x - 1$

$= 1 + \tan^2 x - 1$

$= \tan^2 x$

Q.6. $\sin^2 x . \cot^2 x$

Solution:

$\sin^2 x . \cot^2 x$

$= \sin^2 x \dfrac{\cos^2 x}{\sin^2 x}$

$= \cos^2 x$

In problem 7 — 24, verify the identities

Q.7. (1–sinθ) (1+sinθ) = cos²θ

Solution:

L.H.S $= (1 - \sin\theta)(1 + \sin\theta)$

$= (1)^2 - (\sin\theta)^2$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

L.H.S = R.H.S

Q.8. $\dfrac{\sin\theta + \cos\theta}{\cos\theta} = 1 + \tan\theta$

Solution: Let

L.H.S $= \dfrac{\sin\theta + \cos\theta}{\cos\theta}$

$= \dfrac{\cos\theta + \sin\theta}{\cos\theta}$

$= \dfrac{\cos\theta}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= 1 + \tan\theta \qquad \because \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

L.H.S = R.H.S

Q.9. (tanθ + cotθ) tanθ = sec²θ

Solution: Let

L.H.S $= (\tan\theta + \cot\theta)\tan\theta$

$= \tan^2\theta + \cot\theta . \tan\theta$

$= \tan^2\theta + \dfrac{1}{\tan\theta} . \tan\theta$

$= 1 + \tan^2\theta$

$= \sec^2\theta \qquad (1 + \tan^2\theta = \sec^2\theta)$

L.H.S = R.H.S

Q.10. (Cotθ +cosecθ)(tanθ– sinθ) =secθ –cosθ

Solution: Let

L.H.S $= (\cot\theta + \operatorname{cosec}\theta)(\tan\theta - \sin\theta)$

$= \dfrac{1}{\tan\theta} + \dfrac{1}{\sin\theta} (\tan\theta - \sin\theta)$

$= \dfrac{\sin\theta + \tan\theta}{\tan\theta . \sin\theta} (\tan\theta - \sin\theta)$

$= \dfrac{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}{\tan\theta . \sin\theta}$

$= \dfrac{(\tan\theta)^2 - (\sin\theta)^2}{\tan\theta . \sin\theta}$

$= \dfrac{\tan^2\theta - \sin^2\theta}{\tan\theta . \sin\theta}$

$= \dfrac{\tan^2\theta}{\tan\theta . \sin\theta} - \dfrac{\sin^2\theta}{\tan\theta . \sin\theta}$

$= \dfrac{\tan\theta}{\sin\theta} - \dfrac{\sin\theta}{\tan\theta}$

$= (\tan\theta \div \sin\theta) - (\sin\theta \div \tan\theta)$

$= \dfrac{\sin\theta}{\cos\theta} \div \sin\theta - \sin\theta \div \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta} \times \dfrac{1}{\sin\theta} - \sin\theta \times \dfrac{\cos\theta}{\sin\theta}$

$= \sec\theta - \cos\theta$

L.H.S = R.H.S

Q.11. $\dfrac{\sin\theta + \cos\theta}{\tan^2\theta - 1} = \dfrac{\cos^2\theta}{\sin\theta - \cos\theta}$

Solution: Let

L.H.S $= \dfrac{\sin\theta + \cos\theta}{\tan^2\theta - 1}$

$= (\sin\theta + \cos\theta) \div (\tan^2\theta - 1)$

$= (\sin\theta + \cos\theta) \div \dfrac{\sin^2\theta}{\cos^2\theta} - 1$

$= (\sin\theta + \cos\theta) \div \dfrac{\sin^2 - \cos^2}{\cos^2}$

$= (\sin\theta + \cos\theta) \times \dfrac{\cos^2}{(\sin^2\theta - \cos^2\theta)}$

$= \dfrac{(\sin\theta + \cos\theta) \times \cos^2\theta}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}$

$= \dfrac{\cos^2\theta}{\sin\theta - \cos\theta}$

L.H.S = R.H.S

Q.12. $\frac{\cos^2\theta}{\sin\theta}+\sin\theta = \operatorname{cosec}\theta$

Solution: Let

$$\text{L.H.S} = \frac{\cos^2\theta}{\sin\theta}+\sin\theta$$

$$= \frac{\cos^2\theta+\sin^2\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta}$$

$$= \operatorname{cosec}\theta$$

L.H.S = R.H.S

Q.13. $\sec\theta - \cos\theta = \tan\theta.\sin\theta$

Solution: Let

$$\text{L.H.S} = \sec\theta - \cos\theta$$

$$= \frac{1}{\cos\theta} - \cos\theta$$

$$= \frac{1-\cos^2\theta}{\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}\sin\theta$$

$$= \tan\theta.\sin\theta$$

L.H.S = R.H.S

Q.14. $\frac{\sin^2\theta}{\cos\theta}+\cos\theta = \sec\theta$

Solution: Let

$$\text{L.H.S} = \frac{\sin^2\theta}{\cos\theta}+\cos\theta$$

$$= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta$$

L.H.S = R.H.S

Q.15. $\tan\theta + \cot\theta = \sec\theta.\operatorname{cosec}\theta$

Solution: Let

$$\text{L.H.S} = \tan\theta + \cot\theta$$

$$= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta.\sin\theta}$$

$$= \frac{1}{\cos\theta.\sin\theta} \quad (\sin^2\theta+\cos^2\theta=1)$$

$$= \frac{1}{\cos\theta}.\frac{1}{\sin\theta}$$

$$= \sec\theta.\operatorname{cosec}\theta$$

L.H.S = R.H.S

Q.16. $(\tan\theta+\cot\theta)(\cos\theta+\sin\theta) = \sec\theta+\operatorname{cosec}\theta$

Solution: Let

$$\text{L.H.S} = (\tan\theta+\cot\theta)(\cos\theta+\sin\theta)$$

$$= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}(\cos\theta+\sin\theta)$$

$$= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta.\sin\theta}(\cos\theta+\sin\theta)$$

$$= \frac{1}{\cos\theta.\sin\theta}(\cos\theta+\sin\theta)$$

$$= \frac{\cos\theta+\sin\theta}{\cos\theta.\sin\theta}$$

$$= \frac{\cancel{\cos\theta}}{\cancel{\cos\theta}.\sin\theta}+\frac{\cancel{\sin\theta}}{\cos\theta.\cancel{\sin\theta}}$$

$$= \frac{1}{\sin\theta}+\frac{1}{\cos\theta}$$

$$= \operatorname{cosec}\theta + \sec\theta$$

$$= \sec\theta + \operatorname{cosec}\theta$$

L.H.S = R.H.S

Q.17. $\sin\theta(\tan\theta+\cot\theta) = \sec\theta$

Solution: Let

$$\text{L.H.S} = \sin\theta(\tan\theta+\cot\theta)$$

$$= \sin\theta\ \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$$

$$= \cancel{\sin\theta}\ \frac{\sin^2\theta+\cos^2\theta}{\cos\theta.\cancel{\sin\theta}}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta$$

L.H.S = R.H.S

Q.18. $\frac{1+\cos}{\sin}+\frac{\sin}{1+\cos}=2\text{cosec}$

Solution: Let

$$\text{L.H.S} = \frac{1+\cos\theta}{\sin\theta}+\frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{(1+\cos\theta)^2+(\sin\theta)^2}{(\sin\theta)(1+\cos\theta)}$$

$$= \frac{(1)^2+2(1)(\cos\theta)+\cos^2\theta+\sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1+2\cos\theta+1}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2}{\sin\theta}$$

$$= 2\text{cosec}\theta$$

L.H.S = R.H.S

Q.19. $\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta} = 2\text{cosec}^2\theta$

Solution:

$$\text{L.H.S} = \frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}$$

$$= \frac{1+\cos\theta+1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{2}{(1)^2-(\cos^2\theta)}$$

$$= \frac{2}{1-\cos^2\theta}$$

$$= \frac{2}{\sin^2\theta}$$

$$= 2\text{ cosec}^2\theta$$

L.H.S = R.H.S

Q.20. $\frac{1+\sin}{1-\sin}-\frac{1-\sin}{1+\sin}=4\tan\ \sec$

Solution: Let

$$\text{L.H.S} = \frac{1+\sin\theta}{1-\sin\theta}=\frac{1-\sin\theta}{1+\sin\theta}$$

$$= \frac{(1+\sin\theta)^2-(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{(1+\sin^2\theta+2\sin\theta)-(1+\sin^2\theta-2\sin\theta)}{(1)^2-(\sin\theta)^2}$$

$$= \frac{1+\sin^2\theta+2\sin\theta-1-\sin^2\theta+2\sin\theta}{1-\sin^2\theta}$$

$$= \frac{4\sin\theta}{\cos^2\theta}$$

$$= \frac{4\sin\theta}{\cos\theta.\cos\theta}$$

$$= 4\frac{\sin\theta}{\cos\theta}.\frac{1}{\cos\theta}$$

$$= 4\tan\theta.\ \sec\theta$$

L.H.S = R.H.S

Q.21. $\sin^3\theta = \sin\theta - \sin\theta.\cos^2\theta$

Solution: Let

$$\text{L.H.S} = \sin^3\theta$$

$$= \sin\theta\ .\ \sin^2\theta$$

$$= \sin\theta(1-\cos^2\theta)$$

$$= \sin\theta - \sin\theta.\cos^2\theta$$

L.H.S = R.H.S

Q.22. $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

Solution: Let

$$\text{L.H.S} = \cos^4\theta - \sin^4\theta$$

$$= (\cos^2\theta)^2 - (\sin^2\theta)^2$$

$$= (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$$

$$= (1)(\cos^2\theta - \sin^2\theta)$$

$$= \cos^2\theta - \sin^2\theta$$

L.H.S = R.H.S

Q.23. $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}} = \dfrac{\sin\theta}{1-\cos\theta}$

Solution: Let

$$\text{L.H.S} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{(1)^2-(\cos\theta)^2}} = \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} = \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta}\times\frac{1-\cos\theta}{1-\cos\theta}$$

$$= \frac{(1)^2-(\cos\theta)^2}{\sin\theta(1-\cos\theta)}$$

$$= \frac{1-\cos^2\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{\sin^2\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{\sin\theta}{1-\cos\theta}$$

L.H.S = R.H.S

Q.24. $\sqrt{\dfrac{\sec\theta+1}{\sec\theta-1}} = \dfrac{\sec\theta+1}{\tan\theta}$

Solution: Let

$$\text{L.H.S} = \sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$$

$$= \sqrt{\frac{\sec\theta+1}{\sec\theta-1}\times\frac{\sec\theta+1}{\sec\theta+1}}$$

$$= \sqrt{\frac{(\sec\theta+1)^2}{(\sec\theta)^2-(1)^2}}$$

$$= \sqrt{\frac{(\sec\theta+1)^2}{\sec^2\theta-1}}$$

$$= \sqrt{\frac{(\sec\theta+1)^2}{\tan^2\theta}}$$

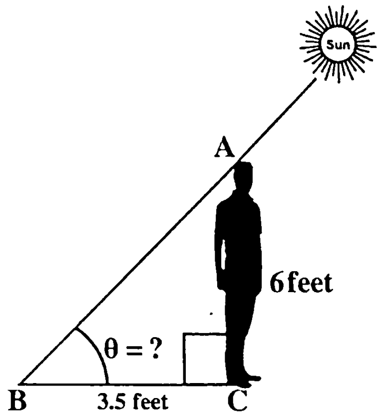
$$= \frac{\sec\theta+1}{\tan\theta}$$

L.H.S = R.H.S

EXERCISE 7.5

Q.1. Find the angle of elevation of the sun if a 6 feet man casts a 3.5 feet shadow.

Solution:



From the figure we observe that

Height of man $= m\overline{AC} = 6$ feet

Length of shadow $= m\overline{BC} = 3.5$ feet

Angle of elevation $= \theta\ ? =$

Using the fact that

$$\tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan\theta = \frac{6}{3.5}$$

$$\theta = \tan^{-1}\frac{6}{3.5}$$

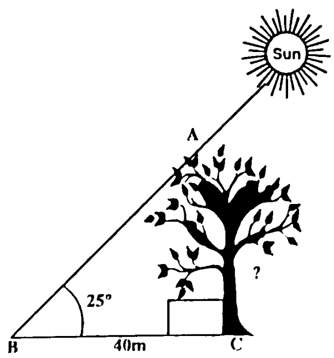
$$\theta = 59.7436$$

$$\theta = 59.74^\circ$$

So, the angle of elevation is $59^\circ\,44'37''$.

Q.2. A tree casts a 40 meters shadow when the angle of elevation of the sun is 25°. Find the height of the tree.

Solution:



From the figure

Height of tree $= m\overline{AC}\ ? =$

Length of shadow $= m\overline{BC} = 40$m

Angle of elevation $= \theta = 25^\circ$

Using the fact that

$$\tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan 25^\circ = \frac{m\overline{AC}}{40}$$

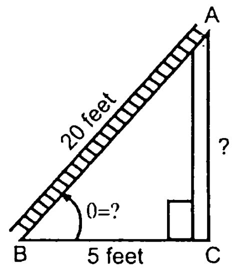
$$m\overline{AC} = 40 \times \tan 25^\circ$$

$$m\overline{AC} = 18.65 \text{ m}$$

So, height of tree is 18.65 m

Q.3. A 20 feet long ladder is leaning against a wall. The bottom of the ladder is 5 feet from the base of the wall. Find the acute angle (angle of elevation) the ladder makes with the ground.

Solution:



From the figure

Length of ladder $= m\overline{AB} = 20$ feet

Distance of ladder from the wall$=m\overline{BC}=5$ feet

Angle of elevation $= \theta\ ? =$

Using the fact that

$$\cos\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\cos\theta = \frac{5\text{ft.}}{20\text{ft.}}$$

$$\cos\theta = 0.25$$

$$\theta = \cos^{-1}(0.25)$$

$$\theta = 75.5225$$

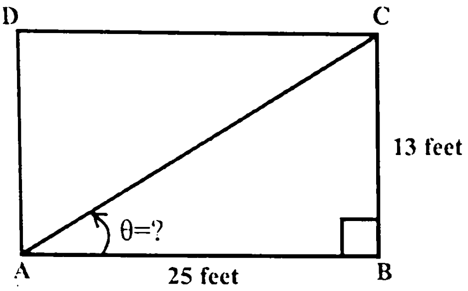
$$\theta = 75.5^\circ$$

or $\theta = 75^\circ\ 30'$

So, angle of elevation is $75^\circ 31' 21''$

Q.4. The base of rectangle is 25 feet and the height of rectangle is 13 feet. Find the angle that the diagonal of the rectangle makes with the base.

Solution:



From the figure

Base of rectangle = $m\overline{AB}$ = 25 feet

Height of rectangle = $m\overline{BC}$ = 13 feet

Diagonal $\overline{AC}$ is taken

Angle between diagonal and base = θ ? =

Using the fact that

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\tan\theta = \frac{13}{25}$$

$$\theta = \tan^{-1}\frac{13}{25}$$

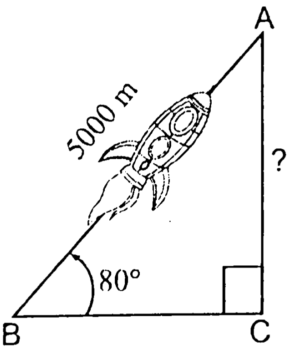
$$\theta = 27.4744$$

$$\theta = 27.47^\circ$$

So, angle between diagonal and base is 27°28′28″.

Q.5. A rocket is launched and climbs at a constant angle of 80°. Find the altitude of the rocket after it travels 5000 meter.

Solution:



From the figure

Distance travelled by rocket = $m\overline{AB}$ = 5000m

Altitude of rocket = $m\overline{AC}$? =

Angle of elevation = θ = 80°

Using $\sin\theta = \frac{m\overline{AC}}{m\overline{AB}}$

$$\sin 80^\circ = \frac{m\overline{AC}}{5000}$$

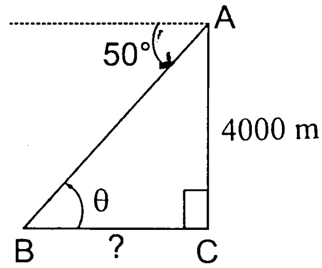
$$m\overline{AC} = 5000 \times \sin 80^\circ$$

$$m\overline{AC} = 4924.04\text{m}$$

So, the altitude of rocket is 4924.04m

Q.6. An aeroplane pilot flying at an altitude of 4000m wishes to make an approach to an airport at an angle of 50° with the horizontal. How far from the airport will the plane be when the pilot begins to descend?

Solution:



From the figure

Altitude of aeroplane = $m\overline{AC}$ = 4000m

Distance of plane from airport = $m\overline{BC}$? =

Angle of depression = 50°

As the alternate angles of parallel lines are equal, so angle

$$\theta = 50^\circ$$

Using the fact that, $\tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$

$$\tan 50^\circ = \frac{4000\text{m}}{m\overline{BC}}$$

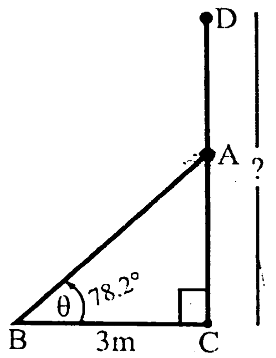
$$m\overline{BC} = \frac{4000\text{m}}{\tan 50^\circ}$$

$$m\overline{BC} = 3356.4\text{ m}$$

So, the distance of aeroplane from airport is 3356.4 m.

Q.7. A guy wire (supporting wire) runs from the middle of a utility pole to the ground. The wire makes an angle of 78.2° with the ground and touch the ground 3 meters from the base of the pole. Find the height of the pole.

Solution:



From the figure

Height of pole $= m\overline{CD}$? =

Distance of wire from the base of the pole

$= m\overline{BC} = 3m$

Angle of elevation $= \theta = 78.2°$

As the wire is attached with the pole at its middle point A, so, first we find $m\overline{AC}$

Using the fact that

$$\tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan 78.2 = \frac{m\overline{AC}}{3}$$

$$m\overline{AC} = 3m \times \tan 78.2°$$

$$m\overline{AC} = 14.36 \text{ m}$$

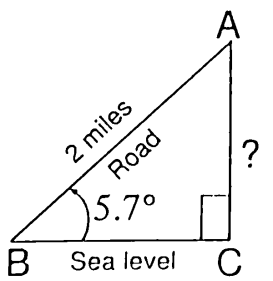
So Height of pole is $= m\overline{DC} = 2(m\overline{AC})$

$= 2 \times 14.36$ m

$= 28.72$ m

Q.8. A road is inclined at an angle 5.7°. Suppose that we drive 2 miles up this road starting from sea level. How high above sea level are we? 07(121)

Solution:



From the figure

Distance covered on road $= m\overline{AB} = 2$ miles

Angle of inclination $= \theta = 5.7°$

Height from sea level $= m\overline{AC}$? =

Using the fact that,

$$\sin\theta = \frac{m\overline{AC}}{m\overline{AB}}$$

$$\sin 5.7° = \frac{m\overline{AC}}{2}$$

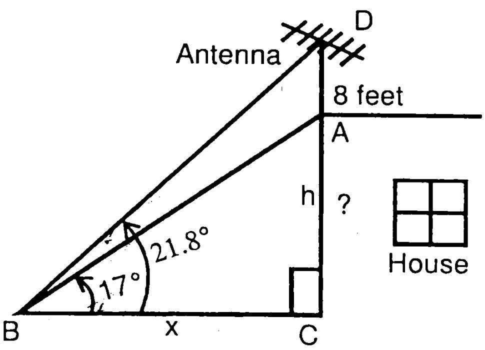
$$m\overline{AC} = 2 \times \sin 5.7°$$

$$m\overline{AC} = 0.199 \text{ mile}$$

Hence, we are at the height of 0.199 mile from the sea level.

Q.9. A television antenna of 8 feet height is located on the top of a house. From a point on the ground the angle of elevation to the top of the house is 17° and the angle of elevation to the top of antenna is 21.8°. Find the height of the house.

Solution:



From the figure

Distance of point from house $= m\overline{BC} = x$

Height of house $= m\overline{AC} = h =$?

Height of antenna $= m\overline{AD} = 8$ feet

Angle of elevation of top of house $= 17°$

Angle of elevation of top of antenna $= 21.8°$

In right angled ΔABC

$$\tan 17° = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan 17° = \frac{h}{x}$$

$$x = \frac{1}{\tan 17°} \times h$$

$x = 3.271 \times h$ ....(i)

Now in right angle ΔDBC

$$\tan 21.8 = \frac{m\overline{CD}}{m\overline{BC}}$$

$$\tan 21.8 = \frac{m\overline{AD}+m\overline{AC}}{m\overline{BC}}$$

$$\tan 21.8 = \frac{8+h}{x}$$

$$0.40 = \frac{8+h}{3.271h} \quad \text{[From (i)]}$$

$$0.40 \times 3.271h = 8 + h$$

$$1.3084\ h - h = 8$$

$$(1.3084 - 1)\ h = 8$$

$$0.3084\ h = 8$$

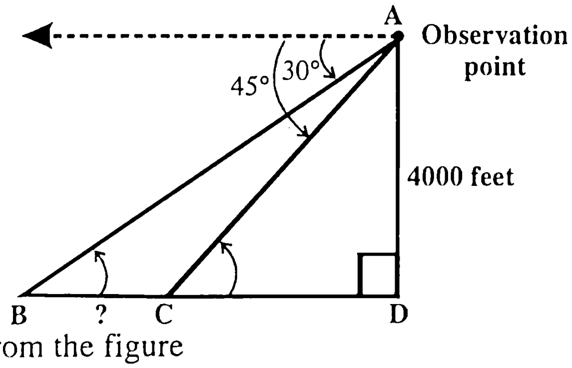
$$h = \frac{8}{0.3084}$$

$$h = 25.94 \text{ feet}$$

So, the height of the house is 25.94 feet

Q.10. From an observation point, the angles of depression of two boats in line with this point are found to 30° and 45°. Find the distance between the two boats if the point of observation is 4000 feet high.

Solution:



From the figure

Height of observation point= $m\overline{AD}$ =4000 feet

Distance between boats = $m\overline{BC}$? =

Angles of depression of points B and C are 30° and 45° respectively from point A.

As the alternate angles of parallel lines are equal, so

$m\angle B = 30°$ and $m\angle C = 45°$

Now in right angled ΔACD

$$\tan 45° = \frac{m\overline{AD}}{m\overline{CD}}$$

$$1 = \frac{4000}{m\overline{CD}}$$

$$m\overline{CD} = 4000 \text{ feet}$$

Now in right angled ΔBCD

$$\tan 30° = \frac{m\overline{AD}}{m\overline{BD}}$$

$$\frac{1}{\sqrt{3}} = \frac{4000}{m\overline{BC}+m\overline{CD}}$$

$$\frac{1}{\sqrt{3}} = \frac{4000}{m\overline{BC}+4000}$$

$$m\overline{BC} + 4000 = 4000\sqrt{3}$$

$$m\overline{BC} = 4000\sqrt{3} - 4000$$

$$m\overline{BC} = 6928.20 - 4000$$

$$m\overline{BC} = 2928.20 \text{ feet}$$

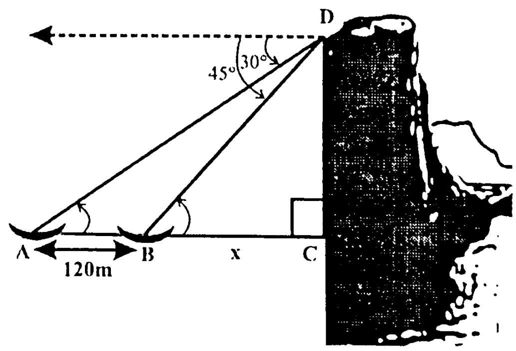
So, the distance between boats is 2928.2 feet.

Q.11. Two ships, which are in line with the base of a vertical cliff are 120 meters apart. The angles of depression from the top of the cliff to the ships are 30° and 45°, as shown in the diagram.

(a) Calculate the distance BC

(b) Calculate the height CD of the cliff.

Solution:



From the figure

Height of cliff = $\overline{CD}$ = h = ?

Distance = $\overline{BC}$ = x = ?

Distance between boats = $\overline{AB}$ = 120 m

Angles of depression from point D to points A and B are 30° and 45° respectively.

As the alternate angles of parallel lines are equal, so $m\angle A = 30°$ and $m\angle B = 45°$

In right angled ΔBCD

$$\tan 45^\circ = \frac{m\overline{CD}}{m\overline{BC}}$$

$$1 = \frac{h}{x}$$

$$x = h \quad \ldots\ldots (i)$$

Now in right angled ΔACD

$$\tan 30^\circ = \frac{m\overline{CD}}{m\overline{AC}}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{m\overline{AB} + m\overline{BC}}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{120 + x}$$

$$120 + x = \sqrt{3}\,h$$

$$120 + h = \sqrt{3}\,h \qquad (\quad x = h)$$

$$120 = \sqrt{3}h - h$$

$$120 = \left(\sqrt{3} - 1\right)h$$

$$120 = (1.7321 - 1)\,h$$

$$120 = 0.7321\ h$$

$$\frac{120}{0.7321} = h$$

$h = 163.91$ m

As $x = h$, so

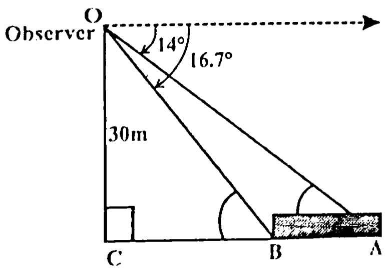
$x = 163.91$ m or 164m

Thus Distance $m\overline{BC} = 164m$

Height of cliff $= m\overline{CD} = 164m$

Q.12. Suppose that we are standing on a bridge 30 meter above a river watching a log (piece of wood) floating towards us. If the angle with the horizontal to the front of the log is 16.7° and angle with the horizontal to the back of the log is 14°, how long is the log?

Solution:



From the figure

Height of observer's position $= m\overline{OC} = 30m$

Length of log of wood $= m\overline{AB} = x = ?$

Angles of depression from point O of the points A and B are 14° and 16.7° respectively.

In right angled ΔOBC

$$\tan 16.7^\circ = \frac{m\overline{OC}}{m\overline{BC}}$$

$$0.30 = \frac{30}{m\overline{BC}}$$

$$m\overline{BC} = \frac{30}{0.30}$$

$$m\overline{BC} = 100m$$

Now in right angled ΔOAC

$$\tan 14^\circ = \frac{m\overline{OC}}{m\overline{AC}}$$

$$0.249 = \frac{30}{m\overline{AB} + m\overline{BC}}$$

$$0.249 = \frac{30}{(x + 100)}$$

$$0.249(x + 100) = 30$$

$$x + 100 = \frac{30}{0.249}$$

$$x + 100 = 120.482$$

$$x = 120.482 - 100$$

$$x = 20.482 \text{ m}$$

So the length of log is 20.482 m.

MISCELLANEOUS EXERCISE –7

Q. 1 Multiple choice questions:
Four possible answers are given for the following questions. Tick ( ) the correct answer.

1. The union of two non–collinear rays, which have common end point is called
   (a) an angle (b) a degree
   (c) a minute (d) a radian
2. The system of measurement in which the angle is measured in radians is called
   (a) CGS system
   (b) sexagesimal system
   (c) MKS system
   (d) circular system
3. $\sec\theta \cot\theta =$
   (a) $\sin\theta$ (b) $\frac{1}{\cos\theta}$
   (c) $\frac{1}{\sin\theta}$ (d) $\frac{\sin\theta}{\cos\theta}$
4. $\operatorname{cosec}^2\theta - \cot^2\theta =$
   (a) -1 (b) 1
   (c) 0 (d) $\tan\theta$
5. If 'r' is the radius of a circle, then its circumference is:
   (a) $\frac{\pi}{2}r$ (b) πr
   (c) $2\pi r$ (d) $4\pi r$
6. $\sec^2\theta =$
   (a) $1 - \sin^2\theta$ (b) $1 + \tan^2\theta$
   (c) $1 + \cos^2\theta$ (d) $1 - \tan^2\theta$
7. $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$
   (a) $2\sec^2\theta$ (b) $2\cos^2\theta$
   (c) $\sec^2\theta$ (d) $\cos\theta$
8. How many right angles are there in 360 degrees?
   (a) two (b) four
   (c) six (d) eight
9. $20° = \ldots\ldots\ldots$
   (a) $360'$ (b) $630'$
   (c) $1200'$ (d) $3600'$
10. $\frac{3\pi}{4}$ radians =
    (a) 115° (b) 135°
    (c) 150° (d) 30°
11. In degree measurement, $1°$ is equal to:
    (a) $1'$ (b) $60'$
    (c) $90'$ (d) $360'$
12. In degree measurement, $1'$ is equal to:
    (a) $1''$ (b) $60''$
    (c) $90''$ (d) $360''$
13. $\frac{1}{2}\operatorname{cosec}45°$
    (a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$
    (c) $\sqrt{2}$ (d) $\frac{\sqrt{3}}{2}$
14. If $\tan\theta = \sqrt{3}$, then θ is equal to
    (a) 90° (b) 45°
    (c) 60° (d) 30°
15. The radian measure of an angle that form a complete circle is:
    (a) $\frac{\pi}{2}$ (b) π
    (c) 2π (d) 4π
16. $\frac{\pi}{2}$ radians =
    (a) 30° (b) 45°
    (c) 60° (d) 90°
17. $\frac{\pi}{3}$ radians =
    (a) 30° (b) 45°
    (c) 60° (d) 90°
18. $1° =$
    (a) 180π radian (b) π radian
    (c) $\frac{\pi}{180}$ radian (d) $\frac{180}{\pi}$ radian

19. Area of a circular sector =

(a) $r\theta$ (b) $r^2\theta$

(c) $\frac{1}{2}r\theta$ (d) $\frac{1}{2}r^2\theta$

20. 2π radians =

(a) 0° (b) 90°

(c) 180° (d) 360°

21. π radians =

(a) 0° (b) 90°

(c) 180° (d) 360°

22. $\frac{1}{\text{Cos}\theta} =$

(a) $\text{Sin}\theta$ (b) $\text{Sec}\theta$

(c) $\text{Cosec}\,\theta$ (d) $\text{Cot}\theta$

23. $\frac{\pi}{6}$ radians =

(a) 30° (b) 45°

(c) 60° (d) 90°

24. $\text{Sin}45^\circ =$

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

25. $1^\circ =$

(a) 0.0175 radians

(b) 0.175 radian

(c) 1.75 radians

(d) 175 radians

26. A part of circumference of a circle is called:

(a) radius (b) chord

(c) sector (d) arc

27. $\tan 45^\circ =$

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

28. 1 radian =

(a) $(180\pi)^\circ$ (b) $(180)^\circ$

(c) $\left(\frac{\pi}{180}\right)^\circ$ (d) $\left(\frac{180}{\pi}\right)^\circ$

29. $\frac{1}{\text{Sin}\theta} =$

(a) $\text{Cos}\theta$ (b) $\text{Sec}\theta$

(c) $\text{Cosec}\,\theta$ (d) $\text{Cot}\,\theta$

30. $\frac{\pi}{4}$ radians =

(a) 30° (b) 45°

(c) 60° (d) 90°

31. $\frac{1}{\tan\theta} =$

(a) $\tan\theta$ (b) $\text{Sec}\theta$

(c) $\text{Cosec}\,\theta$ (d) $\text{Cot}\theta$

32. $\frac{3\pi}{2}$ radians=

(a) 90° (b) 180°

(c) 270° (d) 360°

33. $\text{Cos}45^\circ =$

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

34. Formula for arc length is:

(a) $= r\theta$ (b) $r = \theta$

(c) $\theta = r$ (d) $= \frac{r}{\theta}$

35. $\text{Cosec}\,45^\circ =$

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

36. $\text{Sin}60^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

37. $\text{Cot}45^\circ =$

(a) 1 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 0

38. $\text{Cos}60^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

39. $\text{Cos}30^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

40. $\tan 30^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

41. $\text{Cot}30^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

42. In which quadrant only $\text{Cos}\theta$ and $\text{Sec}\theta$ are positive?

(a) I (b) II

(c) III (d) IV

43. $\text{Cosec}\,30^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

44. $\text{Sec}45^\circ =$

(a) 1 (b) $\sqrt{2}$

$\frac{1}{\sqrt{2}}$

45. $\text{Sin}30^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

46. In which quadrant θ lie when $\text{Cos}\theta < 0, \text{Sin}\theta < 0$?

(a) I (b) II

(c) III (d) IV

47. In which quadrant θ lie when $\text{Sec}\theta > 0, \text{Sin}\theta < 0$?

(a) I (b) II

(c) III (d) IV

48. $\text{Sec}60^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

49. $\text{Cosec}\,60^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

50. In which quadrant only $\text{Sin}\theta$ and $\text{Cosec}\,\theta$ are positive?

(a) I (b) II

(c) III (d) IV

51. $\text{Sec}30^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\frac{2}{\sqrt{3}}$

52. In which quadrant only $\tan\theta$ and $\text{Cot}\theta$ are positive?

(a) I (b) II

(c) III (d) IV

53. In which quadrant θ lie when $\text{Sin}\theta > 0, \tan\theta < 0$?

(a) I (b) II

(c) III (d) IV

54. $\tan 60^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

55. $\text{Cot}60^\circ =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

56. In which quadrant θ lie when $\text{Cos}\theta < 0, \tan\theta < 0$?

(a) I (b) II
(c) III (d) IV

57. Cos . Sec=

(a) 1 (b) tan
(c) 0 (d) Cot

58. In which quadrant θ lie when $\text{Sin}\theta < 0, \text{Sec}\theta < 0$?

(a) I (b) II
(c) III (d) IV

59. $\text{Sin}^2\theta + \text{Cos}^2\theta =$

(a) $\tan^2\theta$ (b) $\text{Cot}^2\theta$
(c) 1 (d) 0

60. $1+\tan^2\theta =$

(a) $\text{Sin}^2\theta$ (b) $\text{Cos}^2\theta$
(c) $\text{Cosec}^2\theta$ (d) $\text{Sec}^2\theta$

61. Angles between 180° and 270° are in which quadrant?

(a) I (b) II
(c) III (d) IV

62. Angles between 0° and 90° are in which quadrant?

(a) I (b) II
(c) III (d). IV

63. Fundamental trigonometric ratios are

(a) 3 (b) 4
(c) 5 (d) 6

64. Which one is a quadrantal angle?

(a) 30° (b) 45°
(c) 60° (d) 90°

65. Sin .cosec=

(a) 1 (b) 0
(c) Sin (d) Cos

66. In which quadrant θ lie when $\text{Cosec}\theta > 0, \text{Cos}\theta > 0$?

(a) I (b) II
(c) III (d) IV

67. $\tan\theta\cot\theta =$

(a) $\sin\theta$ (b) $\text{Sec}\theta$
(c) 1 (d) 0

68. $1+\text{Cot}^2\theta =$

(a) $\text{Sin}^2\theta$ (b) $\text{Cos}^2\theta$
(c) $\text{Cosec}^2\theta$ (d) $\text{Sec}^2\theta$

69. In which quadrate all trigonometric ratios are positive?

(a) I (b) II
(c) III (d) IV

70. Sin (–310°)=…….

(a) Sin310° (b) –Sin310°
(c) Cos310° (d) tan310°

71. Sec (–60°)…….

(a) –Sec60° (b) Sec60°
(c) Cos60° (d) Cot60°

ANSWER KEY

1	a	**2**	d	**3**	c	**4**	b	**5**	c	**6**	b	**7**	a	**8**	b
9	c	**10**	b	**11**	b	**12**	b	**13**	b	**14**	c	**15**	c	**16**	d
17	c	**18**	c	**19**	d	**20**	d	**21**	c	**22**	b	**23**	a	**24**	c
25	a	**26**	d	**27**	a	**28**	d	**29**	c	**30**	b	**31**	d	**32**	c
33	c	**34**	a	**35**	b	**36**	b	**37**	a	**38**	a	**39**	b	**40**	d
41	c	**42**	d	**43**	c	**44**	b	**45**	a	**46**	c	**47**	d	**48**	c
49	d	**50**	b	**51**	d	**52**	c	**53**	b	**54**	c	**55**	d	**56**	b
57	a	**58**	c	**59**	c	**60**	d	**61**	c	**62**	a	**63**	d	**64**	d
65	a	**66**	a	**67**	c	**68**	c	**69**	a	**70**	b	**71**	b		

Q.1. Write short answers of the following question:

(i) Define an angle.

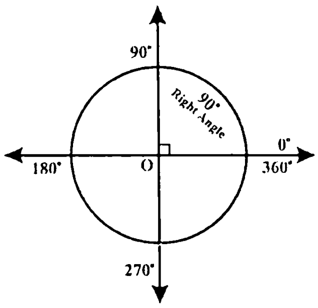
Ans: Angle:

An angle is defined as the union of two non–collinear rays with some common end point. The rays are called arms of the angle and the common end point is known as vertex of the angle

(ii) What is sexagesimal system of measurement of angles?

Ans: Measurement of an angle in sexagesimal system (degree, minute and second)

Degree: We divide the circumference of a circle into 360 equal arcs. The angle subtended at the centre of the circle by one arc is called one degree and is denoted by 1°.



The symbols 1°, 1′ and 1″ are used to denote a degree, a minute and a second respectively.

Thus 60 seconds (60″) make one minute (1′)
60 minutes (60′) make one degree (1°)
90 degrees (90°) make one right angle.
360 degrees (360°) make 4 right angles.
An angle of 360° denotes a complete

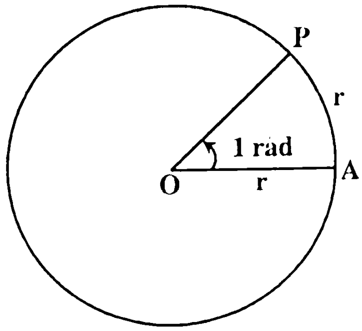
(iii) How many minutes are in two right angles?

Ans: As we know that one right angle = 90°
Two right angles = 180°
Minutes in 1° = 60′
Minutes in 180° = 180×60
Minutes in two right angles = 10800′

(iv) Define radian measure of an angle.

Ans: 07(200)

Radian: The angle subtended at the centre of the circle by an arc, whose length is equal to the radius of the circle is called one Radian.



Consider a circle of radius r whose centre is O. From any point A on the circle cut off an arc AP whose length is equal to the radius of the circle. Join O with A and O with P. The ∠AOP is one radian. This means that when

Length of arc AP = length of radius $\overline{OA}$ then m∠AOP = 1 radian

(v) Convert $\frac{\pi}{4}$ radian to degree measure.

Ans: $\frac{\pi}{4}$ radian $= \frac{\pi}{4} \cdot \frac{180}{\pi}$ degrees

$= \frac{\not\pi}{\not 4} \cdot \frac{\not 4 \times 45}{\not\pi}$ degrees

$= 45°$

(vi) Convert 15° to radians

Ans: $15° = 15 \cdot \frac{\pi}{180}$ radian

$= \not{15} \cdot \frac{\pi}{\not{15} \times 12}$ radian

$= \frac{\pi}{12}$ radian

(vii) What is the radian measure of the central angle of an arc 50m long on the circle of radius 25m.

Solution: Central angle = θ ? =

Arc length = l = 50m

Radius = r = 25m

Using formula

$L = r\theta$

$\theta = \frac{\;}{r}$

$= \frac{50}{25}$

$\theta = 2$ radian

(viii) Find r when l = 56cm and θ = 45°

Solution:

$r = ?$

$L = 56$ cm

$\theta = 45°$

$= 45 \dfrac{\pi}{180}$ radian

$= \dfrac{\pi}{4}$ radian

Using formula

$L = r\,\theta$

$56 = r\,\dfrac{\pi}{4}$

$r = \dfrac{56\times 4}{\pi}$

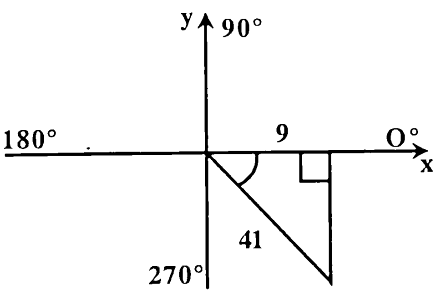
$r = 71.3$ cm

(ix) Find tanθ when cosθ = $\dfrac{9}{41}$ and terminal side of the angle θ is in fourth quadrant.

Solution: tanθ ? =

As $\cos\theta = \dfrac{9}{41}$ i.e Base = 9 and hypotenuse = 41

and terminal side of θ is in quadrant IV.



By Pythagorean theorem.

$(\text{Base})^2 + (\text{Perpendicular})^2 = (\text{Hypotenuse})^2$

$(9)^2 + (\text{Per.})^2 = (41)^2$

$(\text{Per.})^2 = 1681 - 81$

$(\text{Per.})^2 = 1600$

$\text{Per.} = \sqrt{1600}$

$\text{Per.} = \pm 40$

So $\tan\theta = \dfrac{\text{Per.}}{\text{Base}}$

$= \dfrac{-40}{9}$

–ve sign shows tanθ is –ve in quadrant IV.

(x) Prove that (1–sin²θ) (1+tan²θ) = 1

Solution: Let

L.H.S $= (1-\sin^2\theta)(1+\tan^2\theta)$

$= \cos^2\theta.\sec^2\theta$

$= \cancel{\cos^2\theta}.\dfrac{1}{\cancel{\cos^2\theta}}$

$= 1$

L.H.S = R.H.S

Q.2. Fill in the blanks:

(i) π radians =_________ degree.

(ii) The terminal side of angle 235° lies in _______quadrant.

(iii) Terminal side of the angle – 30° lies in _______quadrant.

(iv) Area of a circular sector is ________.

(v) If r = 2cm and θ = 3 radian, then area of the circular sector is______.

(vi) The general form of the angle 480° is ________________

(vii) If sinθ= $\dfrac{1}{2}$, then θ __________=

(viii) If θ = 300°, then see (–300)° = _____

(ix) 1 + cot²θ ________=

(x) secθ – tanθ ___________ =

ANSWER KEY

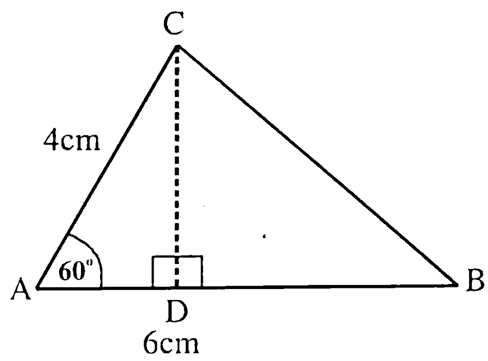
(i)	180°	(ii)	III	(iii)	IV	(iv)	$\frac{1}{2}r^2\theta$	(v)	6cm²
(vi)	2kπ + 120°, where k = 1	(vii)	30° or $\frac{\pi}{6}$ *radian*	(viii)	2	(ix)	cosec²θ	(x)	$\frac{1-\sin\theta}{\cos\theta}$

EXRCISE 8.2

Q. 1 In a ΔABC calculate $m\overline{BC}$

When $m\overline{AB} = 6cm$, $m\overline{AC} = 4cm$ and $m\angle A = 60°$

Solution:



Given: In a ΔABC, $m\overline{AB} = 6cm$, $m\overline{AC} = 4cm$ and $m\angle A = 60°$

To find: $m\overline{BC} = ?$

Calculations:

In acute angled triangle ABC, by theorem 2

$$\left(m\overline{BC}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{AB}\right)^2 - 2\left(m\overline{AB}\right)\left(m\overline{AD}\right) \text{ .... (i)}$$

In right angle Δ ACD

$$\text{Cos } 60° = \frac{m\overline{AD}}{m\overline{AC}}$$

$$\frac{1}{\not{2}} = \frac{m\overline{AD}}{\not{4}_2}$$

$$\boxed{m\overline{AD} = 2cm}$$

Putting the corresponding values in equation (i), we get

$$\left(m\overline{BC}\right)^2 = (4cm)^2 + (6cm)^2 - 2(6cm)(2cm)$$

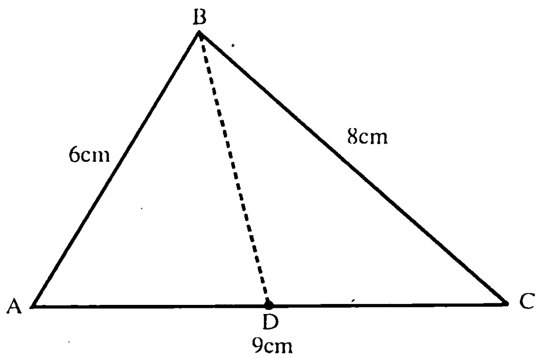
$$\left(m\overline{BC}\right)^2 = 16cm^2 + 36\ cm^2 - 24\ cm^2$$

$$\left(m\overline{BC}\right)^2 = 28\ cm^2$$

$$\sqrt{\left(m\overline{BC}\right)^2} = \sqrt{28cm^2}$$

$$m\overline{BC} = 5.29\ cm$$

Q.2 In a $\Delta ABC, m\overline{AB} = 6cm, m\overline{BC} = 8cm, m\overline{AC} = 9cm$ and D is the mid-point of side $\overline{AC}$. Find length of the median $\overline{BD}$.



Solution:

Given:

In a ABC,

$m\overline{AB} = 6cm$

$m\overline{BC} = 8cm$

$m\overline{AC} = 9cm$

To Find: Length of median i.e. $m\overline{BD} = ?$

Calculations:

By Apollonius' theorem

In a ABC

$$\left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2 = 2\left(m\overline{AD}\right)^2 + 2\left(m\overline{BD}\right)^2 \quad ....(i)$$

As $m\overline{AD} = \frac{1}{2} m\overline{AC}$

$$m\overline{AD} = \frac{1}{2}(9cm) = 4.5cm$$

Now, putting the corresponding value in equation (i)

$$(6cm)^2 + (8cm)^2 = 2(4.5cm)^2 + 2\left(m\overline{BD}\right)^2$$

$$36cm^2 + 64cm^2 = 2(20.25cm^2) + 2\left(m\overline{BD}\right)^2$$

$$100cm^2 + 40.5cm^2 = 2\left(m\overline{BD}\right)^2$$

$$100cm^2 - 40.5cm^2 = 2\left(m\overline{BD}\right)^2$$

$$59.5cm^2 = 2\left(m\overline{BD}\right)^2$$

$$\frac{59.5cm^2}{2} = \left(m\overline{BD}\right)^2$$

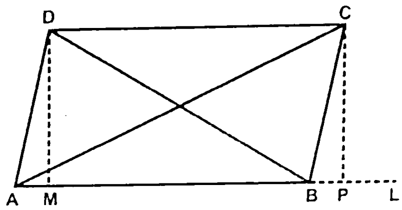
$$29.75cm^2 = \left(m\overline{BD}\right)^2$$

By taking square root

$$\sqrt{\left(m\overline{BD}\right)^2} = \sqrt{29.75cm^2}$$

$$\boxed{m\overline{BD} = 5.45cm}$$

Q.3 In a Parallelogram ABCD prove that $\left(m\overline{AC}\right)^2+\left(m\overline{BD}\right)^2=2\left(m\overline{AB}\right)^2+\left(m\overline{BC}\right)^2$



Given: ABCD is a Parallelogram.

To Prove: $\left(m\overline{AC}\right)^2+\left(m\overline{BD}\right)^2=2\left(m\overline{AB}\right)^2+\left(m\overline{BC}\right)^2$

Construction:

Extend $\overline{AB}$ beyond B. Draw $\overline{DM}\perp\overline{AB}$ and $\overline{CP}\perp\overline{AB}$ extended.

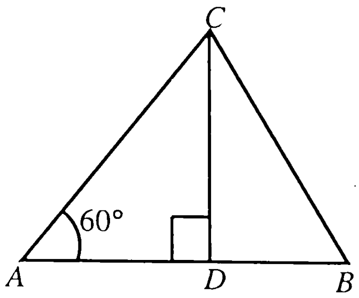
Proof:

Statements	Reasons
In $\Delta ABC, \angle ABC$ is obtuse	
$\left(m\overline{AC}\right)^2+\left(m\overline{AB}\right)^2+\left(m\overline{BC}\right)^2=2\left(m\overline{AB}\right)\left(m\overline{BP}\right)$ ....(i)	By theorem 1
In $\Delta ABD, \angle BAD$ is acute	
$\left(m\overline{BD}\right)^2=\left(m\overline{AB}\right)^2+\left(m\overline{AD}\right)^2-2\left(m\overline{AB}\right)\left(m\overline{AM}\right)$	By theorem 2
$=\left(m\overline{AB}\right)^2+\left(m\overline{BC}\right)^2-2\left(m\overline{AB}\right)\left(m\overline{BP}\right)$ ...(ii)	AMD $\cong$ BPC i.e $m\overline{AM}=m\overline{BP}$
$\left(m\overline{AC}\right)^2+\left(m\overline{BD}\right)^2=2\left(m\overline{AB}\right)^2+2\left(m\overline{BC}\right)^2$	By adding (i) and (ii)
$\left(m\overline{AC}\right)^2+\left(m\overline{BD}\right)^2=2\left(m\overline{AB}\right)^2+\left(m\overline{BC}\right)^2$	

MISCELLANEOUS EXERCISE – 8

Q. 1 In a ΔABC, m∠A = 60°,

Prove that $(m\overline{BC})^2 = (m\overline{AB})^2 + (m\overline{AC})^2 - (m\overline{AB})(m\overline{AC})$



Solution:

Given: In a ABC, m∠A = 60°

To Prove: $(m\overline{BC})^2 = (m\overline{AB})^2 + (m\overline{AC})^2 - (m\overline{AB})(m\overline{AC})$

Proof: In acute angled triangle ABC, by Theorem No. 2

$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})(m\overline{AD}) \ldots\ldots\text{(i)}$$

In right angled Δ ACD

$$\text{Cos}60° = \frac{m\overline{AD}}{m\overline{AC}} \qquad \frac{1}{2} = \frac{m\overline{AD}}{m\overline{AC}} \qquad (\ \text{Cos}60° = \frac{1}{2})$$

$$m\overline{AD} = \frac{1}{2}m\overline{AC}$$

Put it in equation (i)

$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})\ \frac{1}{2}\ m\overline{AC}$$

$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - \cancel{2}(m\overline{AB})\ \frac{1}{\cancel{2}}\ m\overline{AC}$$

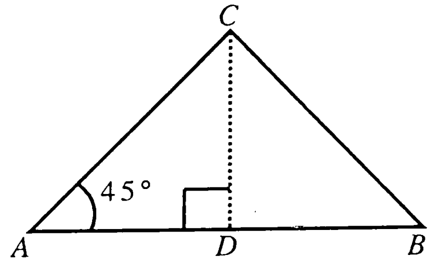
$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - (m\overline{AB})(m\overline{AC})$$ Hence proved

Q. 2 In a ΔABC, $m\angle A = 45°$, prove that $(m\overline{BC})^2 = (m\overline{AB})^2 + (m\overline{AC})^2 - \sqrt{2}(m\overline{AB})(m\overline{AC})$.

Solution:

Given: In a ABC, $m\angle A = 45°$

To prove: $(m\overline{BC})^2 = (m\overline{AB})^2 + (m\overline{AC})^2 - \sqrt{2}(m\overline{AB})(m\overline{AC})$



Proof: In triangle ABC, $\angle A$ is acute so by Theorem 2

$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})(m\overline{AD})$ .... (i)

In right angled ΔACD

$$\text{Cos}45° = \frac{m\overline{AD}}{m\overline{AC}}$$

$$\frac{1}{\sqrt{2}} = \frac{m\overline{AD}}{m\overline{AC}}$$

$$m\overline{AD} = \frac{1}{\sqrt{2}} m\overline{AC}$$

Put it in equation (i)

$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})\frac{1}{\sqrt{2}} m\overline{AC}$$

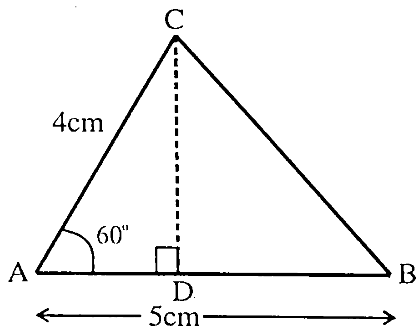
$$(m\overline{BC})^2 = (m\overline{AB})^2 + (m\overline{AC})^2 - \sqrt{2}(m\overline{AB})(m\overline{AC}) \qquad \frac{2}{\sqrt{2}} = \sqrt{2}$$

Q. 3 In a ΔABC, calculate $m\overline{BC}$ when $m\overline{AB} = 5cm, m\overline{AC} = 4cm, m\angle A = 60°$

Solution:

Given: In a ΔABC $m\overline{AB} = 5cm, m\overline{AC} = 4cm, m\angle A = 60°$

To Find: $m\overline{BC} = ?$



Calculations: In triangle ABC, $\angle A$ is acute so by Theorem 2

$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})(m\overline{AD})$$

$$(m\overline{BC})^2 = (4cm)^2 + (5cm)^2 - 2(5cm)(m\overline{AD}) \text{ ......(i)}$$

In right angle ΔACD

$$\text{Cos}60° = \frac{m\overline{AD}}{m\overline{AC}} \qquad \frac{1}{2} = \frac{m\overline{AD}}{4cm} \qquad \left(\text{Cos}60° = \frac{1}{2}\right)$$

$$2cm = m\overline{AD} \qquad \boxed{m\overline{AD} = 2cm}$$

Putting the corresponding values in equation (i)

$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})(m\overline{AD})$$

$$= (4cm)^2 + (5cm)^2 - 2(5cm)(2cm)$$

$$= 16cm^2 + 25cm^2 - 20cm^2$$

$$= 41cm^2 - 20cm^2$$

$$(m\overline{BC})^2 = 21cm^2$$

$$\sqrt{(m\overline{BC})^2} = \sqrt{21cm^2} \qquad \boxed{m\overline{BC} = 4.58 \text{ cm}}$$

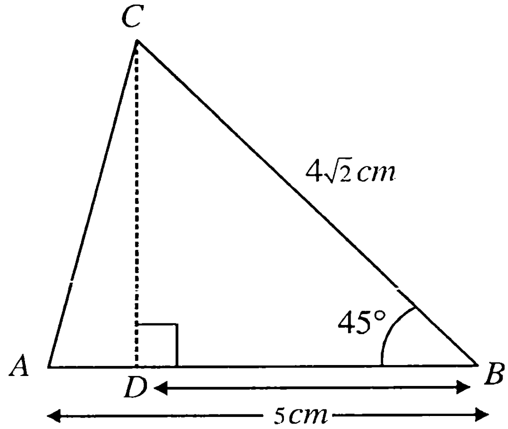
Q. 4 In a ΔABC, calculate $m\overline{AC}$ when $m\overline{AB} = 5cm, m\overline{BC} = 4\sqrt{2}cm$, $m\angle B = 45°$

Solution:

Given: In a ΔABC $m\overline{AB} = 5cm, m\overline{BC} = 4\sqrt{2}cm$, $m\angle B = 45°$

To Find: $m\overline{AC} = ?$

Calculations: In acute angled triangle ABC by theorem 2



$$\left(m\overline{AC}\right)^2 = \left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2 - 2\left(m\overline{AB}\right)\left(m\overline{BD}\right)$$

$$\left(m\overline{AC}\right)^2 = (5cm)^2 + \left(4\sqrt{2}cm\right)^2 - 2(5cm)\left(m\overline{BD}\right) \ldots\ldots.(i)$$

$m\overline{BD} = ?$

In right angle ΔBCD

$$\text{Cos}45° = \frac{m\overline{BD}}{m\overline{BC}}$$

$$\frac{1}{\sqrt{2}} = \frac{m\overline{BD}}{4\sqrt{2}}$$

$$1 = \frac{m\overline{BD}}{4} \qquad \boxed{m\overline{BD} = 4cm}$$

Putting the value of $m\overline{BD}$ in equation (i)

$$\left(m\overline{AC}\right)^2 = (5cm)^2 + \left(4\sqrt{2}cm\right)^2 - 2(5cm)(4cm)$$

$$= 25cm^2 + 16(2cm^2) - 40cm^2$$

$$= 25cm^2 + 32cm^2 - 40cm^2$$

$$= 57cm^2 - 40cm^2$$

$$\left(m\overline{AC}\right)^2 = 17cm^2$$

$$\sqrt{\left(m\overline{AC}\right)^2} = \sqrt{17cm^2} \qquad \boxed{m\overline{AC} = 4.12\text{ cm}}$$

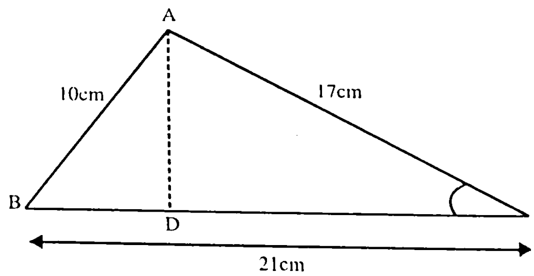
Q. 5 In a triangle ABC, $m\overline{BC} = 21cm$, $m\overline{AC} = 17cm$, $m\overline{AB} = 10cm$. Measure the length of projection of $\overline{AC}$ upon $\overline{BC}$.

Solution:

Given: In a triangle ABC, $m\overline{BC} = 21cm$, $m\overline{AC} = 17cm$, $m\overline{AB} = 10cm$

To Find: Projection of $\overline{AC}$ upon $\overline{BC}$ i.e., $m\overline{DC} = ?$

Calculations: In triangle ABC, $\angle C$ is acute so by theorem 2



$$\left(m\overline{AB}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{BC}\right)^2 - 2\left(m\overline{BC}\right)\left(m\overline{DC}\right)$$

$$(10cm)^2 = (17cm)^2 + (21cm)^2 - 2(21cm)\left(m\overline{DC}\right)$$

$$100cm^2 = 289cm^2 + 441cm^2 - 42cm\left(m\overline{DC}\right)$$

$$100cm^2 - 289cm^2 - 441cm^2 = -42cm\left(m\overline{DC}\right)$$

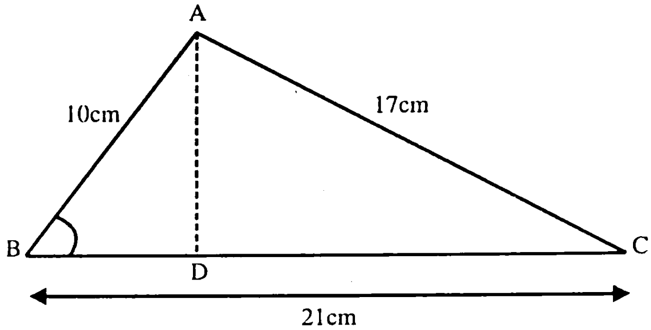
$$-630cm^2 = -42cm\left(m\overline{DC}\right)$$

$$\frac{-630cm^2}{-42cm} = m\overline{DC} \qquad \boxed{m\overline{DC} = 15cm}$$

Q. 6 In a triangle ABC, $m\overline{BC} = 21cm$, $m\overline{AC} = 17cm$, $m\overline{AB} = 10cm$. Calculate the projection of $\overline{AB}$ upon $\overline{BC}$.

Solution:

Given



$m\overline{BC} = 21cm$

$m\overline{AC} = 17cm$

$m\overline{AB} = 10cm$

To Find: Projection of $\overline{AB}$ upon $\overline{BC}$ i.e $m\overline{BD} = ?$

Calculations:

In triangle ABC , $\angle B$ is acute so by theorem 2

$$(m\overline{AC})^2 = (m\overline{AB})^2 + (m\overline{BC})^2 - 2(m\overline{BC})(m\overline{BD})$$

$$(17cm)^2 = (10cm)^2 + (21cm)^2 - 2(21cm)(m\overline{BD})$$

$$289cm^2 = 100cm^2 + 441cm^2 - 42cm(m\overline{BD})$$

$$289cm^2 = 541cm^2 - 42cm(m\overline{BD})$$

$$289cm^2 - 541cm^2 = -42cm(m\overline{BD})$$

$$-252cm^2 = -42cm(m\overline{BD})$$

$$\frac{-252cm^2}{-42cm} = m\overline{BD}$$

$$\boxed{m\overline{BD} = 6cm}$$

Q. 7 In a ΔABC, a = 17cm, b = 15cm and c = 8cm. Find $m\angle A$.

Solution:

Given: In a ΔABC, a = 17cm, b = 15cm, c = 8cm

To Find: $m\angle A = ?$

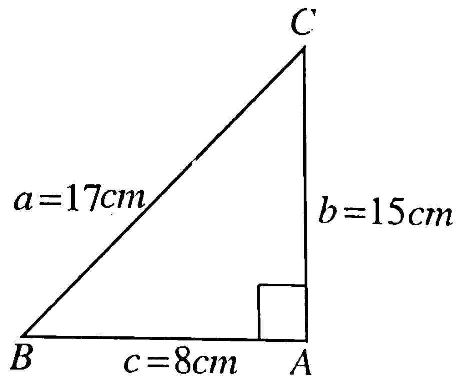
Calculations:

Sum of squares of two sides $= b^2 + c^2$

$= (15cm)^2 + (8cm)^2$

$= 225cm^2 + 64\ cm^2$

$= 289\ cm^2$ ….. (i)



Square of length of third side $= a^2$

$= (17cm)^2$

$= 289cm^2$ ….. (ii)

From (i) and (ii)

$$a^2 = b^2 + c^2$$

The result show that the triangle ABC is right angled triangle with side a = 17cm as hypotenuse. The angle opposite to the hypotenuse is right angle i.e $m\angle A = 90°$

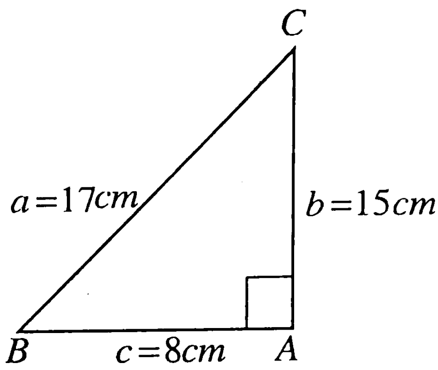
8. In a ΔABC , a = 17cm , b = 15cm and c = 8cm find m∠B.

Solution:

Given: In a ΔABC, a = 17cm, b = 15cm, c = 8cm

To Find: m∠B = ?

Calculations:



Sum of squares of two sides = $b^2 + c^2$

$= (15cm)^2 + (8cm)^2$

$= 225cm^2 + 64\ cm^2$

$= 289\ cm^2$ ..... (i)

Square of length of third side = a^2

$= (17cm)^2$

$= 289cm^2$ ..... (ii)

From (i) and (ii)

$$a^2 = b^2 + c^2$$

The result show that the triangle ABC is right angled triangle with m∠A = 90°

In triangle ABC, $\tan m\angle B = \frac{Per}{Base} = \frac{15cm}{8cm}$

$m\angle B = tan^{-1}\frac{15}{8}$ $\quad$ $m\angle B = (61.9)^{\circ}$

Q.9 Whether the triangle with sides 5cm, 7cm, 8cm is acute, obtuse or right angled.

Solution:

In a triangle ABC , let a = 5cm , b = 7cm , c = 8cm

Sum of squares of two sides = $a^2 + b^2$

$= (5cm)^2 + (7cm)^2$

$= 25cm^2 + 49cm^2$

$= 74cm^2$ .... (i)

Square of length of 3rd side = c^2

$= (8cm)^2$

$= 64cm^2$ .... (ii)

From (i) and (ii) $\quad 74cm^2 > 64cm^2$ i.e.

$$a^2 + b^2 > c^2$$

The result shows that the triangle with sides 5cm, 7cm, 8cm is acute angled triangle.
It is acute angled triangle.

Q.10 Whether the triangle with sides 8cm, 15cm, 17cm is acute, obtuse or right angled.

Solution:

In a triangle ABC let a = 8cm, b = 15cm, c = 17cm

Sum of squares of two sides = $a^2 + b^2$

$= (8cm)^2 + (15cm)^2$

$= 64cm^2 + 225\ cm^2$

$= 289\ cm^2$ ....(i)

Square of length of 3rd side = c^2

$= (17cm)^2$

$= 289cm^2$ ....(ii)

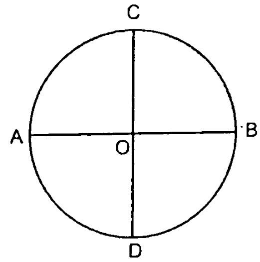
From (i) and (ii)

i.e $\quad a^2 + b^2 = c^2$

Result shows that triangle with sides a = 8cm, b = 15cm and c = 17cm is right angled triangle.

EXERCISE 9.1

Q. 1 Prove that, the diameters of a circle bisect each other.



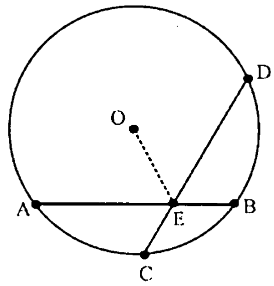
Given: A circle with centre O. Two diameters $\overline{AB}$ and $\overline{CD}$.

To Prove: Two diameters $\overline{AB}$ and $\overline{CD}$ bisect each other.

Proof:

Statements	Reasons
$\overline{AB}$ and $\overline{CD}$ intersect each other at point O.	$\overline{AB}$ and $\overline{CD}$ are non-parallel.
$\overline{OA} \cong \overline{OB}$ ---------------- (i)	Radii of the same circle
O is the midpoint of $\overline{AB}$ thus $\overline{CD}$ bisects the $\overline{AB}$ at O. Similarly	from (i)
$\overline{OC} \cong \overline{OD}$ ------------------- (ii)	Radii of the same circle
O is the midpoint of $\overline{CD}$ thus $\overline{AB}$ bisects the $\overline{CD}$ at O.	from (ii)
Hence, two diameters $\overline{AB}$ and $\overline{CD}$ bisect each other.	

Q. 2 Two chords of a circle do not pass through the centre. Prove that they cannot bisect each other.



Given:

A Circle with centre "Q". Two different chords $\overline{AB}$ and $\overline{CD}$ not passing through the centre, intersect each other at point E.

To Prove:

$\overline{AB}$ and $\overline{CD}$ do not bisect each other, i.e.

E is not midpoint of $\overline{AB}$ and $\overline{CD}$.

Construction:

Suppose chords $\overline{AB}$ and $\overline{CD}$ bisect each other at point E i.e. E is the common midpoint of $\overline{AB}$ and $\overline{CD}$. Join O to E.

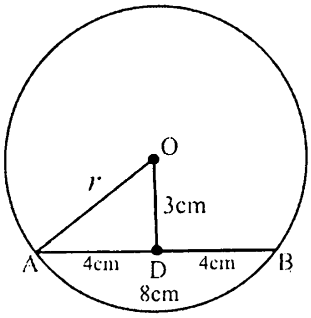
Proof:

Statements	Reasons
As $\overline{OE}$ is perpendicular from "O" to the midpoint E of $\overline{AB}$ and $\overline{CD}$ so ,	A line segment from the centre "O" to the midpoint of a chord is ⊥ on the chord. (Construction)
$m\angle OEA = 90°$ .... (i)	
$m\angle OED = 90°$ .... (ii)	
$m\angle OEA + m\angle OED = 180°$ .... (iii)	Adding (i) and (ii)
$m\angle AED = 180°$	
It is only possible when A and D are on the same line segment.	
But A and D are not on the same line segment.	Given
So, our supposition, $\overline{AB}$ and $\overline{CD}$ bisect each other, is wrong.	
Thus chords $\overline{AB}$ and $\overline{CD}$ do not bisect each other	

Q. 3 If length of the chord $\overline{AB} = 8$ cm. Its distance from the centre is 3 cm, then find the diameter of such circle.

Solution:

Given:



Length of chord = $m\overline{AB} = 8cm$

Distance from centre = $m\overline{OD} = 3cm$

To Find: Diameter = 2r

Calculations:

$m\overline{AB} = 8cm$ (Given)

Perpendicular from the centre to the chord bisects the chord ($\overline{OD} \perp \overline{AB}$)

$$m\overline{AD} = \frac{1}{2}m\overline{AB} = \frac{1}{2}(8cm) = 4cm$$

In right angled ΔADO by Pythagoras theorem

$$(m\overline{OA})^2 = (m\overline{AD})^2 + (m\overline{OD})^2$$

$$r^2 = (4cm)^2 + (3cm)^2$$

$$r^2 = 16cm^2 + 9cm^2$$

$$r^2 = 25cm^2$$

$$\sqrt{r^2} = \sqrt{25cm^2}$$

$$r = 5cm$$

We know that diameter = 2r = 2(5cm) = 10cm

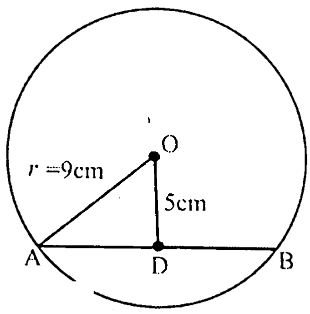
Q.4 Calculate the length of a chord which stands at a distance 5cm from the centre of a circle whose radius is 9cm.

Given:

In a circle with centre O radius = 9cm,

$m\overline{OD} = 5cm$

To find:



Length of chord $\overline{AB}$

Calculations:

In right angled ΔADO, by Pythagoras theorem

$$\left(m\overline{OA}\right)^2 = \left(mAD\right)^2 + \left(mOD\right)^2$$

$$(9cm)^2 = \left(m\overline{AD}\right) + (5cm)^2$$

$$81cm^2 = \left(m\overline{AD}\right)^2 + 25cm^2$$

$$81cm^2 - 25cm^2 = \left(m\overline{AD}\right)^2$$

$$56cm^2 = \left(m\overline{AD}\right)^2$$

$$\sqrt{\left(m\overline{AD}\right)^2} = \sqrt{56cm^2}$$

$$m\overline{AD} = \sqrt{56}cm$$

We know that

$m\overline{AD} = \frac{1}{2}m\overline{AB}$ (Perpendicular from the centre to the cord bisects the chord)

$$\Rightarrow \quad m\overline{AB} = 2m\overline{AD}$$

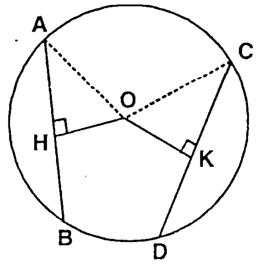
$$= 2\times\sqrt{56}cm$$

$$m\overline{AB} = 14.966cm$$

or $\quad m\overline{AB} \simeq 14.97cm$

THEOREM 4

If two chords of a circle are congruent then they will be equidistant from the centre.



Given: $\overline{AB}$ and $\overline{CD}$ are two equal chords of a circle with centre at O.

So that $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$.

To Prove: $m\overline{OH} = m\overline{OK}$

Construction: Join O with A and O with C.

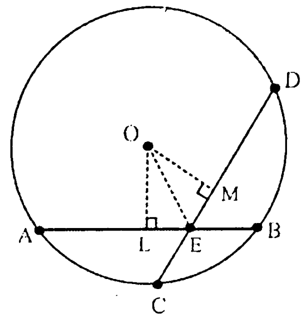
So that we have $\angle$rtΔ^s OAH and OCK.

Proof:

Statements	Reasons
$\overline{OH}$ bisects chord $\overline{AB}$	
i.e., $m\overline{AH} = \frac{1}{2}m\overline{AB}$ .... (i)	$\overline{OH} \perp \overline{AB}$ By Theorem 3
Similarly $\overline{OK}$ bisects chord $\overline{CD}$	
i.e., $m\overline{CK} = \frac{1}{2}m\overline{CD}$ .... (ii)	$\overline{OK} \perp \overline{CD}$ By Theorem 3
But $m\overline{AB} = m\overline{CD}$ .... (iii)	Given
Hence $m\overline{AH} = m\overline{CK}$ .... (iv)	Using (i) , (ii) and (iii) Both are half of equal segment
Now in $\angle$rt Δ^s OAH $\leftrightarrow$ OCK	Given $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$
hyp. $\overline{OA}$ = hyp. $\overline{OC}$	Radii of the same circle
$m\overline{AH} = m\overline{CK}$	Already proved in (iv)
$\Delta OAH \cong \Delta OCK$	H.S $\cong$ H.S
$\Rightarrow$ $m\overline{OH} = m\overline{OK}$	Corresponding sides of congruent triangles.

EXERCISE 9.2

Q.1 Two equal chords of a circle intersect, show that the segments of the one are equal corresponding to the segments of the other.



Given: A circle with centre "O". Two equal chords $\overline{AB}$ and $\overline{CD}$ (i.e. $m\overline{AB} = m\overline{CD}$) intersect each other at point E.

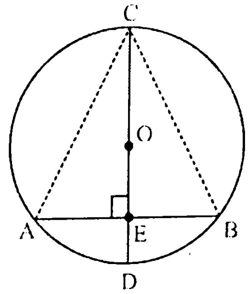
To prove: $m\overline{AE} = m\overline{ED}$ and $m\overline{EB} = m\overline{EC}$

Construction: Draw perpendiculars $\overline{OL}$ and $\overline{OM}$ from the centre "O" to the chords $\overline{AB}$ and $\overline{CD}$ respectively. L and M are midpoints of $\overline{AB}$ and $\overline{CD}$ respectively.

Proof:

Statements	Reasons
In $\Delta OLE \leftrightarrow \Delta OME$	
$\overline{OL} \cong \overline{OM}$	Two equal chords of a circle are equidistant from the centre.
$m\angle OLE = m\angle OME = 90^\circ$	$\overline{OL} \perp \overline{AB}$ and $\overline{OM} \perp \overline{CD}$
$m\overline{OE} \cong m\overline{OE}$	Common side
$\therefore$ $\Delta OLE \cong \Delta OME$	H.S ≅ H.S
$\overline{LE} \cong \overline{ME}$ .... (i)	Corresponding sides of congruent triangles.
$m\overline{AL} = \frac{1}{2} m\overline{AB}$	
$m\overline{DM} = \frac{1}{2} m\overline{CD}$	
$m\overline{AL} = m\overline{DM}$ .... (ii)	Both are half of equal chords.
$m\overline{AL} + m\overline{LE} = m\overline{DM} + m\overline{ME}$	Adding (i) and (ii).
$m\overline{AE} = m\overline{DE}$ .... (iii)	
Now, $m\overline{AB} = m\overline{CD}$	Given
$m\overline{AE} + m\overline{EB} = m\overline{DE} + m\overline{EC}$	
$m\cancel{\overline{AE}} + m\overline{EB} = m\cancel{\overline{AE}} + m\overline{EC}$	From (iii)
$m\overline{EB} = m\overline{EC}$	By cancellation property.

Q.2 AB is the chord of a circle and diameter CD is perpendicular bisector of $\overline{AB}$. Prove that $m\overline{AC} = m\overline{BC}$



Given: A circle with centre "O" diameter $\overline{CD} \perp$ chord $\overline{AB}$ i.e

$m\angle CEA = m\angle CEB = 90°$ and $\overline{AE} \cong \overline{EB}$

To prove: $m\overline{AC} = m\overline{BC}$

Construction: Join C to A and B.

Proof:

Statements	Reasons
In $\Delta ACE \leftrightarrow \Delta BCE$	
$\overline{AE} \cong \overline{EB}$	A diameter $\overline{CD} \perp$ on chord AB bisect it.
$m\angle CEA = m\angle CEB$	Given
$\overline{CE} \cong \overline{CE}$	Common side
$\Delta ACE \cong \Delta BCE$	S.A.S ≅ S.A.S
$\overline{AC} \cong \overline{BC}$	Corresponding sides of congruent triangles.
$m\overline{AC} = m\overline{BC}$	

Q.3 As shown in fig. find the distance between two parallel chords $\overline{AB}$ and $\overline{CD}$.

Given: A fig. as shown.

To find: Distance between two parallel chords $\overline{AB}$ and $\overline{CD}$ i.e $m\overline{EF} = ?$

Construction: Join O to A.

Calculations:

$$m\overline{AE} = \frac{1}{2} m\overline{AB} = \frac{1}{2}(6cm) = 3cm$$

$$m\overline{CF} = \frac{1}{2} m\overline{CD} = \frac{1}{2}(8cm) = 4cm$$

$$m\overline{OA} = m\overline{OC} = 5cm$$

In right triangle AOE, by Pythagoras theorem

$$\left(m\overline{OA}\right)^2 = \left(m\overline{OE}\right)^2 + \left(m\overline{AE}\right)^2$$

$$(5cm)^2 = \left(m\overline{OE}\right)^2 + (3cm)^2$$

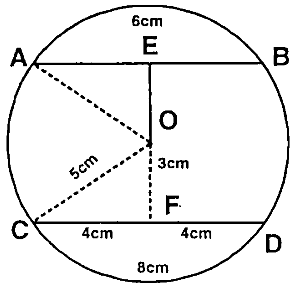
$$25cm^2 = \left(m\overline{OE}\right)^2 + (9cm^2)$$

$$m25cm^2 - 9cm^2 = \left(m\overline{OE}\right)^2$$

$$16cm^2 = \left(m\overline{OE}\right)^2$$

$$\sqrt{\left(m\overline{OE}\right)^2} = \sqrt{16cm^2}$$

$$m\overline{OE} = 4cm$$



In right triangle COF, by Pythagoras theorem

$$\left(m\overline{OC}\right)^2 = \left(m\overline{CF}\right)^2 + \left(m\overline{OF}\right)2$$

$$(5cm)^2 = (4cm)^2 + \left(m\overline{OF}\right)^2$$

$$25cm^2 - 16cm^2 + \left(m\overline{OF}\right)^2$$

$$9cm^2 = \left(m\overline{OF}\right)^2$$

$$\sqrt{\left(m\overline{OF}\right)^2} = \sqrt{9cm^2}$$

$$\boxed{m\overline{OF} = 3cm}$$

Now,

$$m\overline{EF} = m\overline{OE} + m\overline{OF}$$

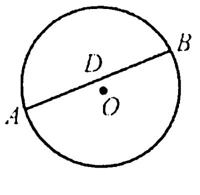
$$m\overline{EF} = 4cm + 3cm$$

$$\boxed{m\overline{EF} = 7cm}$$

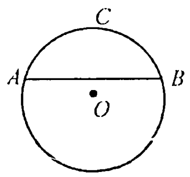
MISCELLANEOUS EXERCISE – 9

Q.1 Four possible answers are given for the following questions.

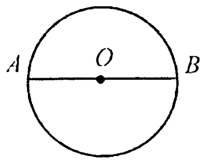
1. In the circular figure. ADB is called
   (a) an arc
   (b) a secant
   (c) a chord
   (d) a diameter



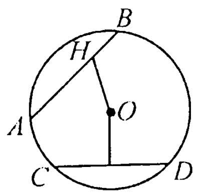
2. In the circular figure, $\overset{\frown}{ABC}$ is called
   (a) an arc
   (b) a secant
   (c) a chord
   (d) a diameter



3. In the circular figure, AOB is called
   (a) an arc
   (b) secant
   (c) a chord
   (d) diameter



4. In a circular figure, two chords $\overline{AB}$ and $\overline{CD}$ are equidistant from the centre. They will be
   (a) parallel
   (b) non congruent
   (c) congruent
   (d) perpendicular



5. Radii of a circle are
   (a) all equal
   (b) double of the diameter
   (c) all unequal
   (d) half of any chord
6. A chord Passing through the centre of a circle is called
   (a) radius
   (b) diameter
   (c) circumference
   (d) secant
7. Right bisector of the chord of a circle always passes through the
   (a) radius (b) circumference
   (c) centre (d) diameter
8. The circular region bounded by two radii and the corresponding arc is called
   (a) circumference of a circle
   (b) sector of a circle
   (c) diameter of a circle
   (d) segment of a circle
9. The distance of any point of the circle to its centre is called
   (a) radius (b) diameter
   (c) a chord (d) an arc
10. Line segment joining any point of the circle to the centre is called
    (a) circumference
    (b) diameter
    (c) radial segment
    (d) perimeter
11. Locus of a point in a plane equidistant from a fixed point is called
    (a) radius
    (b) circle
    (c) circumference
    (d) diameter
12. The symbol for a triangle is denoted by
    (a) $\angle$ (b) Δ
    (c) $\perp$ (d) $\odot$
13. A complete circle is divided into
    (a) 90 degree (b) 180 degree
    (c) 270 degree (d) 360 degree
14. Through how many non-collinear points, a circle can pass?
    (a) one (b) two
    (c) three (d) None
15. The vertex of central angle is at.........
    (a) circumference
    (b) center
    (c) any point of radius
    (d) any point of diameter

16. The line segment joining the centre and any point of circle is called.
   (a) circumference
   (b) radial segment
   (c) chord
   (d) diameters

17. The length of boundary traced by a moving point in a circular path is called...
   (a) circumference
   (b) radial segment
   (c) chord
   (d) diameter

18. The ling segment joining any two points of circle is called.
   (a) circumference
   (b) radial segment
   (c) chord
   (d) diameter

19. The central chord of circle is its.........
   (a) circumference
   (b) radial segment
   (c) chord
   (d) diameter

20. The largest chord of a circle is its......
   (a) circumference
   (b) radial segment
   (c) chord
   (d) diameter

21. A circle of radius 4cm has a chord few cm away from its centre, which of the following length of chord may be? 09(028)
   (a) 6cm (b) 8cm
   (c) 10cm (d) 12cm

22. Circumference is the ratio of:
   (a) radius and diameter
   (b) diameter and circumference
   (c) circumference and diameter
   (d) circumference and radius

23. $\pi \approx \frac{22}{7}$ is an .... number.
   (a) rational (b) irrational
   (c) natural (d) prime

24. If radius of a circle is "r", then its diameter is.......
   (a) r^2 (b) $2 + r$
   (c) $2r$ (d) $r - 2$

25. If central chord of a circle is 12cm, then its radius is....
   (a) 6cm (b) 8cm
   (c) 12cm (d) 24cm

ANSWER KEY

1.	c	**2.**	a	**3.**	d	**4.**	c	**5.**	a
6.	b	**7.**	c	**8.**	b	**9.**	a	**10.**	c
11.	b	**12.**	b	**13.**	d	**14.**	c	**15.**	b
16.	b	**17.**	a	**18.**	c	**19.**	d	**20.**	d
21.	a	**22.**	c	**23.**	b	**24.**	c	**25.**	a

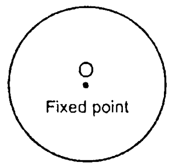
Q. 2 Differentiate between the following terms and illustrate them by diagram.

(i) A circle and a circumference.

Ans.

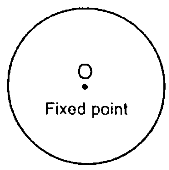
Circle:

A circle is the locus of a moving point P in a plane which is always equidistant from the fixed point O. This fixed point O not lying on the circle is called the centre of the circle.



Circumference:

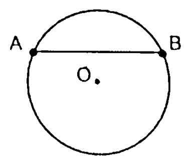
The length of the boundary traced by a moving point P in a circular path is called circumference of the circle. Circumference is calculated by $C = 2\pi r$. Here r is a radius and π is an irrational number.



(ii) A chord and the diameter of circle

Ans.

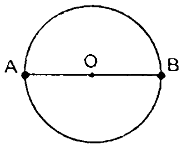
Chord: The Line segment joining any two points of the circle with each other is called chord of the circle.



In figure $\overline{AB}$ is chord of the circle.

Diameter:

The Chord passing through the centre of the circle is called diameter of the circle. Evidently diameter bisects a circle.



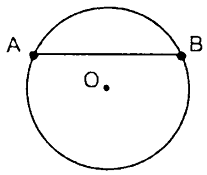
In figure $\overline{AB}$ is diameter of the circle.

(iii) A chord and an arc of a circle.

Ans.

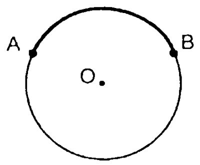
Chord

The Line segment joining any two points of the circle with each other is called chord of the circle.



Arc

Any part or portion of a circle is called its arc. An arc AB, as shown in figure, is denoted by $\overset{\frown}{AB}$.



There are two types of Arc:

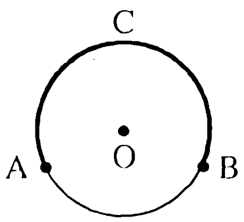
i. Major Arc (Arc greater than semi circle)
ii. Minor Arc (Arc less than semi circle)

(iv) Minor arc and major arc a circle.

Ans.

Major arc:

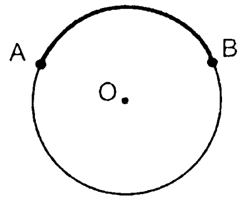
An arc greater than semi circle is called major arc.



In figure ACB is major arc. It is denoted by $\overset{\frown}{ACB}$

Minor Arc:

An arc less than semi circle is called minor arc.

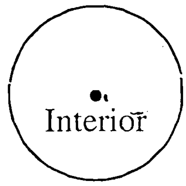


In fig. AB is minor arc. It is denoted by $\overset{\frown}{AB}$

(v) Interior and exterior of a circle.

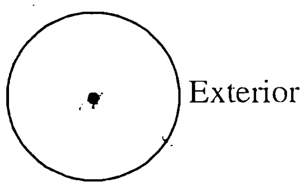
Ans. Interior:

The set of all the points lying inside the boundary of a circle is called interior of a circle.



Exterior:

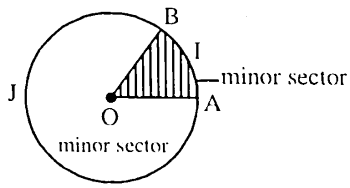
The set of all the points lying outside the boundary of a circle is called interior of a circle.



(vi) A sector and a segment of a circle.

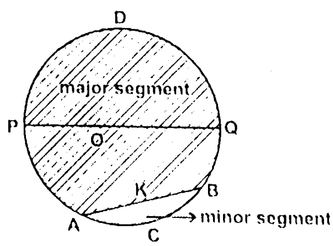
Ans. Sector:

A sector of a circle is the plane figure founded by two radii and the arc intercepted between them.



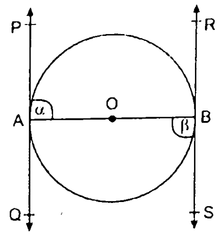
Segment of circle:

The circular region bounded by an arc and a corresponding chord is called segment of the circle.



Exercise 10.1

Q.1 Prove that the tangents drawn at the ends of a diameter in a given circle must be parallel.



Given: A circle with centre O, has $\overline{AB}$ as diameter. $\overleftrightarrow{PAQ}$ is tangent at point A. $\overleftrightarrow{RBS}$ is tangent at point B

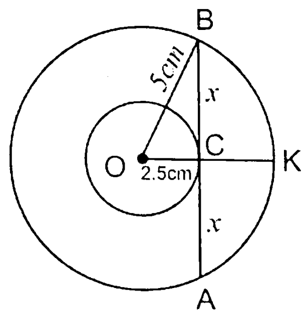
To Prove: $\overleftrightarrow{PAQ} \parallel \overleftrightarrow{RBS}$

Proof:

Statements	Reasons
$\overleftrightarrow{PAQ} \perp \overline{OA}$	Tangent is ⊥ at the outer end of radial segment.
$\Rightarrow$ $\overleftrightarrow{PAQ} \perp \overline{AB}$	
$\therefore$ $m\angle\alpha = 90^\circ$ .... (i)	
$\overleftrightarrow{RBS} \perp \overline{OB}$	
$\Rightarrow$ $\overleftrightarrow{RBS} \perp \overline{AB}$	
$\therefore$ $m\angle\beta = 90^\circ$ .... (ii)	
Thus, $m\angle\alpha = m\angle\beta$	
Therefore $\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$	If alternate Angles are equal in measurement, then lines are parallel.

Q.2 The diameters of two concentric circles are 10cm and 5cm respectively. Look for the length of any chord of the outer circle which touches the inner one.

Solution: Let $\overline{AB}$ be any chord of the outer circle that touches the inner circle.



Diameter of outer circle = 10 cm

Radius of outer circle = $m\overline{OB} = \dfrac{10\text{cm}}{2} = 5\text{cm}$

Diameter of inner circle = 5 cm

Radius of inner circle = $m\overline{OC} = \dfrac{5\text{cm}}{2} = 2.5\text{cm}$

ΔOCB is right angled triangle with right angle at C. $\left(\because \overline{OC} \perp \overline{AB}\right)$

By Pythagoras theorem

$$\left(m\overline{OB}\right)^2 = \left(m\overline{BC}\right)^2 + \left(m\overline{OC}\right)^2$$

$$(5\text{ cm})^2 = (x)^2 + (2.5\text{cm})^2$$

$$\Rightarrow \quad x^2 = (5\text{cm})^2 - (2.5\text{cm})^2$$

$$x^2 = 25\text{cm}^2 - 6.25\text{ cm}^2$$

$$x^2 = 18.75\text{cm}^2$$

Taking Square root of both sides.

$$\sqrt{x^2} = \sqrt{18.75\text{cm}^2}$$

$$x = \sqrt{18.75}\text{cm}$$

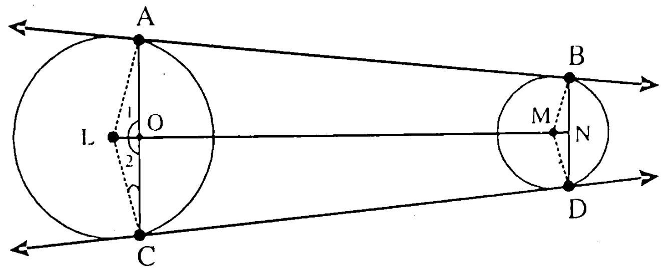
Length of Chord = $m\overline{AB} = 2x$

$$m\overline{AB} = 2(\sqrt{18.75}\text{cm})$$

$$m\overline{AB} = 8.66\text{cm} \quad \Rightarrow \quad \boxed{m\overline{AB} \simeq 8.7\text{cm}}$$

Q.3 $\overleftrightarrow{AB}$ **and** $\overleftrightarrow{CD}$ **are the common tangents drawn to the pair of circles. If A and C are the points of tangency of 1st circle where B and D are the points of tangency of 2nd circle, then prove that** $\overline{AC} \parallel \overline{BD}$**.**

Given: Two circles with centre L and M. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are their common tangents. A is joined with C and B is joined with D.



To prove:

$\overline{AC} \parallel \overline{BD}$

Construction:

Join L to A and C. Join M to B and D. Join L to M and produce it to meet the $\overline{BD}$ at N.

Proof:

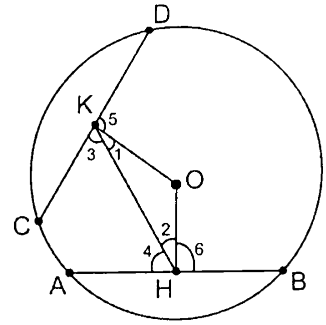
Statements	Reasons
In $\Delta AOL \leftrightarrow \Delta COL$	
$\overline{AL} \cong \overline{CL}$	Radii of the same circle
$\angle A \cong \angle C$	angles opposite to congruent sides
$\overline{LO} \cong \overline{LO}$	common side.
$\therefore$ $\Delta AOL \cong \Delta COL$	S.A.S ≅ S.A.S
$m\angle 1 = m\angle 2$ ....(i)	Corresponding angles of congruent triangle.
$m\angle 1 + m\angle 2 = 180^\circ$ ....(ii)	O is the point on line segment $\overline{AC}$.
$\Rightarrow$ $m\angle 1 = m\angle 2 = 90^\circ$	
$\overline{LO} \perp \overline{AO}$	
or $\overline{LO} \perp \overline{AC}$	
or $\overline{AC} \perp \overline{LOMN}$ ....(iii)	
Similarly in the circle with centre M, it can be proved that	
$\overline{BD} \perp \overline{MN}$	
or $\overline{BD} \perp \overline{LOMN}$ ....(iv)	
Both $\overline{AC}$ and $\overline{BD}$ are $\perp$ to the same line segment	
$\therefore$ $\overline{AC} \parallel \overline{BD}$	Two line segments making same angle with a line are parallel to each other.

Exercise 10.2

Q. 1 $\overline{AB}$ and $\overline{CD}$ are two equal chords in a circle with centre O. H and K are respectively the mid points of the chords. Prove that $\overline{HK}$ makes equal angles with $\overline{AB}$ and $\overline{CD}$.

Given:

A circle with centre 'O'. Two chords such that $m\overline{AB} = m\overline{CD}$. H and K are mid points of chords AB and CD respectively. H is joined with K



To Prove:

(i) $m\angle AHK = m\angle CKH$

(ii) $m\angle BHK = m\angle DKH$

Proof:

Statements	Reasons
In ΔHOK	
$m\overline{OH} = m\overline{OK}$	Two equal chords are equidistant from the center.
$\therefore$ $m\angle 1 = m\angle 2$ .... (i)	Angles opposite to the equal line segments
And $m\angle 5 = m\angle 6$ .... (ii)	Each 90^o
$m\angle 1 + m\angle 5 = m\angle 2 + m\angle 6$	Adding (i) and (ii)
Thus, $m\angle DKH = m\angle BHK$	
or $m\angle BHK = m\angle DKH$ Proved	
$m\angle AHO = m\angle CKO$	Each 90^o
$m\angle 2 + m\angle 4 = m\angle 1 + m\angle 3$	
But $m\angle 2 = m\angle 1$	
$\cancel{m\angle 1} + m\angle 4 = \cancel{m\angle 1} + m\angle 3$	Proved in (i)
$m\angle 4 = m\angle 3$	By cancellation property
$m\angle AHK = m\angle CKH$	

Q.2 The radius of a circle is 2.5 cm. $\overline{AB}$ and $\overline{CD}$ are two chords 3.9cm apart. If $m\overline{AB} = 1.4$ cm, then measure the other chord.

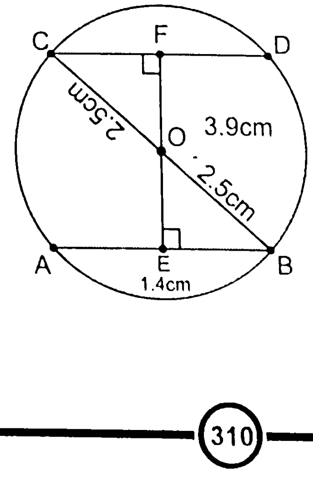
Given:

O is the centre of a circle.

(i) $m\overline{OB} = m\overline{OC} = 2.5cm$

(ii) $m\overline{AB} = 1.4cm$

(iii) $m\overline{EF} = 3.9cm$



To Find:

$m\overline{CD} = ?$

Construction:

Join O with B and C.

Calculations:

Steps	Reasons
In ΔOEB	Given
$m\overline{EB} = \frac{1}{2}m\overline{AB} = \frac{1}{2}(1.4cm) = 0.7\text{ cm}$ ....(i)	
$m\overline{OB} = 2.5cm$ ....(ii)	
$(m\overline{OB})^2 = (m\overline{OE})^2 + (m\overline{EB})^2$	By Pythagoras theorem in right angled ΔOEB
$(2.5cm)^2 = (m\overline{OE})^2 + (0.7cm)^2$	From (i) and (ii)
$\Rightarrow$ $(m\overline{OE})^2 = (2.5cm)^2 - (0.7cm)^2$	
$(m\overline{OE})^2 = 6.25cm^2 - 0.49cm^2$	
$(m\overline{OE})^2 = 5.76cm^2$	
$\sqrt{(m\overline{OE})^2} = \sqrt{5.76cm^2}$	
$m\overline{OE} = 2.4cm$ .... (iii)	
Now	
$m\overline{OF} = m\overline{EF} - m\overline{OE}$	
$m\overline{OF} = 3.9cm - 2.4cm$	
$m\overline{OF} = 1.5cm$ .... (iv)	
In right angled triangle ΔOCF	
$(m\overline{OC})^2 = (m\overline{OF})^2 + (m\overline{CF})^2$	
$\Rightarrow$ $(m\overline{CF})^2 = (2.5cm)^2 - (1.5cm)^2$	From (iv)
$(m\overline{CF})^2 = 6.25cm^2 - 2.25cm^2$	
$(m\overline{CF})^2 = 4cm^2$	
$\therefore$ $\sqrt{(m\overline{CF})^2} = \sqrt{4cm^2}$	
$(m\overline{CF}) = 2\text{ cm}$ .... (v)	
$\therefore$ $(m\overline{CD}) = 2(m\overline{CF})$	$\because m\overline{CF} = \frac{1}{2}m\overline{CD}$
$m\overline{CD} = 2(2cm)$	From (v)
$m\overline{CD} = 4cm$	

Q.3. The radii of two intersecting circles are 10cm and 8cm. If the length of their common chord is 6cm then find the distance between the centres.

Solution:

Given: Two intersecting circles with centers O and C having radius $m\overline{OA} = 10cm$, radius $m\overline{AC} = 8cm$ respectively.

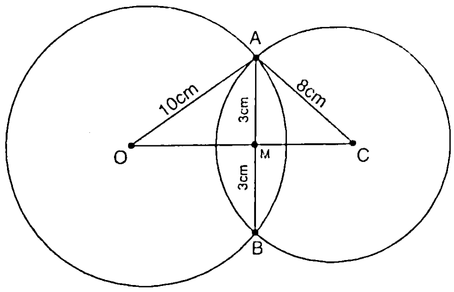
Length of common chord $m\overline{AB} = 6cm$

To Find:

Distance between the centers $m\overline{OC} = ?$

Construction:

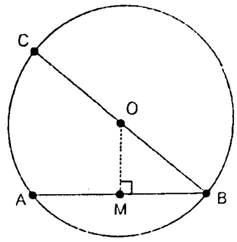
Join the point A to the centers O and C. Joint O to C which meets the chord $\overline{AB}$ at its midpoint.



Calculations:

Steps	Reasons
$\overline{OM} \perp \overline{AB}$	
$m\overline{AM} = \frac{1}{2} m\overline{AB} = \frac{1}{2}(6cm) = 3cm$ ....(i)	$\perp$ from the center to the chord bisect it.
By Pythagoras theorem	
$(m\overline{OM})^2 = (m\overline{OA})^2 - (m\overline{AM})^2$	
$= (10cm)^2 - (3cm)^2$	From given and from (i)
$= 100cm^2 - 9cm^2$	
$(m\overline{OM})^2 = 91\,cm^2$	
$\sqrt{(m\overline{OM})^2} = \sqrt{91cm^2}$	
$m\overline{OM} = 9.54cm$ ....(ii)	
In ΔAMC	
By Pythagoras theorem	
$(m\overline{MC})^2 = (m\overline{CA})^2 - (m\overline{AM})^2$	
$= (8cm)^2 - (3cm)^2$	$m\overline{CA} = 8cm$, $m\overline{AM} = 3cm$
$= 64cm^2 - 9cm^2$	
$(m\overline{MC})^2 = 55\ cm^2$	
$\sqrt{(m\overline{MC})^2} = \sqrt{55cm^2}$	
$m\overline{MC} = 7.42\,cm$ ....(iii)	
We know that the distance between the centers	
$(m\overline{OC}) = (m\overline{OM}) + (m\overline{MC})$	From (ii) and (iii)
$= 9.54mc + 7.42cm$	
$= 16.96cm$	
Distance between the centres is 16.96cm	

Q.4. Show that greatest chord in a circle is its diameter.



Given: O be the centre of the circle, $m\overline{BC}$ is central chord and $\overline{AB}$ be any chord of the circle

To prove: Central chord $m\overline{CB}$ > Any chord $m\overline{AB}$

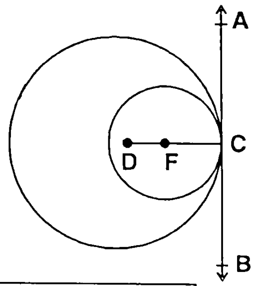
Construction: Draw $\overline{OM} \perp \overline{AB}$ to make right angled triangle OMB.

Proof:

Statements	Reasons
In right angled triangle OMB $(m\overline{OB})^2 = (m\overline{OM})^2 + (m\overline{MB})^2$	By Pythagoras theorem
It means $m\overline{OB} > m\overline{MB}$ $\therefore \quad 2(m\overline{OB}) > 2(m\overline{MB})$ As $2(m\overline{OB})$ is length of the central chord and $2(m\overline{MB})$ is length of the chord $\overline{AB}$ thus, Central chord $m\overline{CB}$ > Any chord $m\overline{AB}$.	The length of hypotenuse is greater than the length of other two sides.
It means central chord of the circle i.e. diameter is greater than any other chord of the circle, which proved that the greatest chord in a circle is its diameter.	

THEOREM 4(B)

If two circles touch each other internally, then the point of contact lies on the line segment through their centres and distance between their centres is equal to the difference of their radii.



Given: Two circles with centres D and F touch each other internally at point C.

So that $\overline{CD}$ and $\overline{CF}$ are the radii of two circles.

To Prove: Point C lies on the join of centres D and F extended, and $m\overline{DF} = m\overline{DC} - m\overline{CF}$

Construction: Draw $\overleftrightarrow{ACB}$ as the common tangent to the pair of circles at C.

Proof:

Statements	Reasons
Both circles touch internally at C whereas $\overleftrightarrow{ACB}$ is the common tangent and $\overline{CD}$ is the radial segment of the first circle. $\therefore m\angle ACD = 90^\circ$ ....(i)	Radial segment $\overline{CD} \perp$ the tangent line $\overline{AB}$
Similarly $\overleftrightarrow{ACB}$ is the common tangent and $\overline{CF}$ is the radial segment of the second circle. $\therefore m\angle ACF = 90^\circ$ .... (ii)	Radi al segment $\overline{CF} \perp$ the tangent line $\overline{AB}$.
$\Rightarrow \quad m\angle ACD = m\angle ACF = 90^\circ$ Where $\angle ACD$ and $\angle ACF$ coincide each other with point F between D and C. Hence $m\overline{DC} = m\overline{DF} + m\overline{FC}$ i.e., $m\overline{DC} - m\overline{FC} = m\overline{DF}$ or $m\overline{DF} = m\overline{DC} - m\overline{FC}$	Using (i) and (ii)

EXERCISE 10.3

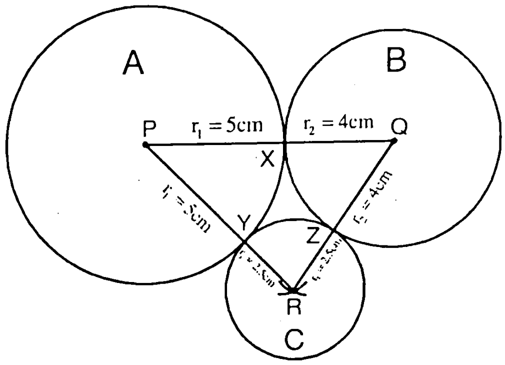
Q.1 Two circles with radii 5cm and 4cm touch each other externally. Draw another circle with radius 2.5cm touching the first pair, externally.

Solution:

Radius of Circle A = $r_1 = 5cm$

Radius of Circle B = $r_2 = 4cm$

Radius of Circle C = $r_3 = 2.5cm$



Steps of construction:

Step 1: Draw a line segment $m\overline{PQ}$ $5cm + 4cm = 9cm$ long.

Step 2: Take 'P' as a centre and draw a circle of radius 5cm.

Step 3: Take 'Q' as a centre and draw a circle of radius 4cm, which intersects the circle of radius 4cm at point x.

Step 4: Take P as a centre and draw an arc of radius $(5cm + 2.5cm = 7.5cm)$

Step 5: Take Q as a centre and draw an arc of radius $(4cm + 2.5cm = 6.5cm)$, which intersects the previous arc at point R.

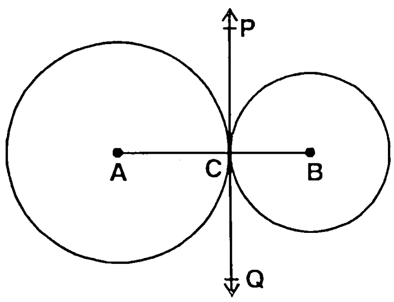
Step 6: Take R as centre and draw a circle of radius 2.5cm which touches externally the circles of centre P and Q at the points Y and Z respectively.

Q.2 If the distance between the centres of two circles is the sum or the difference of their radii they will touch each other.

Given: Two circles with centre A and B. $\overline{AC}$ and $\overline{BC}$ are radial segments of these circles such that $m\overline{AB} = m\overline{AC} + m\overline{BC}$ or

To Prove: Both circle touch each other.

Construction: Join A to B. Draw a tangent $\overleftrightarrow{PQ}$ of circle A at point C i.e. $m\angle PCA = 90^\circ$



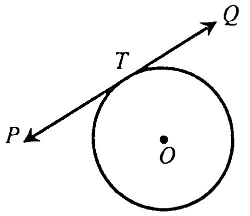
Proof:

Statements	Reasons
$m\overline{AB} = m\overline{AC} + m\overline{BC}$ ....(i)	Given
Points A, C and B are collinear such that C is between A and B.	From (i)
$m\angle PCA + m\angle PCB = 180^\circ$ ....(ii)	Supplementary angles
As $m\angle PCA = 90^\circ$ ....(iii)	Construction
$\therefore m\angle PCB = 90^\circ$ ....(iv)	From (ii) and (iii)
$\overline{PC} \perp \overline{BC}$ at C i.e. $\overline{PQ}$ is $\perp$ $\overline{BC}$ at C	From (iv)
$\overline{PQ}$ is also a tangent of circle B	
$\overline{PQ}$ is common tangent of both circles. i.e. Both circles have a common point C. Thus both circles touch each other at a point C. Similarly the same results can be proved when distance between the centers of two circles is equal to the difference of their radii	

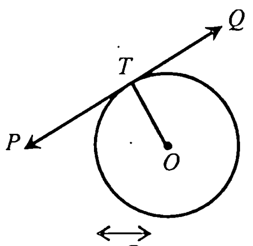
MISCELLANEOUS EXERCISE – 10

Q. 1 Four possible answers are given for the following questions.

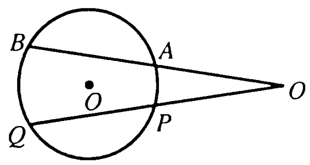
1. In the adjacent figure of the circle, the line $\overleftrightarrow{PQ}$ is named as.
   - (a) an arc
   - (b) a chord
   - (c) a tangent
   - (d) a secant



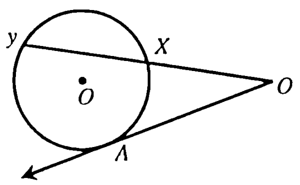
2. In a circle with centre O, $\overline{OT}$ is the radical segment and $\overleftrightarrow{PTQ}$ is the tangent line, then
   - (a) $\overline{OT} \perp \overleftrightarrow{PQ}$
   - (b) $\overline{OT} \not\perp \overleftrightarrow{PQ}$
   - (c) $\overline{OT} \parallel \overleftrightarrow{PQ}$
   - (d) $\overline{OT}$ is the right bisector of $\overleftrightarrow{PQ}$



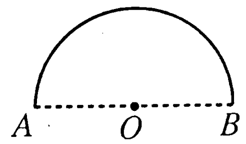
3. In the given diagram find m $\overline{OA}$ if m $\overline{OB}$ =8cm, m $\overline{OP}$ =4cm and m$\overline{OQ}$=12cm
   - (a) 2cm
   - (b) 2.67
   - (c) 2.8 cm
   - (d) 3cm



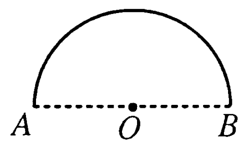
4. In the given diagram find m $\overline{OX}$ if m$\overline{OA}$=6cm and m$\overline{OY}$=9cm
   - (a) 4cm
   - (b) 6cm
   - (c) 9cm
   - (d) 12cm



5. In the adjacent figure find semicircular area if $\pi \simeq 3.1416$ and m$\overline{OA}$=20cm.
   - (a) 62.83sq cm
   - (b) 314.16sq cm
   - (c) 436.20sq cm
   - (d) 628.32sq cm



6. In the adjacent figure find half the perimeter of circle with center O if $\pi = 3.1416$ and m$\overline{OA}$=20cm.
   - (a) 31.42 cm
   - (b) 62.832 cm
   - (c) 125.65 cm
   - (d) 188.50 cm

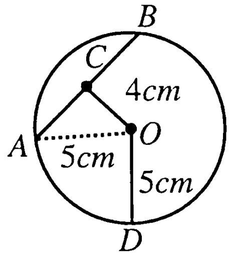


7. A line which has two points in common with a circle is called.
   - (a) sine of a circle
   - (b) cosine of a circle
   - (c) tangent of a circle
   - (d) secant of a circle
8. A line which has only one point in common with a circle is called
   - (a) sine of a circle
   - (b) cosine of a circle
   - (c) tangent of a circle
   - (d) secant of a circle
9. Two tangents drawn to a circle from a point outside it are ....in length
   - (a) half (b) equal
   - (c) double (d) triple
10. A circle has only one.
    - (a) secant (b) chord
    - (c) diameter (d) centre
11. A tangent line intersects the circle at.
    - (a) three points (b) two points
    - (c) single point (d) no point at all

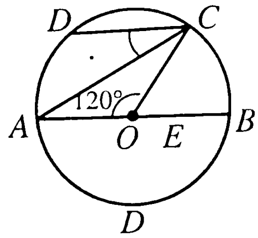
12. Tangents drawn at the ends of diameter of a circle are...... to each other.
(a) parallel (b) non-parallel
(c) collinear (d) perpendicular

13. The distance between the centres of two congruent touching circles externally is
(a) of zero length
(b) the radius of each circle
(c) the diameter of each circle
(d) twice the diameter of each circle

14. In the adjacent circular figure with centre O and radius 5cm. The length of the chord intercepted at 4cm away from the centre of this circle is
(a) 4cm
(b) 6cm
(c) 7cm
(d) 9cm



15. In the adjoining figure there is a circle with centre O. If $\overline{DC} \parallel$ diameter $\overline{AB}$ and $m\angle AOC = 120°$, then $m\angle ACD$ is
(a) 40°
(b) 30°
(c) 50°
(d) 60°

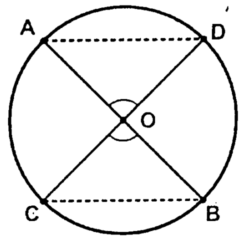


ANSWER KEY

1.	c	**2.**	a	**3.**	b	**4.**	a	**5.**	d
6.	b	**7.**	d	**8.**	c	**9.**	b	**10.**	d
11.	c	**12.**	a	**13.**	c	**14.**	b	**15.**	b

EXERCISE 11.1

Q.1 In a circle two equal diameters $\overline{AB}$ and $\overline{CD}$ intersect each other. Prove that $m\overline{AD} = m\overline{BC}$.



Given: A circle with centre "O". Two diameters $\overline{AB}$ and $\overline{BC}$, intersecting at point O.

To Prove: $m\overline{AD} = m\overline{BC}$

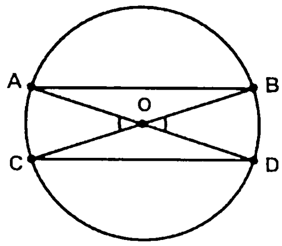
Construction:

Join A to D and C to B

Proof:

Statements	Reasons
In $\Delta AOD \leftrightarrow \Delta BOC$	
$\overline{OA} \cong \overline{OB}$	Radii of the same circle
$\angle AOD \cong \angle BOC$	Vertical angles are congruent
$\overline{OD} \cong \overline{OC}$	Radii of the same circle
$\therefore \Delta AOD \cong \Delta BOC$	S. A. S ≅ S. A. S
$\overline{AD} \cong \overline{BC}$	Corresponding sides of congruent triangle
Or $m\overline{AD} = m\overline{BC}$	

Q.2. In a circle prove that the arcs between two parallel and equal chords are equal.



Given: A circle with centre O. Two chords $\overline{AB}$ and $\overline{CD}$ Such that $\overline{AB} \parallel \overline{CD}$ and $m\overline{AB} = m\overline{CD}$

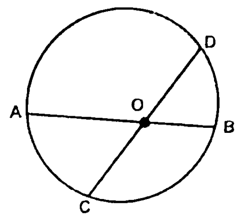
To Prove: $m\overparen{AC} = m\overparen{BD}$

Construction: Join A to D and B to C. Such that $\overline{AD}$ and $\overline{CD}$ intersect each other at central point O.

Proof:

Statements	Reasons
$\overline{AD}$ and $\overline{BC}$ are line segment intersecting at centre O.	
$\angle AOC$ and $\angle BOD$ are central angles.	Angle subtended at centre.
$m\angle AOC = m\angle BOD$	Vertical angles
$m\overparen{AC} = m\overparen{BB}$	Within the same circle arcs opposite to the equal central angles are equal.

Q.3. Give a geometric proof that a pair of bisecting chords are the diameters of a circle.



Given: A circle and two chords $\overline{AB}$ and $\overline{CD}$ bisecting each other at point O. i.e.

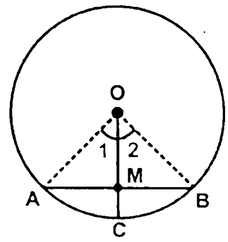
$m\overline{AO} = m\overline{OB}$ and $m\overline{CO} = m\overline{OD}$

To Prove: Chords $\overline{AB}$ and $\overline{CD}$ are diameters.

Proof:

Statements	Reasons
$m\overline{AB} = m\overline{CD}$ ....(i)	Two chords can bisect each other only when they are equal (given)
∴ O is the mid point of $\overline{AB}$ and $\overline{CD}$	Given
$m\overline{AO} = m\overline{BO} = \frac{1}{2}m\overline{AB}$ ....(ii)	
$m\overline{DO} = m\overline{CO} = \frac{1}{2}m\overline{CD}$ ....(iii)	
$m\overline{AO} = m\overline{BO} = m\overline{CO} = m\overline{DO}$ ....(iv)	From (i), (ii) and (iii)
The points of circle A, B, C and D are equidistant form the fixed point "O".	From (iv)
This fixed pint O is the centre of the circle having the points A, B, C and D.	By definition
As chords $\overline{AB}$ and $\overline{CD}$ pass through the centre "O" therefore chords $\overline{AB}$ and $\overline{CD}$ are diameters.	

Q.4. If C is the midpoint of an arc ACB in a circle with centre O. Show that line segment OC bisects the chord AB.



Given: A circle with centre "O" $\overset{\frown}{ACB}$ is an arc with C as its midpoint and $m\overset{\frown}{AC} = m\overset{\frown}{CB}$. Center "O" is joined with C such that $\overline{OC}$ meets $\overline{AB}$ at M.

To Prove: $m\overline{AM} = m\overline{BM}$

Construction: Join center "O" with A and B to make central angle AOB.

Proof:

Statements	Reasons
$\angle AOB$ is central angle	Construction
∴ $m\angle 1 = m\angle 2$ ....(i)	C is the midpoint of $\overset{\frown}{ACB}$ (Given)
In $\Delta AOM \longleftrightarrow \Delta BOM$	
$\overline{OM} \cong \overline{OM}$	Common
$\angle 1 \cong \angle 2$	Proved
$\overline{OA} \cong \overline{OB}$	Radii of the same Circle
$\Delta AOM \cong \Delta BOM$	S.A.S ≅ S.A.S
$\overline{AM} \cong \overline{BM}$	Corresponding sides of congruent triangles.
Hence $m\overline{AM} = m\overline{BM}$	

MISCELLANEOUS EXERCISE – 11

Q.1 Multiple Choice Questions
Four possible answers are given for the following questions.

1. A 4 cm long chord subtends central angle of 60°. The radial segment of this, circle
(a) 1 (b) 2
(c) 3 (d) 4

2. If an arc of a circle subtends a central angle of 60°, then the corresponding chord of the arc will make the central angle of:
(a) 20° (b) 40°
(c) 60° (d) 80°

3. The semi circumference and the diameter of a circle both subtend a central angle of
(a) 90° (b) 180°
(c) 270° (d) 360°

4. The arcs opposite to incongruent central angles of a circle arc always:
(a) Congruent (b) incongruent
(c) parallel (d) perpendicular

5. If a chord of a circle subtends a central angle of 60°, then the length of the chord and the radial segment are:
(a) congruent (b) incongruent
(c) parallel (d) perpendicular

6. The length of a chord and the radial segment of a circle are congruent, the central angle made by the chord will be:
(a) 30° (b) 45°
(c) 60° (d) 75°

7. Out of two congruent arcs of a circle, if one arc makes a central angle of 30° then the other arc will subtend the central angle of:
(a) 15° (b) 30°
(c) 45° (d) 60°

8. The chord length of a circle subtending a central angle of 180° is always:
(a) less than radial segment
(b) equal to the radial segment
(c) double of the radial segment
(d) none of these

9. A pair of chords of a circle subtending two congruent central angles is:
(a) congruent (b) incongruent
(c) over lapping (d) parallel

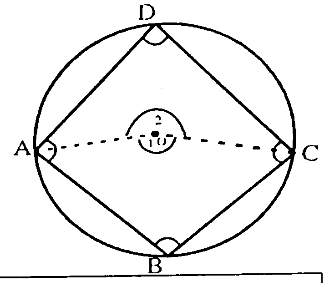
10. An arc subtends a central angle of 40° then the corresponding chord will subtended a central angle of:
(a) 20° (b) 40°
(c) 60° (d) 80°

ANSWER KEY

1.	d	**2.**	c	**3.**	b	**4.**	b	**5.**	a
6.	c	**7.**	b	**8.**	c	**9.**	a	**10.**	b

EXERCISE 12.1

Q.1 Prove that in a given cyclic quadrilateral, sum of opposite angles is two right angles and conversely.



Given: A circle with centre "O"
ABCD is a cyclic quadrilateral

To Prove: $m\angle B + m\angle D = 180^\circ$

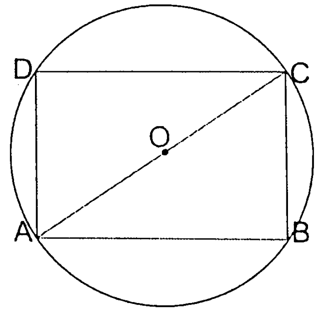
$m\angle BCD + m\angle DAB = 180^\circ$

Construction: Join O with A and C

Proof:

Statements	Reasons
$m\angle 1 = 2m\angle D$..........(i)	$\angle 1$, $\angle 2$ are central angles and $\angle D$, $\angle B$ are circum angles in Arcs
$m\angle 2 = 2m\angle B$..........(ii)	
$m\angle 1 + m\angle 2 = 2m\angle D + 2m\angle B$	Adding (i) and (ii)
$m\angle 1 + m\angle 2 = 2(m\angle D + m\angle B)$	
or $2(m\angle D + m\angle B) = m\angle 1 + m\angle 2$	By symmetric property
$2(m\angle D + m\angle B) = 360^\circ$	Sum of all central angles is 360°
$m\angle D + m\angle B = \frac{360^\circ}{2}$	Dividing by 2
$m\angle D + m\angle B = 180^\circ$	
Similarly $m\angle BCD + m\angle DAB = 180^\circ$	

Q.2 Show that parallelogram inscribed in a circle will be a rectangle.



Given: ABCD is a parallelogram inscribed in the circle with centre "O"

$m\overline{AB} = m\overline{DC}$ and $\overline{AB} \parallel \overline{DC}$

$m\overline{AD} = m\overline{BC}$ and $\overline{AD} \parallel \overline{BC}$

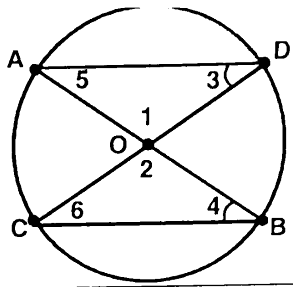
To Prove: ABCD is a rectangle

Construction: Join A with C

Proof:

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta ADC$	
$m\overline{AC} = m\overline{AC}$	Common
$m\overline{AB} = m\overline{DC}$	Given
$m\overline{BC} = m\overline{AD}$	Given
$\therefore \Delta ABC \cong \Delta ADC$	S.S. S $\cong$ S. S. S
Thus, $m\angle B = m\angle D$........................(i)	Corresponding angles of congruent triangles
$m\angle B + m\angle D = 180^\circ$..........(ii)	Opposite angles of parallelogram
$\Rightarrow m\angle B = m\angle D = 90^\circ$	From (i)
Similarly $m\angle BAD = m\angle BCD = 90^\circ$	From (i) and (ii)
Hence ABCD is rectangle	

Q.3 AOB and COD are two intersecting chords of a circle. Show that Δ^sAOD and BOC are equiangular.



Given: In a circle $\overline{AOB}$ and $\overline{COD}$ are two intersecting chords at point O.

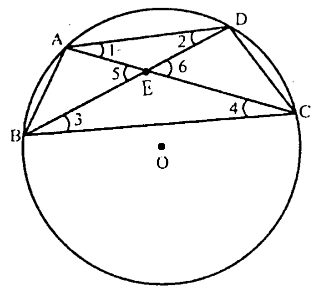
To Prove: ΔAOD and ΔBOC are equiangular

Construction: Join A with C and D. Join B with C and D.

Proof:

Statements	Reasons
$m\angle 1 \cong m\angle 2$........(i)	Vertical angles
$\overline{AC}$ is chord and angles $\angle 3$, $\angle 4$ are in the same segment. $\angle 3 \cong \angle 4$............(ii)	
Now $\overline{BD}$ is chord and angles $\angle 5$, $\angle 6$ are in the same segments. Therefore $\angle 5 \cong \angle 6$............(iii)	
Thus, ΔAOD and ΔBOC are equiangular	From (i), (ii) and (iii)

Q.4 $\overline{AD}$ and $\overline{BC}$ are two parallel chords of a circle prove that arc AB $\cong$ are CD and arc AC $\cong$ arc BD.



Given: A circle with centre "O". Two chords $\overline{AD}$ and $\overline{BC}$ are such that $\overline{AD} \parallel \overline{BC}$.

To Prove: arc AB $\cong$ arc CD and arc AC $\cong$ arc BD

Construction: Join A to B and C. Join D to B and C. $\overline{AC}$ and $\overline{BD}$ intersect each other at point E. some angles are named as $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6$.

Proof:

Statements	Reasons
$m\angle 1 = m\angle 3$ ....(i) $m\angle 2 = m\angle 4$ ....(ii)	Angles inscribed by an arc in the same segment are equal.
$m\angle 1 = m\angle 4$ ....(iii)	Alternate angles are congruent $(\overline{AD} \parallel \overline{BC})$
$m\angle 3 = m\angle 4$ ....(iv)	From (i) and (iii)
$m\angle 1 = m\angle 2$ ....(v)	From (ii) and (iii)
In $\Delta AEB \leftrightarrow \Delta DEC$	
$\overline{AE} \cong \overline{ED}$	Side opposite to equal angles (v)
$m\angle 5 = m\angle 6$	vertical angles
$\overline{BE} \cong \overline{EC}$	Sides opposite to equal angles (iv)
$\therefore$ $\Delta AED \cong \Delta DEC$	S.A.S $\cong$ S.A.S
$\overline{AB} \cong \overline{CD}$	Corresponding sides of congruent.
Thus arc AB $\cong$ arc CD (Hence Proved)	Arcs corresponding to congruent chords are congruent.
$m\widehat{BC} \cong m\widehat{CB}$	Self congruent
$m\widehat{BA} + m\widehat{AC} = m\widehat{CD} + m\widehat{DB}$	
$\cancel{m\widehat{AB}} + m\widehat{AC} = \cancel{m\widehat{AB}} + m\widehat{BD}$	$\because$ arc $\widehat{AB} \cong$ arc CD proved
$m\widehat{AC} = m\widehat{BD}$	
or arc AC $\cong$ arc BD (Hence proved)	

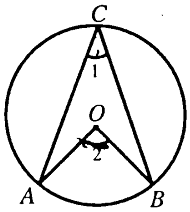
MISCELLANEOUS EXERCISE – 12

Q. 1 Multiple Choice Questions

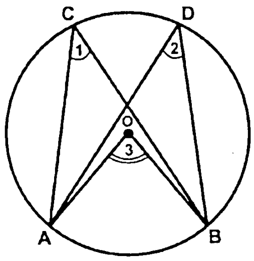
Four possible answers arc given for the following questions.

1. A circle passes through the vertices of a right angled ΔABC with $m\overline{AC} = 3$cm and $m\overline{BC} = 4$cm, $m\angle C = 90^\circ$, Radius of the circle is:
   (a) 1.5 cm (b) 2.0 cm
   (c) 2.5 cm (d) 3.5 cm

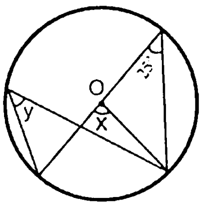
2. In the adjacent circular figure, central and inscribed angles stand on the same are AB.
   (a) $m\angle 1 = m\angle 2$
   (b) $m\angle 1 = 2m\angle 2$
   (c) $m\angle 2 = 3m\angle 1$
   (d) $m\angle 2 = 2m\angle 1$



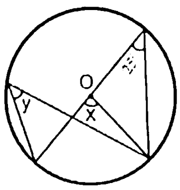
3. In the adjacent figure if $m\angle 3 = 75^\circ$, then find $m\angle 1$ and $m\angle 2$
   (a) $37\frac{1}{2}^\circ, 37\frac{1}{2}^\circ$
   (b) $37\frac{1}{2}^\circ, 75^\circ$
   (c) $75^\circ, 37\frac{1}{2}^\circ$
   (d) $75^\circ, 75^\circ$



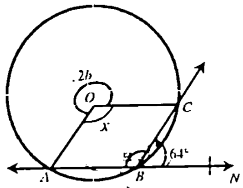
4. Given that O is the centre of the circle, the angle marked x will be.
   (a) $12\frac{1}{2}^\circ$ (b) 25°
   (c) 50° (d) 75°



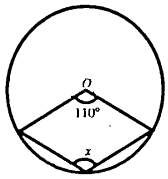
5. Given that O is the centre of the circle the angle marked y will be.
   (a) $12\frac{1}{2}^\circ$ (b) 25°
   (c) 50° (d) 75°



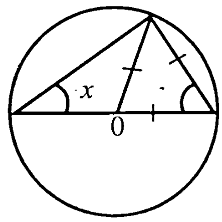
6. In the figure, O is the centre of the circle and $\overleftrightarrow{ABN}$ is a straight line. The obtuse angle AOC = x is.
   (a) 32°
   (b) 64°
   (c) 96°
   (d) 116°



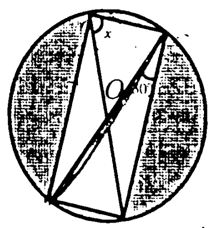
7. In the figure , O is the centre of the circle, then the angle x is
   (a) 55°
   (b) 110°
   (c) 220°
   (d) 125°



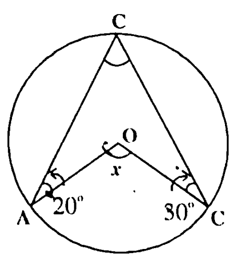
8. In the figure, O is the centre of the circle then angle x is.
   (a) 15°
   (b) 30°
   (c) 45°
   (d) 60°



9. In the figure ,O is the centre of the circle then the angle x is
   (a) 15°
   (b) 30°
   (c) 45°
   (d) 60°



10. In the figure, O is the centre of the circle then the angle x is.
    (a) 50°
    (b) 75°
    (c) 100°
    (d) 125°



ANSWER KEY

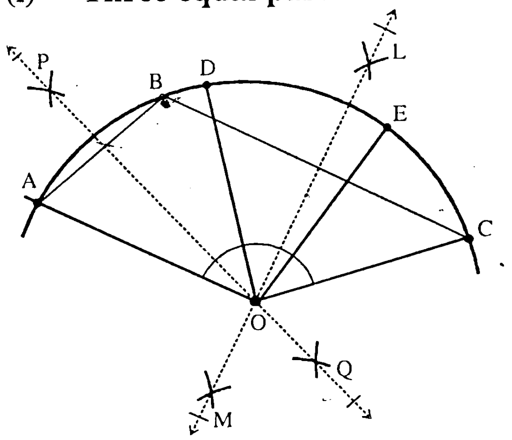
1.	c	2.	d	3.	a	4.	c	5.	b
6.	d	7.	b	8.	b	9.	d	10.	c

EXERCISE 13.1

Q.1 Divide an arc of any length
(i) Into three equal parts
(ii) Into four equal parts
(iii) Into six equal parts

Solution:

(i) Three equal parts



Steps of Construction:

i. Take an arc AC of any length.

ii. Take any point B on the arc AC and join A to B and B to C.

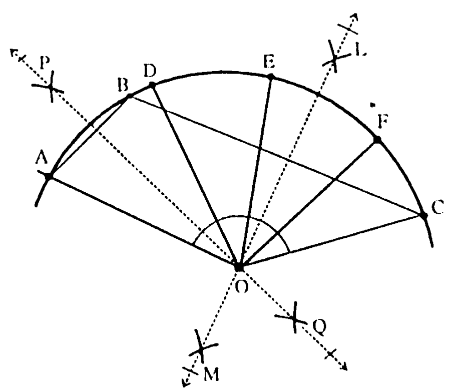
iii. Draw right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{LM}$ of $\overline{AB}$ and $\overline{BC}$ respectively, which meet each other at point "O". Point O is the centre of circle having the arc AC.

iv. Join end points of arc AC with centre O to form central angle AOC.

v. Measure the central angle and divide it into three equal central angles cutting the arc AC at points D and E.

vi. Arcs of same radii corresponding to equal central angles are equal. Thus three equal parts of the arc ABC are
$m\widehat{AD} = m\widehat{DE} = m\widehat{EC}$.

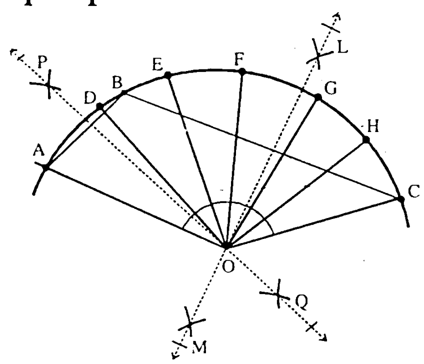
(ii) Four equal parts



Steps of Construction:

i. Take an arc AC of any length.
ii. Take any point B on the arc AC and join A to B and B to C.
iii. Draw right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{LM}$ of $\overline{AB}$ and $\overline{BC}$ respectively, which meet each other at point "O". Point O is the centre of circle having the arc AC.
iv. Join end points of arc AC with centre O to form central angle AOC.
v. Measure the central angle and divide it into four equal central angles cutting the arc AC at points D, E and F.
vi. Arcs of same radii corresponding to equal central angles are equal. Thus four equal parts of the arc ABC are $m\overset{\frown}{AD} = m\overset{\frown}{DE} = m\overset{\frown}{EF} = m\overset{\frown}{FC}$.

(iii)Six equal parts

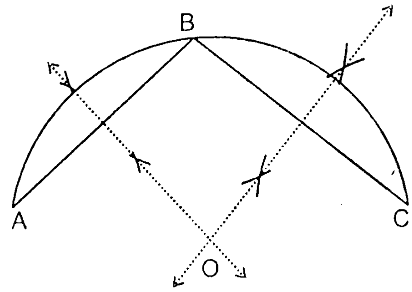


Steps of Construction:

i. Take an arc AC of any length.
ii. Take any point B on the arc AC and join A to B and B to C.
iii. Draw right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{LM}$ of $\overline{AB}$ and $\overline{BC}$ respectively, which meet each other at point "O". Point O is the centre of circle having the arc AC.
iv. Join end points of arc AC with centre O to form central angle AOC.
v. Measure the central angle and divide it into six equal central angles cutting the arc AC at points D, E, F, G and H.

Arcs of same radii corresponding to equal central angles are equal. Thus six equal parts of the arc ABC are $m\overset{\frown}{AD} = m\overset{\frown}{DE} = m\overset{\frown}{EF} = m\overset{\frown}{FG} = m\overset{\frown}{GH} = m\overset{\frown}{HC}$

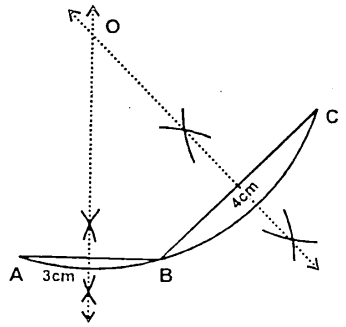
Q.2 Practically find the centre of an arc ABC



Steps of Construction:

i. We draw an arc ABC of any length.
ii. We draw line segments $\overline{AB}$ and $\overline{BC}$.
iii. We draw right bisectors of $\overline{AB}$ and $\overline{BC}$, intersecting each other at point O.
iv. Point 'O' is the required centre of arc ABC.

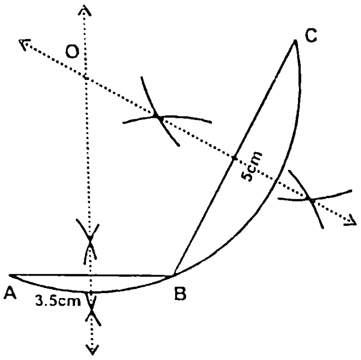
Q. 3 (i) If $|\overline{AB}| = 3cm$ and $|\overline{BC}| = 4cm$ are the lengths of two chords of an arc, then locate the centre of the arc.



Steps of Construction:

i. We draw $|\overline{AB}| = 3cm$ and $|\overline{BC}| = 4cm$, inclined at any angle.
ii. We draw right bisectors of $\overline{AB}$ and $\overline{BC}$ intersecting each other at point O, which is the required centre of arc ABC.
iii. Taking centre 'O', we draw an arc ABC of radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$.

(ii) If $|\overline{AB}| = 3.5cm$ and $|\overline{BC}| = 5cm$ are the lengths of two chords of an arc, then locate the centre of the arc.



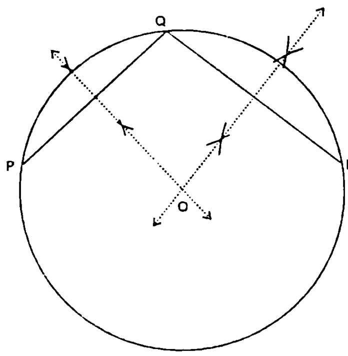
Steps of Construction:

i. We draw $|\overline{AB}| = 3.5cm$ and $|\overline{BC}| = 5cm$, inclined at any angle.

ii. We draw right bisectors of $\overline{AB}$ and $\overline{BC}$ intersecting each other at point O, which is the required centre of arc ABC.

iii. Taking centre 'O', we draw an arc ABC of radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$.

4. For an arc draw two perpendicular bisectors of the chords $\overline{PQ}$ and $\overline{QR}$ of this arc, construct a circle through P, Q and R.



Steps of construction:

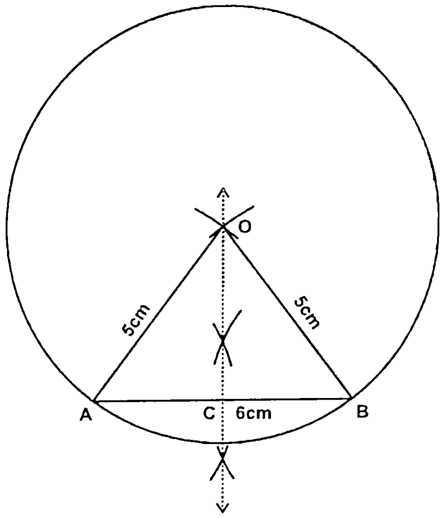
i. We take an arc PQR of any length.

ii. We take two chords $\overline{PQ}$ and $\overline{QR}$ of any lengths of arc PQR.

iii. We draw right bisectors of $\overline{PQ}$ and $\overline{QR}$, intersecting each other at point 'O', which is the centre of arc PQR.

iv. Taking 'O' as centre, we complete the required circle passing through P, Q and R.

5. Describe a circle of radius 5 cm passing through points A and B, 6 cm apart. Also find distance from the centre to line AB.



Steps of Construction:

i. We draw a line segment $\overline{AB}$ of length 6cm.

ii. We draw right bisector of $\overline{AB}$ intersecting it at point 'C'.

iii. From points A and B we draw arcs of radius 5cm each, intersecting the bisector at point O.

iv. Taking 'O' as centre we draw a circle of radius 5 cm passing through the points A and B.

v. To find the distance of centre O from $\overline{AB}$, we consider right angle ΔOAC.

By Pythagorean Theorem

$(m\overline{OC})^2 + (m\overline{AC})^2 = (m\overline{OA})^2$

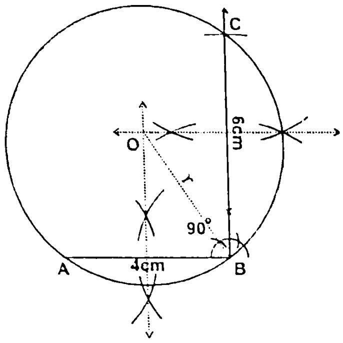
$(m\overline{OC})^2 + (3)^2 = (5)^2$

$(m\overline{OC})^2 = 25 - 9$

$(m\overline{OC})^2 = 16$

$= 4 \text{ cm} \, m\overline{OC}$

6. If $|\overline{AB}| = 4cm$ and $|\overline{BC}| = 6cm$, such that $\overline{AB}$ is perpendicular to $\overline{BC}$, construct a circle through points A, B and C. Also measure its radiu



Steps of construction:

i. We draw $\overline{AB}$ and $\overline{BC}$, 4 cm and 6 cm long respectively, perpendicular to each other.

ii. We draw right bisectors of $\overline{AB}$ and $\overline{BC}$, intersecting each other at point 'O'.

iii. Taking 'O' as centre we draw a circle of radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$ passing through the points A, B and C.

iv. The radius of this circle is measured to be 3.6 cm.

v. By Pythagoras theorem

$r^2 = 2^2 + 3^2$

$r^2 = 4 + 9$

$\sqrt{r^2} = \sqrt{13}$

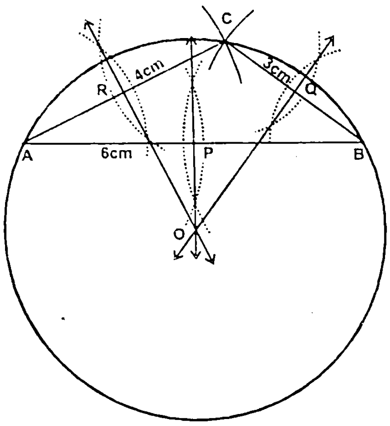
r = 3.6cm

EXERCISE 13.2

Q. 1 Circumscribe a circle about a triangle ABC with sides $|\overline{AB}| = 6cm$, $|\overline{BC}| = 3cm$ and $|\overline{CA}| = 4cm$. Also measure its circum radius.

Solution:

Data: $|\overline{AB}| = 6cm, |\overline{BC}| = 3cm, |\overline{CA}| = 4cm$



Steps of construction:

i. We construct triangle ABC according to given condition.

ii. We draw right bisectors $\overleftrightarrow{OP}, \overleftrightarrow{OQ}$ and $\overleftrightarrow{OR}$ of sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively, concurrent at point 'O'.

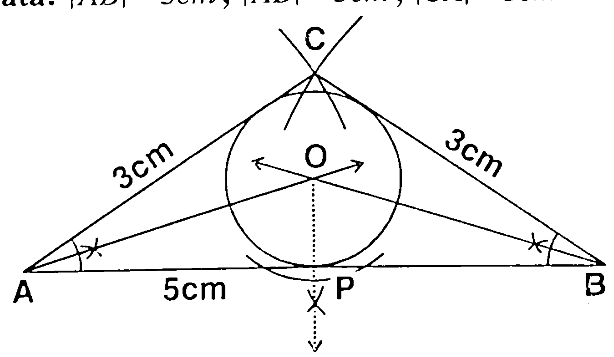
iii. Taking 'O' as centre and radius equal to $m\overline{OA}$ or $m\overline{OB}$ or $m\overline{OC}$, we draw a circle passing through the vertices A, B and C.

iv. This is the required circum circle, whose radius is measured to be 3.3 cm.

Q. 2 Inscribe a circle in a triangle ABC with side $|\overline{AB}| = 5cm$, $|\overline{BC}| = 3cm$ and $|\overline{CA}| = 3cm$. Also measure its in-radius.

Solution:

Data: $|\overline{AB}| = 5cm$, $|\overline{AB}| = 5cm$, $|\overline{CA}| = 3cm$



Steps of construction:

i. We construct triangle ABC according to given condition.

ii. We draw bisectors of $\angle A$ and $\angle B$ intersecting each other at point 'O'.

iii. From point O, we draw $\overrightarrow{OP}$ perpendicular to $\overline{AB}$.

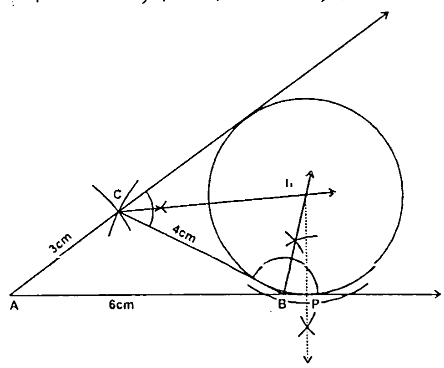
iv. Taking 'O' as centre and radius equal to $m\overline{OP}$, we draw a circle, touching three sides of triangle internally.

v. This is the required in-circle whose radius is measured to be 0.8 cm.

Q. 3 Escribe a circle opposite to vertex A to a triangle ABC with sides $|AB| = 6cm$, $|BC| = 4cm$ and $|CA| = 3cm$. Find its radius also.

Solution:

Data: $|AB| = 6cm$, $|BC| = 4cm$, $|CA| = 3cm$



Steps of construction:

i. We construct a triangle ABC according to given condition.

ii. We produce the sides $\overline{AB}$ and $\overline{AC}$ beyond B and C respectively.

iii. We draw, bisectors of exterior angles at points B and C, intersecting each other at point I_1.

iv. From point I_1, we draw $\overrightarrow{I_1P}$ perpendicular to $\overline{AB}$ produced.

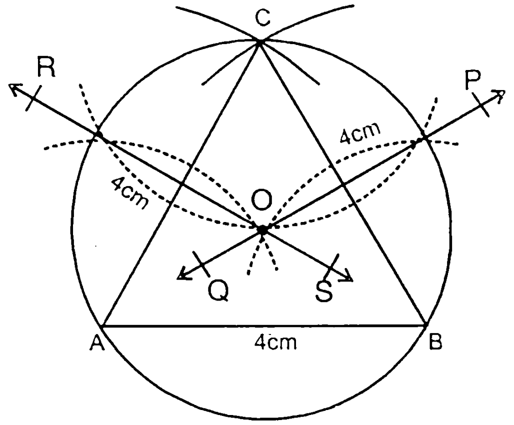
v. Taking I_1, as centre and radius equal to I_1P, we draw a circle, touching one side of ΔABC externally and other two produced sides internally.

vi. This is the required escribed circle, whose radius is measured to be 2.2 cm.

Q. 4 Circumscribe a circle about an equilateral triangle ABC with each side of length 4cm.

Solution:

Data: $m\overline{AB} = m\overline{BC} = m\overline{CA} = 4cm$



Steps of construction:

i. We construct equilateral triangle ABC with each side 4cm long.

ii. We draw right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ of side $\overline{BC}$ and $\overline{AC}$ respectively intersecting each other at point O.

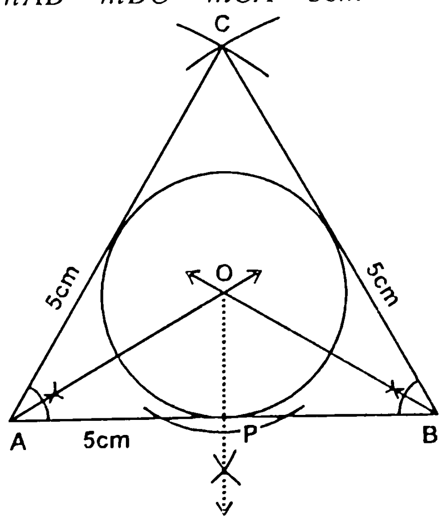
iii. Taking O as centre and radius equal to $m\overline{OA}$ or $m\overline{OB}$ or $m\overline{OC}$, we draw a circle passing through the points A, B and C.

iv. This is our required circum circle whose radius is measured to be 2.3 cm.

Q. 5 Inscribe a circle in an equilateral triangle ABC with each side of length 5cm.

Solution:

Data: $m\overline{AB} = m\overline{BC} = m\overline{CA} = 5cm$



Steps of construction:

i. We construct equilateral triangle ABC with each side 5cm long.

ii. We draw bisectors of $\angle A$ and $\angle B$ intersecting each other at point 'O'.

iii. From point O, we draw $\overrightarrow{OP}$ perpendicular to $\overline{AB}$.

iv. Taking 'O' as centre and radius equal to $\overline{OP}$, we draw a circle, touching three sides of triangle internally.

v. This is the required in-circle whose radius is measured to be 1.4 cm.

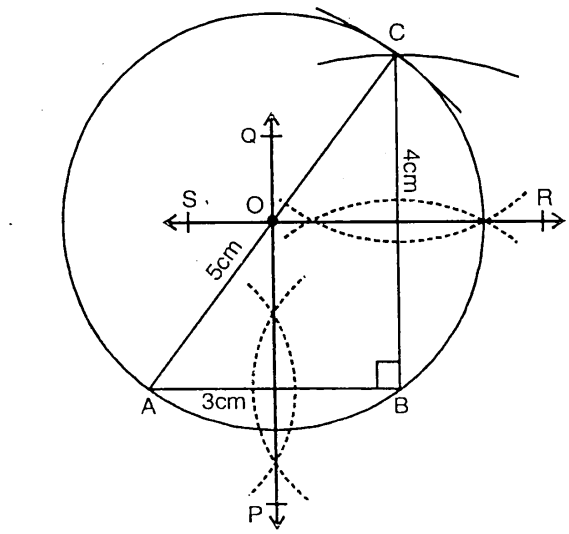
Q. 6 Circumscribe and inscribe circles with regard to a right angle triangle with sides 3cm, 4cm and 5cm.

Solution:

Let

$m\overline{AB} = 3cm, m\overline{BC} = 4cm$ and $m\overline{CA} = 5cm$

a. Circum Circle:



Steps of construction:

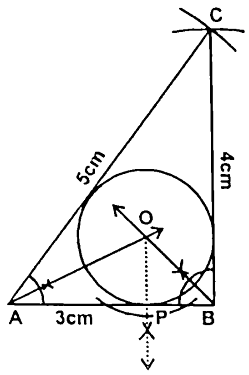
i. We construct right angle triangle ABC with sides 3cm, 4cm and 5cm.

ii. We draw right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ of side $\overline{AB}$ and $\overline{BC}$ respectively intersecting each other at point O.

iii. Taking O as centre and radius equal to $m\overline{OA}$ or $m\overline{OB}$ or $m\overline{OC}$, we draw a circle passing through the points A, B and C.

iv. This is our required circum circle whose radius is measured to be 2.5 cm.

b. Inscribed Circle



Steps of construction:

i. We construct right angle triangle ABC according to given condition.

ii. We draw bisectors of ∠A and ∠B intersecting each other at point 'O'.

iii. From point O, we draw $\overrightarrow{OP}$ perpendicular to $\overline{AB}$.

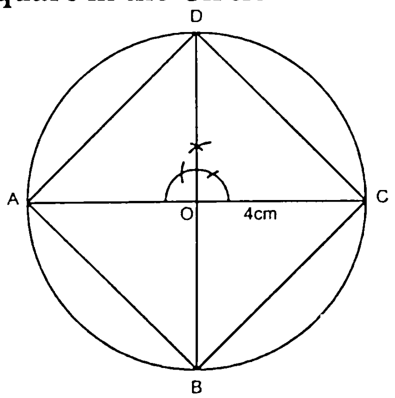
iv. Taking 'O' as centre and radius equal to $\overline{OP}$, we draw a circle, touching three sides of triangle internally.

v. This is the required in-circle whose radius is measured to be 1 cm.

Q. 7 In and about a circle of radius 4 cm describe a square.

Solution:

a. Square in the Circle



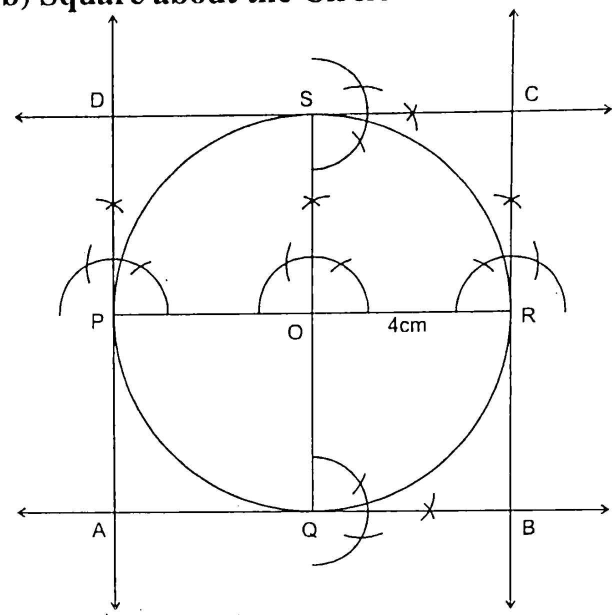
Steps of construction:

i. We draw a circle with centre 'O' of radius 4cm.

ii. We draw two diameters $\overline{AC}$ and $\overline{BD}$ of circle perpendicular to each other.

iii. By joining points A with B, B with C, C with D and D with A, we get the required square inscribed in the given circle.

(b) Square about the Circle



Steps of Construction:

i. We draw a circle with centre "O' and a radius 4cm.

ii. We draw two diameters $\overline{PR}$ and $\overline{QS}$ of circle perpendicular to each other.

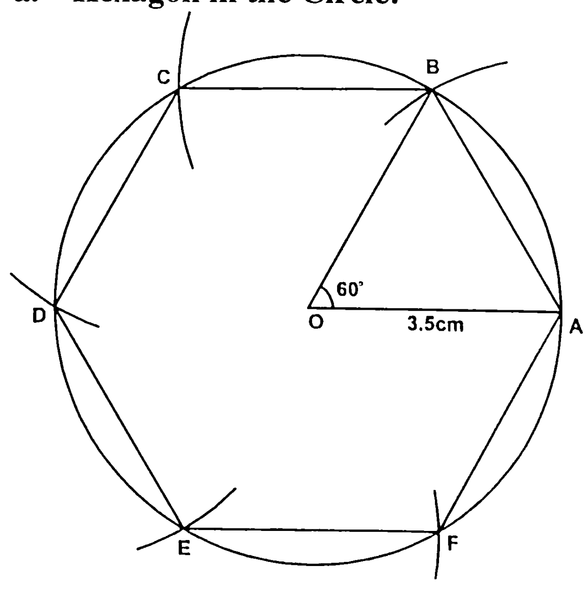
iii. We draw tangents to the circle at points P, Q, R and S.

iv. We produce the tangents to meet each other at point A, B, C and D.

v. ABCD is the required circumscribed square.

Q. 8 In and about a circle of radius 3.5 cm describe a hexagon.

a. Hexagon in the Circle:



Steps of Construction:

i. We draw a circle with centre 'O' of radius 3.5 cm.

ii. We take a point A anywhere on the circle and draw the radial segment $\overline{OA}$.

iii. From point A, we draw an arc of radius $\overline{OA}$ which intersects the circle at point B.

iv. By joining 'O' with A and B we get an equilateral triangle OAB, so that the angle subtended by the chord at the centre is 60°.

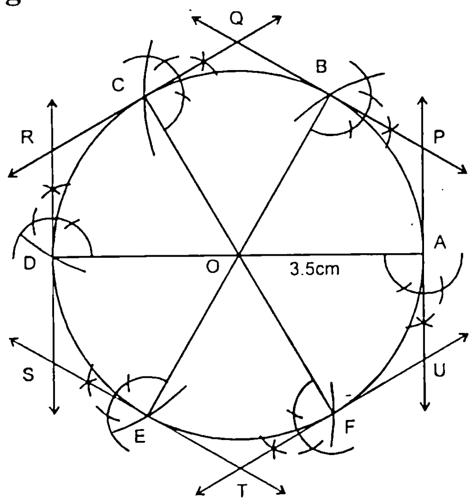
v. From point B, we draw an arc of same radius intersecting the circle at point C, then joining B to C we get another chord $\overline{BC}$.

vi. We continue to draw the arcs, which cut the circle at points D, E and F, such that

$m\overline{OA} = m\overline{AB} = m\overline{BC} = m\overline{CD}$
$= m\overline{DE} = m\overline{EF} = m\overline{FA}$

vii. We draw end to end on the circle the six chords $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$ and $\overline{FA}$, which completes the required hexagon

b. Hexagon about the Circle:



Steps Construction:

i. We draw a circle with centre 'O' of radius 3.5 cm.

ii. We take a point A anywhere on the circle and draw the radial segment $\overline{OA}$.

iii. From point A, we draw an arc of radius $\overline{OA}$, which intersects the circle at point B.

iv. From point B, we draw an arc of same radius intersecting the circle at point C.

v. We continue to draw the arcs, which cut the circle at points D, E and F.

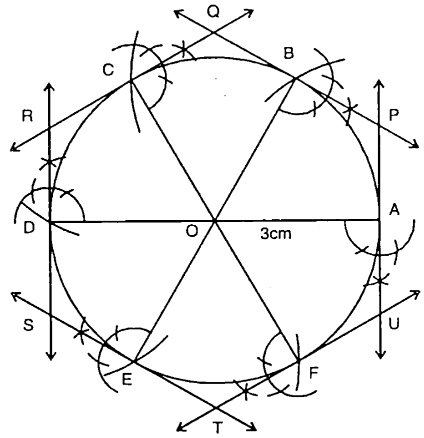
vi. We draw the diameters $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$.

vii. We draw tangents at points A, B, C, D, E and F to the circle.

viii. We produce the tangents to meet each other at points P, Q, R, S, T and U.

ix. PQRSTU is the required circumscribed hexagon.

Q. 9 Circumscribe a regular hexagon about a circle of radius 3 cm.



Steps Construction:

i. We draw a circle with centre 'O' of radius 3 cm.

ii. We take a point A anywhere on the circle and draw radial segment $\overline{OA}$.

iii. From point A, we draw an arc of radius $\overline{OA}$, which intersects the circle at point B.

iv. From point B, we draw an arc of same radius intersecting the circle at point C.

v. We continue to draw the arcs, which cut the circle at points D, E and F.

vi. We draw diameter $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$.

vii. We draw tangents at points A, B, C, D, E and F to the circle.

viii. We produce the tangents to meet each other at points P, Q, R, S, T and U.

ix. PQRSTU is the required circumscribed hexagon.